Sample Questions Exam II, FS2009

Paulette Saab

Calculators are neither needed nor allowed.

Part A: (SHORT ANSWER QUESTIONS) Do the following problems. Write the answer in the space provided. Only the answers will be graded; **there is no partial credit.**

1. If $f'(9) = 10$, find $\frac{d}{dx} f(x^2)$ when $x = 3$.

2. If $y = \cos^3 x$, find $\frac{dy}{dx}$.

3. $\frac{d^2y}{dx^2} = (\frac{dy}{dx})^2$

   Circle the appropriate answer **True** **False**

4. If f is a positive differentiable function, then $\frac{d}{dx}\sqrt{f(x)} =$

   Circle the appropriate answer $\frac{1}{2\sqrt{f(x)}}$ $\frac{f'(x)}{2\sqrt{f(x)}}$

5. Find $\lim_{\theta \to 0} \frac{\sin(5\theta)}{\sin(8\theta)}$.

6. If $h(x) = \sqrt{x^2 + 16}$, then

$$\lim_{x \to 3} \frac{h(x) - h(3)}{x - 3} =$$

Circle the appropriate answer $\frac{3}{10}$ $\frac{3}{5}$ $\frac{1}{10}$

7. $\frac{d}{d\theta} \tan^2 \theta = \frac{d}{d\theta} \sec^2 \theta$.

Circle the appropriate answer: True False

8. If $f'(a)$ exists, then $\lim_{x \to a} f(x) = f(a)$.

Circle the appropriate answer Always True Can be False

9. Find $\lim_{\theta \to 0} \frac{\tan(4\theta)}{7\theta}$.

10. If $y = x^3 \sin(x)$, find $\frac{dy}{dx}$.

11. If $y = \sqrt{x^4 + 1}$, find $\dfrac{dy}{dx}$.

12. If $f(t) = \dfrac{t^2}{t^2 + 1}$, find $f'(t)$.

13. If a function f is differentiable, then $\dfrac{d}{dx} f(\sqrt{x}) =$

Circle the appropriate answer: $f'(\sqrt{x})$ $f'(x)\dfrac{1}{2\sqrt{x}}$ $f'(\sqrt{x})\dfrac{1}{2\sqrt{x}}$

14. if $f(x) = \sin(|x|)$, find $f'(x)$ for $x \neq 0$.

15. $\dfrac{d}{dx}|x^2 - x| = |2x - 1|$

Circle the appropriate answer: True False

16. $\dfrac{d}{dx}(\sin^2 x) = -\dfrac{d}{dx}(\cos^2 x)$.

Circle the appropriate answer: True False

17. Find the x**-coordinate** of **all points** on the curve $y = x^3 - 3x$ where the tangent line is horizontal.

18. What is $\lim_{x \to \frac{\pi}{3}} \dfrac{\sin(x) - \frac{\sqrt{3}}{2}}{x - \frac{\pi}{3}}$?

19. What is $\lim_{x \to a} \dfrac{\cos(x^2) - \cos(a^2)}{x - a}$?

20. What is $\lim_{h \to 0} \dfrac{\tan(3+h)^3 - \tan(27)}{h}$?

21. There are no points on the curve $y = x^3 + 2x - 10$ where the tangent is horizontal.

    Circle the appropriate answer True False

22. If a function f is continuous at $x = a$, then f is differentiable at $x = a$.

    Circle the appropriate answer Always True Can be False

Part B: For the following problems give a complete solution. Partial credit is possible and you must **SHOW ALL YOUR WORK.**

I) (a) If $f(x) = x\tan(x^2)$, find $f'(x)$.

(b) If $g(x) = \sin^2(3x) + \sec(\sqrt{x});\ \ x > 0,\ \ \text{find } g'(x)$.

(c) If $f(t) = \sqrt{1 + \sin^2(5t)};\ \ \text{find } f'(t)$

II) On a distant planet a ball is thrown vertically upward with a velocity of 100 ft/s. Its height after t seconds is $100t - 10t^2$.

(a) What is the **maximum height** reached by the ball?

(b) What is the velocity of the ball when it is **210 ft** above the ground on its way up? On its way down?

(c) Compute $\lim_{\theta \to 0} \dfrac{1 - \cos(4\theta)}{\theta^2}$.

III) (a) Find the **linearization** of $\sqrt[3]{x}$ at $\mathbf{a = 27}$ and use it to approximate $\sqrt[3]{26}$.

(b) If $y = \cos^2(5\theta) + \tan(3\theta)$, find $\dfrac{dy}{d\theta}$.

(c) If $f(x) = \sin(\dfrac{\cos(x)}{x})$, find $\dfrac{dy}{dx}$.

IV) (a) The volume of a cube is increasing at a rate of 10 cm^3/min. How fast is the **surface area** increasing when the length of an edge is 30 cm?

(b) Compute $\lim_{\theta \to 0} \dfrac{\sin(3\theta) + \sin(\theta)}{\theta + \sin(\theta)\cos(\theta)}$

(c) Compute $\lim_{x \to 2} \dfrac{\sqrt{x^2+1} - \sqrt{5}}{x - 2}$

V) (a) (10 points) Find the linear approximation of the function $f(x) = \sqrt{9+x}$ at $a = 0$ and use it to approximate the number $\sqrt{9.01}$.

(b) A boat is pulled into a dock by a rope attached to the bow of the boat and passing through a pulley on the dock that is **1m higher** than the bow of the boat. If the rope is pulled in at a **rate of 1m/s**, **how fast is the boat approaching the dock** when it is **8m from the dock**?

VI) (a) Find the equation of the **tangent line** to the curve $xy+x^3 = y^2$ at the point $(2, -2)$.

(b) Find the points on the curve $x^2 + xy + y^2 = 3$, the tangent line is horizontal.

(c) Find the equation to the tangent line to the curve $x^3 - 2xy + y^4 = \sin(y - 1)$ at the point $(1, 1)$.

VII) (a) Find the equation of the tangent line from the point $(0, 1)$ to the curve $y = \frac{1}{x}$.

(b) Show that the family of circles $x^2 + y^2 = ax$ and $x^2 + y^2 = by$ are orthogonal.

(c) Show that the parabola $x = y^2$ and the ellipse $2x^2 + y^2 = 3$ are orthogonal.

VIII) (a) The cost function for production of a commoditity is

$$C(x) = 2000 + 100x - 0.1x^2.$$

1) Find the **average cost per machine** of producing the **first 100 washing machines**.

2) Find the **marginal cost** when 100 machines are produced.

3) Show that the marginal cost when 100 washing machines are produced is approximately the cost of producing one more machine after the first 100 have been made, by calculating the later cost directly.

IX) (a) A water tank has the shape of an inverted circular cone with base radius $2m$ and height $4m$. If water is being pumped into the tank at a rate of $2m^3$/min, find the rate at which the water level is rising when the water is $3m$ deep.

(b) Bike A is traveling **west** at 10 mi/h and bike B is traveling **north** at 12 mi/h. Bike A is headed **toward** the intersection of the two roads while bike B is headed **away** from the intersection of the two roads. At what rate is the distance between bike A and bike B changing when bike A is 6 mi and bike B is 8 mi from the intersection?