Sample Questions Exam I, FS2009

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Calculators are neither needed nor allowed.

Part A: (SHORT ANSWER QUESTIONS) Do the following problems. Write the answer in the space provided. Only the answers will be graded; **there is no partial credit.**

1. What is the domain of the function $f(x) = \dfrac{1}{\sqrt{8-5x}}$?

2. Determine whether the function $f(x) = x^2 \sin(x) + x^3$ is **even**, **odd** or **neither**.

3. Compute $\displaystyle\lim_{x\to 1^+} \frac{\sqrt{x-1}+x}{1-2x}$

4. The composition of an even function and an odd function is an even function.

   Circle the appropriate answer: Always true Can be false

5. If $f(x) = x^4 + 1$ and $g(x) = \dfrac{1}{\sqrt{x-1}}$, what is $g(f(x))$?

6. Express the **surface area** A of a **cube** as a function of **its volume** V.

7. Compute $\lim_{x\to 4} \dfrac{x^2 - 3x - 4}{x^2 - 6x + 8}$.

8. Compute $\lim_{x\to 3^-} \dfrac{2(x-3)}{|x-3|}$

9. $\lim_{x\to 0} \sin\left(\dfrac{1}{x}\right)$ is:

   Circle the appropriate answer: 0 ∞ does not exist

10. Compute $\lim_{x\to 5^-} \dfrac{x-6}{5-x}$

11. If $f(2) = 5$, $g(2) = 3$, $f'(2) = 6$ and $g'(2) = 1$, find $\left(\frac{f}{g}\right)'(2)$.

12. If $f(2) = 5$, $g(2) = 3$, $f'(2) = 6$ and $g'(2) = 1$, find $(f.g)'(2)$.

13. Find the equation of the **tangent line** to the curve $y = x^3$ at the point $(2, 8)$.

14. If f is a continuous function on $[0, 1]$, $f(0) < 0$ and $f(1) > 0$, then there is $0 < c < 1$ such that $f(c) = 0$.

    Circle the appropriate answer Always True Can be False

15. Find $\lim_{\theta \to 0} \frac{\sin(5\theta)}{\sin(8\theta)}$.

16. If the tangent line to $y = f(x)$ at $(3, 5)$ passes through the point $(1, 4)$ find $f'(3)$.

17. Find $\lim_{x \to 1} \frac{x^9 - 1}{x - 1}$.

18. **True or False.** If $h(x) = x^5$, then

$$\lim_{x \to 2} \frac{h(x) - h(2)}{x - 2} = 80.$$

19. Find the x**-coordinate** of **all points** on the curve $y = x^3 - 3x$ where the tangent line is horizontal.

20. If a function f is continuous at $x = a$, then f is differentiable at $x = a$.

Circle the appropriate answer Always True Can be False

Part B: For the following problems give a complete solution. Partial credit is possible and you must **SHOW ALL YOUR WORK.**

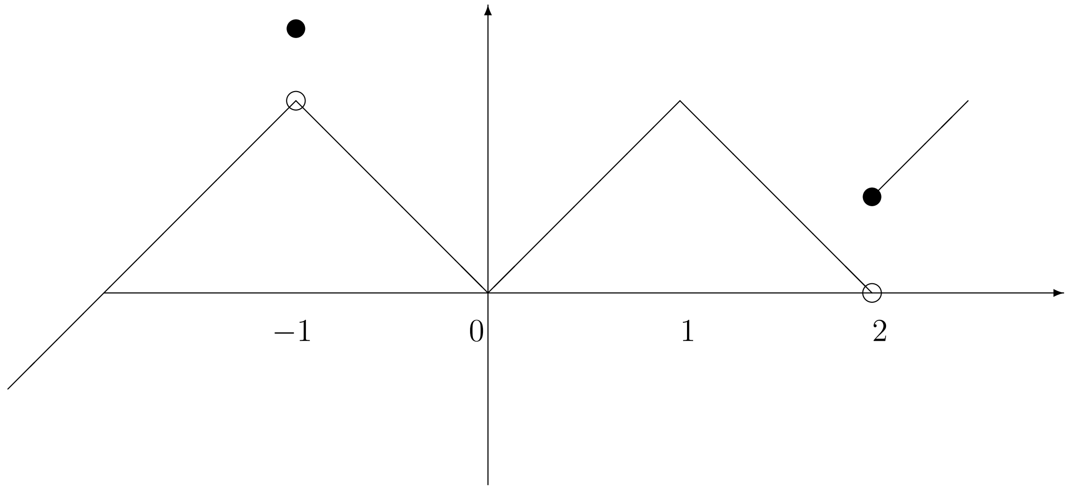
I) (a) Sketch the graph of the function $y = \sqrt{x}$. **Make sure to label the x-axis, as well as the y-axis.**

(b) Sketch the graph of the function $y = |\sqrt{x+1} - 2|$. **Make sure to label the x-intercept, as well as the y-intercept**.

II) (a) If $f(x) = \sqrt{4 + 3x}$ and $g(x) = \dfrac{x^2 - 4}{3}$, find $f \circ g$ and $g \circ f$, and **make sure to specify their domain.**

(b) **Use the Squeeze Theorem** to explain why $\lim_{x \to 0} x^2 \cos\left(\dfrac{1}{x}\right) = 0$.

III) The graph of a function f is given below.



(a) At what numbers is f discontinuous? Give reasons for your answers.

(b) At what numbers is f not differentiable? Give reasons for your answers.

IV) (a) If

$$g(x) = \begin{cases} \dfrac{x^2 - x}{x - 1} & \text{if } x > 1 \\ \sin(x-1) + 2 & \text{if } x \leq 1 \end{cases}$$

(1) Find $\lim_{x \to 1^+} g(x)$

(2) Find $\lim_{x \to 1^-} g(x)$

(3) Is the function g continuous at $x = 1$? Explain.

(b) Compute $\lim_{t \to 0} \dfrac{t}{\sqrt{1+5t} - 1}$

V) (a) (8 points) Find the equation of the **tangent line** to the curve $y = \sqrt{x}$ that is parallel to the line $x - 4y = 1$.

VI) On a distant planet a ball is thrown vertically upward with a velocity of 100 ft/s. Its height after t seconds is $100t - 10t^2$.

(a) What is the **maximum height** reached by the ball?

(b) What is the velocity of the ball when it is **210 ft** above the ground on its way up? On its way down?

VII Compute $\lim_{\theta \to 0} \dfrac{1 - \cos(4\theta)}{\theta^2}$.

VIII) (a) Find the **points** on the curve $y = \dfrac{x}{x-1}$ where the **tangent lines are parallel** to the line $x + y = 4$.

(b) Use the definition given in Section 3.1 to find the slope of the tangent line to the graph of $f(x) = \dfrac{1}{x}$ at the point $\left(2, \dfrac{1}{2}\right)$. Then find an equation of the tangent line there.

IX) (a) (8 points) Show that there are **two tangent lines** to the curve $y = x^2$ through $(2, 3)$. Find an equation of each of these lines.