Sample Questions Exam IV, FS2009

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Calculators are neither needed nor allowed.

Part A: (SHORT ANSWER QUESTIONS) Do the following problems. Write the answer in the space provided. Only the answers will be graded; **there is no partial credit.**

1. Evaluate $\displaystyle\int_1^4 \frac{dt}{2\sqrt{t}}$.

2. Evaluate $\displaystyle\int_0^1 (y^9 - 2y^5)dy$.

3. Evaluate $\displaystyle\int 3\cos(3\theta\ )d\theta$.

4. Evaluate $\displaystyle\int \tan^3(t)\sec^2(t)\ dt$.

5. Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sin^4(\theta)\cos(\theta)d\theta$.

6. Evaluate $\displaystyle\int_{-1}^1 \frac{\sin x}{(1+x^6)^2}dx$.

7. If $F(x) = \displaystyle\int_1^x \frac{1}{\sqrt{1+t^4}}dt$, find $F'(x)$.

8. If $g(x) = \displaystyle\int_3^x \cos(1+t^3)\ dt$, find $g'(x)$.

9. $\displaystyle\int_{-1}^1 \left(x^5 - 6x^9 + \frac{\sin(x)}{(1+x^4)^2}\right) dx = 0.$

   Circle the appropriate answer **True** **False**

10. $\displaystyle\int_0^2 (x - x^3)dx$ represents the area under the curve $y = x - x^3$ from $x = 0$ to $x = 2$.

    Circle the appropriate answer **True** **False**

11. If $F(x) = \int_1^{4x} \sqrt{1+\sin t}\, dt$, find $F'(x)$.

12. If f is continuous on $[a, b]$, then

$$\int_a^b f(x)dx = \text{ area under the graph of } y = f(x) \text{ from } x = a \text{ to } x = b.$$

Circle the appropriate answer Always True Can be False

13. $\int_{-1}^{1} \frac{1}{x^2} dx = -2.$

Circle the appropriate answer **True** **False**

14. Evaluate $\int_{-a}^{a} \sqrt{a^2 - x^2}\, dx$.

15. Evaluate $\int_{-1}^{1} \frac{\sin x}{(1+x^6)^2} dx$.

16. Evaluate $\int_0^3 \sqrt{9 - x^2}\, dx$.

17. If $\int_2^8 f(x)dx = 7$ and $\int_2^5 f(x)dx = 3$, find $\int_5^8 f(x)dx$.

18. If f is continuous on $[a, b]$, then $\int_a^b xf(x)dx = x\int_a^b f(x)dx$.

Circle the appropriate answer: **True** **False**

19. Evaluate $\int \sin^6(t)\cos(t)\, dt$.

20. Evaluate $\int \tan^4(3\theta)\sec^2(3\theta)d\theta$.

21. Evaluate $\int_{-1}^{1} x\cos(x^3)dx$.

22. $\int_{-5}^{5} (ax^2 + bx + c)\, dx = 2\int_0^5 (ax^2 + c)\, dx$

Circle the appropriate answer **True** **False**

Part B: For the following problems give a complete solution. Partial credit is possible and you must **SHOW ALL YOUR WORK.**

I) (a) Evaluate $\int (t^2+1)^3 \, dt$.

(b) (Evaluate $\int_0^2 \frac{2x}{(1+x^2)^2} dx$.

(c) Evaluate $\int_4^9 (\sqrt{x}-1)^3 \frac{dx}{\sqrt{x}}$.

(d) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\cos\theta}{\sqrt{5+4\sin\theta}} d\theta$.

II) (a) Find the area of the region bounded by $y = x^3$ and $y = x$.

(b) Find the area bounded by $x = y^2$ and $x = 3 - 2y$.

(c) Find the area bounded by $y = \sin(x)$, $y = 0$, $x = 0$ and $x = 2\pi$.

(d) Find the area bounded by $y = \sin(2x)$, $y = \cos(2x)$, $x = 0$, and $x = \pi$.

III) (a) If $f(x) = \sqrt{x}$; $1 \le x \le 9$, find the **Riemann sum** with $\mathbf{n} = \mathbf{4}$, taking the sample points to be the **midpoints**.

(b) If $f(x) = \dfrac{1}{x}$; $0 \le x \le 10$, find the **Riemann sum** with $\mathbf{n} = \mathbf{5}$, taking the sample points to be the **right endpoints**.

(c) Write the following limit as a definite integral and compute its value

$$\lim_{n\to\infty} \frac{1}{n}\left(\sqrt{\frac{1}{n}} + \sqrt{\frac{2}{n}} + \cdots + \sqrt{\frac{n}{n}}\right).$$

(d) Write the following limit as a definite integral and compute its value

$$\lim_{n\to\infty} \frac{\pi}{n}\left(\cos\frac{\pi}{n} + \cos\frac{2\pi}{n} + \cdots + \cos\frac{n\pi}{n}\right).$$

IV) Let $\mathcal{R}$ be the region bounded by $\mathbf{y} = \sin \mathbf{x}$**,** $\mathbf{y} = \mathbf{0}$**,** $\mathbf{x} = \mathbf{0}$ **and** $\mathbf{x} = \pi$.

(a) Set up the integral **(DO NOT EVALUATE IT)** that would give you the volume of the solid obtained by rotating the region $\mathcal{R}$ **about the** y**-axis.**

(b) Set up the integral **(DO NOT EVALUATE IT)** that would give you the volume of the solid obtained by rotating the region $\mathcal{R}$ **about the** x**-axis.**

(c) Set up the integral **(DO NOT EVALUATE IT)** that would give the volume of the solid obtained by rotating the region $\mathcal{R}$ **about** $\mathbf{x} = \pi$**.**

(d) Set up the integral **(DO NOT EVALUATE IT)** that would give you the volume of the solid obtained by rotating the region $\mathcal{R}$ **about** $\mathbf{y} = \mathbf{2}$**.**

V) (a) Find the **volume** of the solid generated when the region bounded by $y = x^2$, $y = 0$, $x = 0$, and $x = 2$ is revolved about the x-axis.

(b) Find the **volume** of the solid generated when the region bounded by $y = x^2$, $y = 0$, $x = 0$, and $x = 2$ is revolved about the y-axis.

(c) Find the **volume** of the solid generated when the region in the first quadrant, bounded by $y = x^2$, and $y = 1$ is revolved about the x-axis.

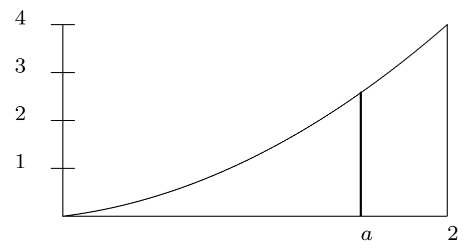
(d) Find the **volume** of the solid generated when the region in the first quadrant, bounded by $y = x^2$, and $y = 1$ is revolved about the y-axis.

VII) (a) If $F(x) = \int_{2x}^{x^3} \sqrt{1+t^2}\, dt$, find the derivative $F'(x)$ of the function F.

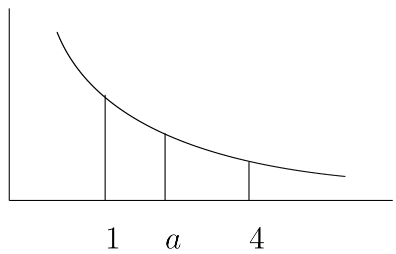
(b) If $F(x) = \int_{x^2}^{\sin(x)} \frac{t}{2+t}\, dt$, find the derivative $F'(x)$ of the function F.

(c) If f is a continuous function such that $\int_0^x f(t)dt = x^2 \sin x + \int_x^0 f(t)\sqrt{t}\, dt$ for all x, find an explicit formula for $f(x)$.

VIII (a) (10 points) Find the number a such that the line $x = a$ partitions the region bounded by $y = x^2$, the x-axis, $x = 0$, and $x = 2$, into two regions of **equal** area.



(b) Find the number a such that the line $x = a$ partitions the region bounded by $y = \frac{1}{x^2}$, the x-axis, $x = 1$, and $x = 4$, into two regions of **equal** area.



IX (a) Evaluate $\int_3^4 x\sqrt{x-3}\,dx$. (**Hint:** Let $u = x - 3$.)

(b) Use the substitution $\mathbf{u} = \mathbf{x} + \mathbf{3}$ to evaluate the indefinite integral

$$\int x\sqrt{x+3}\,dx$$