****BEGINNING OF EXAMINATION****

1. For a fully discrete 3-year endowment insurance of 1000 on (x), you are given:

(i) ${}_kL$ is the prospective loss random variable at time k.

(ii) $i = 0.10$

(iii) $\ddot{a}_{x:\overline{3}|} = 2.70182$

(iv) Premiums are determined by the equivalence principle.

Calculate ${}_1L$, given that (x) dies in the second year from issue.

(A) 540

(B) 630

(C) 655

(D) 720

(E) 910

2. For a double-decrement model:

(i) ${}_{t}p'^{(1)}_{40} = 1 - \frac{t}{60}$, $0 \le t \le 60$

(ii) ${}_{t}p'^{(2)}_{40} = 1 - \frac{t}{40}$, $0 \le t \le 40$

Calculate $\mu^{(\tau)}_{40}(20)$.

(A) 0.025

(B) 0.038

(C) 0.050

(D) 0.063

(E) 0.075

3. For independent lives (35) and (45):

(i) ${}_5p_{35} = 0.90$

(ii) ${}_5p_{45} = 0.80$

(iii) $q_{40} = 0.03$

(iv) $q_{50} = 0.05$

Calculate the probability that the last death of (35) and (45) occurs in the 6^{th} year.

(A) 0.0095

(B) 0.0105

(C) 0.0115

(D) 0.0125

(E) 0.0135

4. For a fully discrete whole life insurance of 100,000 on (35) you are given:

(i) Percent of premium expenses are 10% per year.

(ii) Per policy expenses are 25 per year.

(iii) Per thousand expenses are 2.50 per year.

(iv) All expenses are paid at the beginning of the year.

(v) $1000P_{35} = 8.36$

Calculate the level annual expense-loaded premium using the equivalence principle.

(A) 930

(B) 1041

(C) 1142

(D) 1234

(E) 1352

5. Kings of Fredonia drink glasses of wine at a Poisson rate of 2 glasses per day.

Assassins attempt to poison the king's wine glasses. There is a 0.01 probability that any given glass is poisoned. Drinking poisoned wine is always fatal instantly and is the only cause of death.

The occurrences of poison in the glasses and the number of glasses drunk are independent events.

Calculate the probability that the current king survives at least 30 days.

(A) 0.40

(B) 0.45

(C) 0.50

(D) 0.55

(E) 0.60

6. Insurance losses are a compound Poisson process where:

(i) The approvals of insurance applications arise in accordance with a Poisson process at a rate of 1000 per day.

(ii) Each approved application has a 20% chance of being from a smoker and an 80% chance of being from a non-smoker.

(iii) The insurances are priced so that the expected loss on each approval is -100.

(iv) The variance of the loss amount is 5000 for a smoker and is 8000 for a non-smoker.

Calculate the variance for the total losses on one day's approvals.

(A) 13,000,000

(B) 14,100,000

(C) 15,200,000

(D) 16,300,000

(E) 17,400,000

7. Z is the present-value random variable for a whole life insurance of b payable at the moment of death of (x).

You are given:

(i) $\delta = 0.04$

(ii) $\mu_x(t) = 0.02, \quad t \geq 0$

(iii) The single benefit premium for this insurance is equal to Var(Z).

Calculate b.

(A) 2.75

(B) 3.00

(C) 3.25

(D) 3.50

(E) 3.75

8. For a special 3-year term insurance on (30), you are given:

(i) Premiums are payable semiannually.

(ii) Premiums are payable only in the first year.

(iii) Benefits, payable at the end of the year of death, are:

k	b_{k+1}
0	1000
1	500
2	250

(iv) Mortality follows the Illustrative Life Table.

(v) Deaths are uniformly distributed within each year of age.

(vi) $i = 0.06$

Calculate the amount of each semiannual benefit premium for this insurance.

(A) 1.3

(B) 1.4

(C) 1.5

(D) 1.6

(E) 1.7

9. A loss, *X*, follows a 2-parameter Pareto distribution with $\alpha = 2$ and unspecified parameter θ. You are given:

$$\mathrm{E}\left[X - 100 \middle| X > 100\right] = \frac{5}{3}\mathrm{E}\left[X - 50 \middle| X > 50\right]$$

Calculate $\mathrm{E}\left[X - 150 \middle| X > 150\right]$.

(A) 150

(B) 175

(C) 200

(D) 225

(E) 250

10. The scores on the final exam in Ms. B's Latin class have a normal distribution with mean θ and standard deviation equal to 8. θ is a random variable with a normal distribution with mean equal to 75 and standard deviation equal to 6.

Each year, Ms. B chooses a student at random and pays the student 1 times the student's score. However, if the student fails the exam (score ≤ 65), then there is no payment.

Calculate the conditional probability that the payment is less than 90, given that there is a payment.

(A) 0.77

(B) 0.85

(C) 0.88

(D) 0.92

(E) 1.00

11. For a Markov model with three states, Healthy (0), Disabled (1), and Dead (2):

(i) The annual transition matrix is given by

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & \quad 1 & \quad 2 \end{array} \\ \begin{bmatrix} 0.70 & 0.20 & 0.10 \\ 0.10 & 0.65 & 0.25 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

(ii) There are 100 lives at the start, all Healthy. Their future states are independent.

Calculate the variance of the number of the original 100 lives who die within the first two years.

(A) 11

(B) 14

(C) 17

(D) 20

(E) 23

12. An insurance company issues a special 3-year insurance to a high risk individual. You are given the following homogenous Markov chain model:

(i) State 1: active
State 2: disabled
State 3: withdrawn
State 4: dead

Transition probability matrix:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0.4 & 0.2 & 0.3 & 0.1 \\ 0.2 & 0.5 & 0 & 0.3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(ii) Changes in state occur at the end of the year.

(iii) The death benefit is 1000, payable at the end of the year of death.

(iv) $i = 0.05$

(v) The insured is disabled at the end of year 1.

Calculate the actuarial present value of the prospective death benefits at the beginning of year 2.

(A) 440

(B) 528

(C) 634

(D) 712

(E) 803

13. For a fully discrete whole life insurance of b on (x), you are given:

(i) $q_{x+9} = 0.02904$

(ii) $i = 0.03$

(iii) The initial benefit reserve for policy year 10 is 343.

(iv) The net amount at risk for policy year 10 is 872.

(v) $\ddot{a}_x = 14.65976$

Calculate the terminal benefit reserve for policy year 9.

(A) 280

(B) 288

(C) 296

(D) 304

(E) 312

14. For a special fully discrete 2-year endowment insurance of 1000 on (x), you are given:

(i) The first year benefit premium is 668.

(ii) The second year benefit premium is 258.

(iii) $d = 0.06$

Calculate the level annual premium using the equivalence principle.

(A) 469

(B) 479

(C) 489

(D) 499

(E) 509

15. For an increasing 10-year term insurance, you are given:

(i) $b_{k+1} = 100,000(1+k)$, $k = 0, 1, \ldots, 9$

(ii) Benefits are payable at the end of the year of death.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) The single benefit premium for this insurance on (41) is 16,736.

Calculate the single benefit premium for this insurance on (40).

(A) 12,700

(B) 13,600

(C) 14,500

(D) 15,500

(E) 16,300

16. For a fully discrete whole life insurance of 1000 on (x):

(i) Death is the only decrement.

(ii) The annual benefit premium is 80.

(iii) The annual contract premium is 100.

(iv) Expenses in year 1, payable at the start of the year, are 40% of contract premiums.

(v) $i = 0.10$

(vi) $1000\,{}_1V_x = 40$

Calculate the asset share at the end of the first year.

(A) 17

(B) 18

(C) 19

(D) 20

(E) 21

17. For a collective risk model the number of losses, X, has a Poisson distribution with $\lambda = 20$. The common distribution of the individual losses has the following characteristics:

(i) $\mathrm{E}\left[X\right] = 70$

(ii) $\mathrm{E}\left[X \wedge 30\right] = 25$

(iii) $\Pr\left(X > 30\right) = 0.75$

(iv) $\mathrm{E}\left[X^2 \middle| X > 30\right] = 9000$

An insurance covers aggregate losses subject to an ordinary deductible of 30 per loss.

Calculate the variance of the aggregate payments of the insurance.

(A) 54,000

(B) 67,500

(C) 81,000

(D) 94,500

(E) 108,000

18. For a collective risk model:

(i) The number of losses has a Poisson distribution with $\lambda = 2$.

(ii) The common distribution of the individual losses is:

x	$f_x(x)$
1	0.6
2	0.4

An insurance covers aggregate losses subject to a deductible of 3.

Calculate the expected aggregate payments of the insurance.

(A) 0.74

(B) 0.79

(C) 0.84

(D) 0.89

(E) 0.94

19. A discrete probability distribution has the following properties:

(i) $p_k = c\left(1+\frac{1}{k}\right)p_{k-1}$ for $k = 1, 2, \ldots$

(ii) $p_0 = 0.5$

Calculate c.

(A) 0.06

(B) 0.13

(C) 0.29

(D) 0.35

(E) 0.40

20. A fully discrete 3-year term insurance of 10,000 on (40) is based on a double-decrement model, death and withdrawal:

(i) Decrement 1 is death.

(ii) $\mu_{40}^{(1)}(t) = 0.02$, $t \geq 0$

(iii) Decrement 2 is withdrawal, which occurs at the end of the year.

(iv) $q'^{(2)}_{40+k} = 0.04$, $k = 0, 1, 2$

(v) $v = 0.95$

Calculate the actuarial present value of the death benefits for this insurance.

(A) 487

(B) 497

(C) 507

(D) 517

(E) 527

21. You are given:

(i) $\overset{\circ}{e}_{30:\overline{40}|} = 27.692$

(ii) $s(x) = 1 - \frac{x}{\omega}$, $\quad 0 \le x \le \omega$

(iii) $T(x)$ is the future lifetime random variable for (x).

Calculate $\text{Var}(T(30))$.

(A) 332

(B) 352

(C) 372

(D) 392

(E) 412

22. For a fully discrete 5-payment 10-year decreasing term insurance on (60), you are given:

(i) $b_{k+1} = 1000\left(10 - k\right), \quad k = 0, 1, 2, \ldots, 9$

(ii) Level benefit premiums are payable for five years and equal 218.15 each.

(iii) $q_{60+k} = 0.02 + 0.001\,k, \quad k = 0, 1, 2, \ldots, 9$

(iv) $i = 0.06$

Calculate ${}_2V$, the benefit reserve at the end of year 2.

(A) 70

(B) 72

(C) 74

(D) 76

(E) 78

23. You are given:

(i) $T(x)$ and $T(y)$ are not independent.

(ii) $q_{x+k} = q_{y+k} = 0.05$, $k = 0, 1, 2, \ldots$

(iii) ${}_k p_{xy} = 1.02\ {}_k p_x\ {}_k p_y$, $k = 1, 2, 3 \ldots$

Into which of the following ranges does $e_{\overline{x:y}}$, the curtate expectation of life of the last survivor status, fall?

(A) $e_{\overline{x:y}} \le 25.7$

(B) $25.7 < e_{\overline{x:y}} \le 26.7$

(C) $26.7 < e_{\overline{x:y}} \le 27.7$

(D) $27.7 < e_{\overline{x:y}} \le 28.7$

(E) $28.7 < e_{\overline{x:y}}$

24. Subway trains arrive at your station at a Poisson rate of 20 per hour. 25% of the trains are express and 75% are local. The types and number of trains arriving are independent. An express gets you to work in 16 minutes and a local gets you there in 28 minutes. You always take the first train to arrive. Your co-worker always takes the first express. You are both waiting at the same station.

Calculate the conditional probability that you arrive at work before your co-worker, given that a local arrives first.

(A) 37%

(B) 40%

(C) 43%

(D) 46%

(E) 49%

25. Beginning with the first full moon in October deer are hit by cars at a Poisson rate of 20 per day. The time between when a deer is hit and when it is discovered by highway maintenance has an exponential distribution with a mean of 7 days. The number hit and the times until they are discovered are independent.

Calculate the expected number of deer that will be discovered in the first 10 days following the first full moon in October.

(A) 78

(B) 82

(C) 86

(D) 90

(E) 94

26. You are given:

(i) $\mu_x(t) = 0.03$, $t \ge 0$

(ii) $\delta = 0.05$

(iii) $T(x)$ is the future lifetime random variable.

(iv) g is the standard deviation of $\bar{a}_{\overline{T(x)}|}$.

Calculate $\Pr\left(\bar{a}_{\overline{T(x)}|} > \bar{a}_x - g\right)$.

(A) 0.53

(B) 0.56

(C) 0.63

(D) 0.68

(E) 0.79

27. (50) is an employee of XYZ Corporation. Future employment with XYZ follows a double decrement model:

(i) Decrement 1 is retirement.

(ii) $\mu_{50}^{(1)}(t) = \begin{cases} 0.00 & 0 \le t < 5 \\ 0.02 & 5 \le t \end{cases}$

(iii) Decrement 2 is leaving employment with XYZ for all other causes.

(iv) $\mu_{50}^{(2)}(t) = \begin{cases} 0.05 & 0 \le t < 5 \\ 0.03 & 5 \le t \end{cases}$

(v) If (50) leaves employment with XYZ, he will never rejoin XYZ.

Calculate the probability that (50) will retire from XYZ before age 60.

(A) 0.069

(B) 0.074

(C) 0.079

(D) 0.084

(E) 0.089

28. For a life table with a one-year select period, you are given:

(i)

x	$l_{[x]}$	$d_{[x]}$	l_{x+1}	$\mathring{e}_{[x]}$
80	1000	90	–	8.5
81	920	90	–	–

(ii) Deaths are uniformly distributed over each year of age.

Calculate $\mathring{e}_{[81]}$.

(A) 8.0

(B) 8.1

(C) 8.2

(D) 8.3

(E) 8.4

29. For a fully discrete 3-year endowment insurance of 1000 on (x):

(i) $i = 0.05$

(ii) $p_x = p_{x+1} = 0.7$

Calculate the second year terminal benefit reserve.

(A) 526

(B) 632

(C) 739

(D) 845

(E) 952

30. For a fully discrete whole life insurance of 1000 on (50), you are given:

(i) The annual per policy expense is 1.

(ii) There is an additional first year expense of 15.

(iii) The claim settlement expense of 50 is payable when the claim is paid.

(iv) All expenses, except the claim settlement expense, are paid at the beginning of the year.

(v) Mortality follows De Moivre’s law with $\omega = 100$.

(vi) $i = 0.05$

Calculate the level expense-loaded premium using the equivalence principle.

(A) 27

(B) 28

(C) 29

(D) 30

(E) 31

31. The repair costs for boats in a marina have the following characteristics:

Boat type	Number of boats	Probability that repair is needed	Mean of repair cost given a repair	Variance of repair cost given a repair
Power boats	100	0.3	300	10,000
Sailboats	300	0.1	1000	400,000
Luxury yachts	50	0.6	5000	2,000,000

At most one repair is required per boat each year.

The marina budgets an amount, Y, equal to the aggregate mean repair costs plus the standard deviation of the aggregate repair costs.

Calculate Y.

(A) 200,000

(B) 210,000

(C) 220,000

(D) 230,000

(E) 240,000

32. For an insurance:

(i) Losses can be 100, 200 or 300 with respective probabilities 0.2, 0.2, and 0.6.

(ii) The insurance has an ordinary deductible of 150 per loss.

(iii) Y^P is the claim payment per payment random variable.

Calculate $\operatorname{Var}\left(Y^P\right)$.

(A) 1500

(B) 1875

(C) 2250

(D) 2625

(E) 3000

33. You are given:

$$\mu(x)=\begin{cases}0.05 & 50\le x<60\\ 0.04 & 60\le x<70\end{cases}$$

Calculate ${}_{4|14}q_{50}$.

(A) 0.38

(B) 0.39

(C) 0.41

(D) 0.43

(E) 0.44

34. The distribution of a loss, X, is a two-point mixture:

(i) With probability 0.8, X has a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 100$.

(ii) With probability 0.2, X has a two-parameter Pareto distribution with $\alpha = 4$ and $\theta = 3000$.

Calculate $\Pr(X \leq 200)$.

(A) 0.76

(B) 0.79

(C) 0.82

(D) 0.85

(E) 0.88

35. For a special fully discrete 5-year deferred whole life insurance of 100,000 on (40), you are given:

(i) The death benefit during the 5-year deferral period is return of benefit premiums paid without interest.

(ii) Annual benefit premiums are payable only during the deferral period.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) $(IA)^{1}_{40:\overline{5}|} = 0.04042$

Calculate the annual benefit premiums.

(A) 3300

(B) 3320

(C) 3340

(D) 3360

(E) 3380

36. You are pricing a special 3-year annuity-due on two independent lives, both age 80. The annuity pays 30,000 if both persons are alive and 20,000 if only one person is alive.

You are given:

(i)

k	${}_k p_{80}$
1	0.91
2	0.82
3	0.72

(ii) $i = 0.05$

Calculate the actuarial present value of this annuity.

(A) 78,300

(B) 80,400

(C) 82,500

(D) 84,700

(E) 86,800

37. Company ABC sets the contract premium for a continuous life annuity of 1 per year on (x) equal to the single benefit premium calculated using:

(i) $\delta = 0.03$

(ii) $\mu_x(t) = 0.02$, $t \geq 0$

However, a revised mortality assumption reflects future mortality improvement and is given by

$$\mu_x(t) = \begin{cases} 0.02 & \text{for } t \leq 10 \\ 0.01 & \text{for } t > 10 \end{cases}$$

Calculate the expected loss at issue for ABC (using the revised mortality assumption) as a percentage of the contract premium.

(A) 2%

(B) 8%

(C) 15%

(D) 20%

(E) 23%

38. A group of 1000 lives each age 30 sets up a fund to pay 1000 at the end of the first year for each member who dies in the first year, and 500 at the end of the second year for each member who dies in the second year. Each member pays into the fund an amount equal to the single benefit premium for a special 2-year term insurance, with:

(i) Benefits:

k	b_{k+1}
0	1000
1	500

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

The actual experience of the fund is as follows:

k	Interest Rate Earned	Number of Deaths
0	0.070	1
1	0.069	1

Calculate the difference, at the end of the second year, between the expected size of the fund as projected at time 0 and the actual fund.

(A) 840

(B) 870

(C) 900

(D) 930

(E) 960

39. In a certain town the number of common colds an individual will get in a year follows a Poisson distribution that depends on the individual's age and smoking status. The distribution of the population and the mean number of colds are as follows:

	Proportion of population	Mean number of colds
Children	0.30	3
Adult Non-Smokers	0.60	1
Adult Smokers	0.10	4

Calculate the conditional probability that a person with exactly 3 common colds in a year is an adult smoker.

(A) 0.12

(B) 0.16

(C) 0.20

(D) 0.24

(E) 0.28

40. For aggregate losses, S:

(i) The number of losses has a negative binomial distribution with mean 3 and variance 3.6.

(ii) The common distribution of the independent individual loss amounts is uniform from 0 to 20.

Calculate the 95th percentile of the distribution of S as approximated by the normal distribution.

(A) 61

(B) 63

(C) 65

(D) 67

(E) 69

****END OF EXAMINATION****

Exam M
Spring 2005

PRELIMINARY ANSWER KEY

Question #	*Answer*		*Question #*	*Answer*
1	B		21	B
2	E		22	E
3	B		23	D
4	D		24	A
5	D		25	E
6	E		26	E
7	E		27	A
8	A		28	C
9	B		29	A
10	D		30	E
11	C		31	B
12	A		32	B
13	C		33	A
14	B		34	A
15	D		35	D
16	A		36	B
17	B		37	C
18	A		38	C
19	C		39	B
20	C		40	C