Math 1500, Exam IV, WS2005

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Do all the problems

If you need more space, use the opposite blank page
and say so.

Calculators are neither needed nor allowed.

You may not leave before 7:30 p.m.

Make sure you have 8 pages including the cover page.

Name____

Student Number__________________________________

RSD Instructor____________________ Time your RSD meets______

Do not write below this point.

Part A

____________/30 points

Part B

I. ____________/10 points

II. ____________/20 points

III. ____________/20 points

IV. ____________/20 points

SUM ____________ Grade:___

Part A: (SHORT ANSWERS) Do the following problems. Write the answer in the space provided. Only the answers will be graded; **there is no partial credit.**
(30 points; 3 points each).

1. Evaluate $\int cos(4\theta)^4 d\theta$

   Answer: $\sin(4\theta) + c$

2. Evaluate $\int_0^1 (y^3 - 2y)dy$

   Answer: $\frac{1}{4} - 1 = -\frac{3}{4}$

3. Evaluate $\int \frac{1}{2\sqrt{x}}dx$

   Answer: $\sqrt{x} + c$

4. If f' is continuous on $[1, 4]$, then $\int_1^4 f'(x)dx = f(4) - f(1)$.

   Circle the appropriate answer: [Always true] Can be false

5. Evaluate $\displaystyle\int_{-\pi}^{\pi} \frac{\sin x}{(1+x^4)^2} dx$

Answer: 0

6. If f is continuous on $[a, b]$, then
$\displaystyle\int_a^b f(x)dx =$**area under the graph of** $y = f(x)$ **from** $x = a$ **to** $x = b$

Circle the appropriate answer: Always True $\boxed{\text{Can be False}}$

7. $\displaystyle\int_{-1}^{1} \frac{1}{x^2} dx =$

Circle the appropriate answer: 0 -2 $\boxed{\text{Does not exist}}$

8. Evaluate $\displaystyle\int_{-1}^{1} \sqrt{1-x^2} dx$

Answer: $\dfrac{\pi}{2}$

9. $\displaystyle\lim_{x\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right) =$

Circle the appropriate answer: 1 $\dfrac{3}{2}$ $\boxed{\displaystyle\int_0^1 \sqrt{x} dx}$

10. If $g(x) = \displaystyle\int_3^x \sin(t^2) dt$, find the derivative $g'(x)$ of g.

Answer: $\sin(x^2)$

Part B: For the following problems give a complete solution. Partial credit is possible and you must **SHOW ALL YOUR WORK.**

I) (a) (5 points) Evaluate $\int (x^3+1)^2 dx$

Answer:

$$\begin{aligned}\int (x^3+1)^2 dx &= \int x^6 + 2x^3 + 1 dx \\ &= \frac{x^7}{7} + 2\frac{x^4}{4}x + x + c \\ &= \frac{x^7}{7} + \frac{x^4}{2} + x + c\end{aligned}$$

(b) (5 points) Evaluate $\int \tan^4(\theta)\sec^2(\theta) d\theta$

Answer: Let $u = \tan(\theta)$, then $du = \sec^2(\theta) d\theta$

$$\begin{aligned}\int \tan^4(\theta)\sec^2(\theta) d\theta &= \int u^4 du \\ &= \frac{u^5}{5} + c \\ &= \frac{\tan^5(\theta)}{5} + c\end{aligned}$$

II) (a) (10 point) Find the area bounded by $y = x^2$, and $y = 2 - x^2$.

Answer: The curves intersect when

$$x^2 = 2 - x^2 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$$

$$\begin{aligned}
\text{area} &= \int_{-1}^{1} 2 - x^2 - x^2 dx \\
&= \int_{-1}^{1} 2 - 2x^2 dx = 2\int_{0}^{1} 2 - 2x^2 dx \\
&= 2\left(2x - 2\frac{x^3}{3}\right) |_0^1 = 2\left(2 - \frac{2}{3}\right) \\
&= \frac{8}{3}
\end{aligned}$$

(b) (10 points) Evaluate $\int_1^4 (1+\sqrt{t})^3 \frac{dt}{\sqrt{t}}$

Answer: Let $u = 1 + \sqrt{t}$, $du = \frac{dt}{2\sqrt{t}} \Rightarrow$

$$\begin{aligned}
\int_1^4 (1+\sqrt{t})^3 \frac{dt}{\sqrt{t}} &= 2\int_2^3 u^3 du \\
&= 2\frac{u^4}{4} |_2^3 \\
&= \frac{1}{2}(81 - 16) \\
&= \frac{65}{2}
\end{aligned}$$

III) (a) (10 points) You are driving along a rural highway at a steady **80 ft/sec** when you see an accident ahead and slam on the breaks. Assuming that this will produce a constant **decceleration** of **16** ft/sec^2**, how far does your car travel before it comes to a stop?**

(Make sure to solve the initial value problem $s''(t) = \frac{d^2s}{dt^2} = -16$**,** $v(0) = 80$ **and** $s(0) = 0$**.)**

Answer:

$$s''(t) = -16 \Rightarrow v(t) = s'(t) = -16t + c$$

since $v(0) = 80 \Rightarrow v(t) = s'(t) = -16t + 80. \Rightarrow$

$$s(t) = -8t^2 + 80t + k$$

since $s(0) = 0 \Rightarrow s(t) = -8t^2 + 80t$
the car comes to a stop when $v(t) = 0 \Leftrightarrow$

$$-16t + 80 = 0 \Rightarrow t = 5\,\text{sec}$$

The distance traveled by the car before it comes to a stop is

$$s(5) = -8 \times 25 + 80 \times 5 = 200\text{ft}$$

(b) (10 points) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin\theta\cos\theta}{\sqrt{1+\sin^2\theta}} d\theta$

Answer: Let $u = 1 + \sin^2\theta \Rightarrow du = 2\sin\theta\cos\theta d\theta$

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \frac{\sin\theta\cos\theta}{\sqrt{1+\sin^2\theta}} d\theta &= \int_0^{\frac{\pi}{2}} (1+\sin^2\theta)^{-\frac{1}{2}} \sin\theta\cos\theta d\theta \\ &= \frac{1}{2}\int_1^2 u^{-\frac{1}{2}} du = \\ &= \sqrt{u}\,|_1^2 = \sqrt{2} - 1\end{aligned}$$

IV) (a) (5 points) If $F(x) = \int_1^{x^3} \sqrt{1+t^2}dt$, find the derivative $F'(x)$ of the function F.

$$\begin{aligned} F'(x) &= \sqrt{1+(x^3)^2} \cdot 3x^2 \\ &= \sqrt{1+x^6} \cdot 3x^2 \end{aligned}$$

(b) (5 points) If $f(x) = \frac{1}{x}$ $1 \leq x \leq 9$, 7 find the **Riemann sum** with $\mathbf{n} = \mathbf{4}$, taking the sample points to be the **midpoints**.

$$\begin{aligned} \Delta x &= \frac{9-1}{4} = 2 \\ x_1^* &= 2, \quad x_2^* = 4, \\ x_3^* &= 6, \quad x_4^* = 8. \\ R_4 &= \frac{9-1}{4}\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{6}+\frac{1}{8}\right) \end{aligned}$$

(c) (5 points) find the **number a** such that the **vertical line $\mathbf{x} = \mathbf{a}$** divides the region bounded by $y = 3x^2$ and the x-axis, from $x = 0$ to $x = 2$, into two regions with **equal area**.

Answer:

$$\begin{aligned} 2\int_0^a 3x^2dx &= \int_0^2 3x^2dx \Leftrightarrow \\ 2a^3 &= 8 \\ a^3 &= 4 \Rightarrow a = \sqrt[3]{4} \end{aligned}$$

or

$$\begin{aligned} \int_0^a 3x^2dx &= \int_a^2 3x^2dx \Leftrightarrow \\ a^3 &= 8 - a^3 \Leftrightarrow \\ 2a^3 &= 8 \Rightarrow a = \sqrt[3]{4} \end{aligned}$$

(d) (5 points) Use the substitution $\mathbf{u} = \mathbf{1} + \mathbf{x}$ to evaluate the indefinite integral

$$\int \sqrt{1+x}dx$$

Answer: Let $u = 1 + x \Rightarrow du = dx$ and $x = u - 1 \Rightarrow$

$$\begin{aligned} \int x\sqrt{1+x}dx &= \int (u-1)\sqrt{u}du \\ &= \int u^{\frac{3}{2}} - u^{\frac{1}{2}}du \\ &= \frac{2}{5}u^{\frac{5}{2}} - \frac{2}{3}u^{\frac{3}{2}} + c \\ &= \frac{2}{5}(1+x)^{\frac{5}{2}} - \frac{2}{3}(1+x)^{\frac{3}{2}} + c \end{aligned}$$