Sample Questions Exam III, FS2009

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Calculators are neither needed nor allowed.

Part A: (SHORT ANSWER QUESTIONS) Do the following problems. Write the answer in the space provided. Only the answers will be graded; **there is no partial credit.**

1. If $f'(x) = \sec^2(x)$, and $f(0) = 2$, find $f(x)$.

2. If $g'(x) = x\sqrt[3]{x}$, and $g(1) = 1$, find $g(x)$.

3. The function $f(x) = 5x + \sin(x)$ is concave upward on $(0, \pi)$.

   Circle the appropriate answer True False

4. Find the equation of the horizontal asymptote of $y = \dfrac{2x^3 + 5}{3x^3 - 1}$.

5. If f has an absolute minimum value at c, then $f'(c) = 0$.

   Circle the appropriate answer Always True Can be False

6. If $f'(c) = 0$, then f has a local minimum or a local maximum value at c.

Circle the appropriate answer Always True Can be False

7. Find the equation of the vertical asymptotes of $y = \dfrac{2x}{x^2 - 9}$.

8. Find $\displaystyle\lim_{x \to -\infty} \frac{x}{\sqrt{3x^2 + 1}}$.

9. Use Newton's method with $x_1 = 1$ to find x_2, the second approximation to the root of the equation $x^3 - 4 = 0$.

10. $\displaystyle\lim_{x \to \infty} x \sin\left(\frac{1}{3x}\right)$ is:

Circle the appropriate answer: ∞ 0 $\dfrac{1}{3}$

11. The function $f(x) = 5x + \cos(x)$ is increasing on $(-\infty, \infty)$.

Circle the appropriate answer True False

12. If $f''(c) = 0$, then $(c, f(c))$ is an inflection point of the curve $y = f(x)$.

Circle the appropriate answer Always True Can be False

13. If f has an absolute minimum value at c, and f is differentiable at c, then $f'(c) = 0$.

Circle the appropriate answer Always True Can be False

14. Find the equations of the vertical asymptotes of $y = \dfrac{x}{x^2 - 4}$.

15. Does there exist a function f such that $f(0) = -1$, $f(2) = 4$, and $f'(x) \leq 2$ for all x?

Circle the appropriate answer Yes No

16. The point $(0,0)$ is an inflection point of the curve $y = x^4 + 2x^2$.

Circle the appropriate answer True False

17. Find the most general antiderivative of the function $f(x) = 3x^2 - \dfrac{1}{x^2}$.

18. Find $\lim\limits_{x \to -\infty} \dfrac{\sqrt{4x^2+1}}{x+1}$.

19. Find $\lim\limits_{x \to \infty} \dfrac{\sqrt{4x^2+1}}{1-x}$.

20. Find the most general antiderivative of the function $f(x) = 2x + \sqrt{x}$.

Part B: For the following problems give a complete solution. Partial credit is possible and you must **SHOW ALL YOUR WORK.**

I) (a) Let $f(x) = 4x^3 - 3x^2 - 6x + 1$ for $\infty < x < \infty$.

(a) (4 points) Determine the intervals where f is increasing or decreasing.

(b) (2 points) Find the local maximum and minimum values of f.

(c) (4 points) Determine the intervals where the graph of f is concave up or concave down and identify the inflection point(s).

II) (a) Let $f(x) = x^2 + \frac{2}{x}$; $\frac{1}{2} \leq x \leq 2$. Find the absolute maximum value and the absolute minimum value of f on $[\frac{1}{2}, 2]$.

(b) Let $f(x) = (x-2)^{\frac{2}{3}}$; $1 \leq x \leq 10$. Find the absolute maximum value and the absolute minimum value of f on $[1, 10]$.

(c) Compute

$$\lim_{x \to \infty} (\sqrt{9x^2 + 2x} - 3x)$$

(d) **Use the Squeeze Theorem** to explain why $\lim_{x \to \infty} \frac{\cos^2(x)}{x^2} = 0$.

III a) Apply Newton's method to the equation $\boldsymbol{\frac{1}{x} - a = 0}$ to derive the following reciprocal algorithm:

$$x_{n+1} = 2x_n - ax_n^2.$$

III b) Apply Newton's method to the equation $\boldsymbol{x^2 - a = 0}$ to derive the following square-root algorithm:

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right).$$

IV) (a) Show that for any $x > 0$

$$\sqrt{1+x} < 1 + \frac{1}{2}x$$

(b) Show that for any $x > 0$

$$\sqrt{1+x^2} < 1 + x$$

(c) Show that for any real numbers a and b we have

$$|\cos(b) - \cos(a)| \leq |b - a|$$

(d) Use Rolle's theorem to show that the equation $x^5 + 10x + 3 = 0$ has at most one real root.

VI) (a) A car is travelling at 60 mi/h = 88 ft/s, when the brakes are fully applied, producing a constant deceleration of 22 ft/s^2. What is the distance traveled before the car comes to a stop?

(b) A car braked with a constant deceleration of 16 ft/s^2, producing skid marks measuring 200ft before coming to a stop. How fast was the car travelling when the brakes were first applied?

VII) (a) Find the points on the ellipse $4x^2 + y^2 = 4$ that are farthest away from the point $(1, 0)$.

(b) The **sum** of two non negative numbers is 40. Find the numbers if **one of the numbers plus four times the square root of the other** is to be as **large as possible**.

(c) A box with **square base** and **open top** must have a **volume of 32 $\mathbf{in}^3$**. Find the dimensions of the box that **minimize** the amount of material used.
(Neglect thickness of material and waste in construction.)

(b) A 150 m^2 **rectangular** garden spot is to be enclosed by a fence and divided into **two equal** parts by fences parallel to one of the sides. What dimensions for the outer rectangle will require the **smallest total length of fence**?