Math 1500, Exam II, WS2005

Paulette Saab

Do all the problems

If you need more space, use the opposite blank page
and say so.

Calculators are neither needed nor allowed.

You may not leave before 7:30 p.m.

Name____

Student Number____

RSD Instructor______________________________Time your RSD meets________

Do not write below this point.

Part A

________/30 points

Part B

I. ________/10 points

II. ________/20 points

III. ________/20 points

IV. ________/20 points

SUM ________ Grade:____

Part A: (SHORT ANSWERS) Do the following problems. Write the answer in the space provided. Only the answers will be graded; **there is no partial credit.**
(30 points; 3 points each).

1. If $f(3) = 2$, $g(3) = -3$, $f'(3) = 5$ and $g'(3) = 4$, find $(fg)'(3)$.

   Answer: -7

2. If f is differentiable at $x = 5$, $f(5) = 4$ and $f'(5) = 2$, find $\frac{d}{dx}\sqrt{f(x)}$ when $x = 5$.

   Answer: $\frac{1}{2}$

3. If $y = \frac{x^2 - 1}{x^2 + 1}$, find $\frac{dy}{dx}$. **Make sure to simplify your answer**

   Answer: $\frac{4x}{(x^2 + 1)^2}$

4. Find $\lim_{\theta \to 0} \frac{\sin(3\theta)}{\sin(5\theta)}$.

   Answer: $\frac{3}{5}$

5. If $y = \cos(t^4)$, find $\frac{dy}{dt}$.

   Answer: $-\sin(t^4)4t^3$ or $-4t^3\sin(t^4)$

6. The function $y = |\sin(x)|$ is differentiable at **both** $x = \frac{\pi}{2}$ **and** $x = \pi$.

Circle the appropriate answer: True $\boxed{\text{False}}$

7. If $y = \sin^4(x)$, find $\frac{dy}{dx}$.

Answer: $4\sin^3(x)\cos(x)$ or $4(\sin(x))^3\cos(x)$

8. If a function f is differentiable at $x = a$, then f is continuous at $x = a$.

Circle the appropriate answer: $\boxed{\text{Always true}}$ Can be false

9. The curve $y = \frac{x^3}{3} + x + 2$ has **no point** where the **tangent line is horizontal**.

Circle the appropriate answer: $\boxed{\text{True}}$ False

10. If $h(x) = \sqrt{x^4 + 9}$, then

$$\lim_{x \to 2} \frac{h(x) - h(2)}{x - 2} =$$

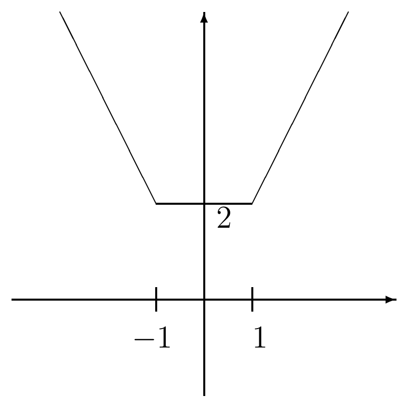
Circle the appropriate answer $\boxed{\frac{16}{5}}$ $\frac{32}{5}$ $\frac{1}{10}$

Part B: For the following problems give a complete solution. Partial credit is possible and you must **SHOW ALL YOUR WORK.**

I) (a) (4 points) Sketch the graph of the function $f(x) = |x+1| + |x-1|$.

Answer:

$$f(x) = \begin{cases} -2x & \text{if } x \leq -1 \\ 2 & \text{if } -1 < x < 1 \\ 2x & \text{if } x \geq 1 \end{cases}$$



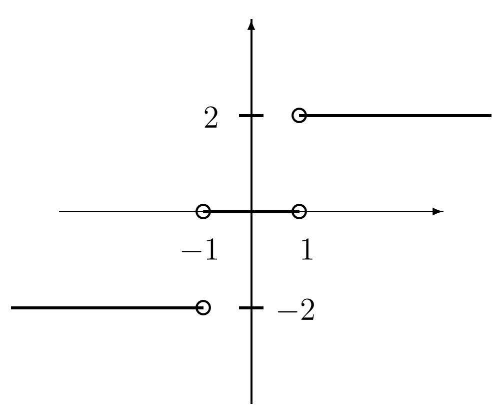
(b) (2 points) Where is the function f differentiable?

Answer: $f'(x)$ exists for all $x \neq \pm 1$
or f is differentiable for all $x \neq \pm 1$
or f is differentiable on $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

(c) (4 points) Give a formula for the derivative function f' and sketch its graph.

Answer:

$$f'(x) = \begin{cases} -2 & \text{if } x < -1 \\ 0 & \text{if } -1 < x < 1 \\ 2 & \text{if } x > 1 \end{cases}$$



II) (a) (10 points) If $f(x) = (x^2+1)(x^3+5)^{\frac{1}{3}}$, find $f'(x)$.

Answer:

$$f'(x) = (x^2+1)\frac{1}{3}(x^3+5)^{-\frac{2}{3}}(3x^2) + (x^3+5)^{\frac{1}{3}}(2x)$$

or

$$= (x^2+1)(x^3+5)^{-\frac{2}{3}}x^2 + (x^3+5)^{\frac{1}{3}}(2x)$$

(b) (10 points) If $g(x) = \sin(x^3) + \frac{1}{3}\tan^3(2x)$, find $g'(x)$

Answer:

$$g'(x) = \cos(x^3)3x^2 + \frac{1}{3}3\tan^2(2x)\sec^2(2x)(2)$$

or

$$g'(x) = 3x^2\cos(x^3) + 2\tan^2(2x)\sec^2(2x)$$

III) (a) (10 points) Compute $\lim_{\theta\to 0} \frac{\sin(5\theta)+\theta}{\sin(\theta)\cos(\theta)}$.

Answer:

$$\lim_{\theta\to 0} \frac{\sin(5\theta)+\theta}{\sin\theta\cos\theta} = \lim_{\theta\to 0} \frac{\frac{\sin(5\theta)+\theta}{\theta}}{\frac{\sin\theta\cos\theta}{\theta}} =$$

$$\lim_{\theta\to 0} \frac{5\frac{\sin(5\theta)}{5\theta}+1}{\frac{\sin(\theta)}{\theta}\cos(\theta)} = \frac{5+1}{1.1} = 6$$

(b) (10 points) Find the **slope of the tangent line** to the curve $x^3+y^3=2xy$ at the point $(1,1)$.

Answer:

$$\begin{aligned}
\frac{d}{dx}(x^3+y^3) &= \frac{d}{dx}(2xy) \Leftrightarrow \\
3x^2+3y^2y' &= 2xy'+2y \Leftrightarrow \\
y'(3y^2-2x) &= 2y-3x^2 \Leftrightarrow \\
y' &= \frac{2y-3x^2}{3y^2-2x}, \quad 3y^2-2x \neq 0 \\
m_{\tan}\big|_{\substack{x=1\\y=1}} &= \frac{2-3}{3-2} = -1
\end{aligned}$$

or

$$\begin{aligned}
3x^2+3y^2y' &= 2xy'+2y \\
3+3y' &= 2y'+2 \Leftrightarrow \\
y' &= -1
\end{aligned}$$

IV) (a) (10 points) Where does the **normal line** to the parabola $\mathbf{y = x + x^2}$ at the point $(\mathbf{-1, 0})$ intersect the parabola a second time? **Illustrate with a sketch**.

Answer:

$\frac{dy}{dx} = 1 + 2x$

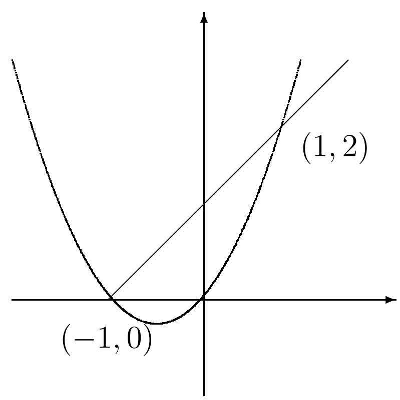
m_{tan} = slope of the tangent line at $(-1, 0) = 1 - 2 = -1$

m_{normal} = slope of the normal line at $(-1, 0 = 1$

The equation of the normal line at $(-1, 0)$ is $y = x + 1$

$x + 1 = x + x^2 \Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$

The normal line intersects the parabola a second time at $(1, 2)$.



(b) (10 points) Find an equation of the **tangent line** through the point $\mathbf{A(-4, 0)}$ to the curve $\mathbf{y = \sqrt{x}}$. **Illustrate with a sketch**.

Answer:

Let $T(x, \sqrt{x})$ be a tangency point, then

slope of $AT = \frac{\sqrt{x} - 0}{x + 4} = \frac{dy}{dx} = \frac{1}{2\sqrt{x}} \Leftrightarrow$

$\frac{\sqrt{x}}{x + 4} = \frac{1}{2\sqrt{x}} \Leftrightarrow$

$2x = x + 4 \Leftrightarrow x = 4$

The tangent point is $(4, 2)$.

The slope of the tangent line is $\frac{1}{2\sqrt{x}} \big|_{x=4} = \frac{1}{4}$

The equation of the tangent line is

$y - 2 = \frac{1}{4}(x - 4)$ or $y = \frac{1}{4}(x + 4)$ or $y = \frac{1}{4}x + 1$.