SOCIETY OF ACTUARIES

EXAM M ACTUARIAL MODELS

EXAM M SAMPLE QUESTIONS

Some of the questions in this study note are taken from past SOA examinations.

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1. For two independent lives now age 30 and 34, you are given:

x	q_x
30	0.1
31	0.2
32	0.3
33	0.4
34	0.5
35	0.6
36	0.7
37	0.8

Calculate the probability that the last death of these two lives will occur during the 3rd year from now (i.e. ${}_{2|}q_{\overline{30:34}}$).

(A) 0.01

(B) 0.03

(C) 0.14

(D) 0.18

(E) 0.24

2. For a whole life insurance of 1000 on (x) with benefits payable at the moment of death:

(i) $\delta_t = \begin{cases} 0.04, & 0 < t \le 10 \\ 0.05, & 10 < t \end{cases}$

(ii) $\mu_x(t) = \begin{cases} 0.06, & 0 < t \le 10 \\ 0.07, & 10 < t \end{cases}$

Calculate the single benefit premium for this insurance.

(A) 379

(B) 411

(C) 444

(D) 519

(E) 594

3. A health plan implements an incentive to physicians to control hospitalization under which the physicians will be paid a bonus B equal to c times the amount by which total hospital claims are under 400 $(0 \le c \le 1)$.

The effect the incentive plan will have on underlying hospital claims is modeled by assuming that the new total hospital claims will follow a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 300$.

$\mathrm{E}(B) = 100$

Calculate c.

(A) 0.44

(B) 0.48

(C) 0.52

(D) 0.56

(E) 0.60

4. Computer maintenance costs for a department are modeled as follows:

(i) The distribution of the number of maintenance calls each machine will need in a year is Poisson with mean 3.

(ii) The cost for a maintenance call has mean 80 and standard deviation 200.

(iii) The number of maintenance calls and the costs of the maintenance calls are all mutually independent.

The department must buy a maintenance contract to cover repairs if there is at least a 10% probability that aggregate maintenance costs in a given year will exceed 120% of the expected costs.

Using the normal approximation for the distribution of the aggregate maintenance costs, calculate the minimum number of computers needed to avoid purchasing a maintenance contract.

(A) 80

(B) 90

(C) 100

(D) 110

(E) 120

5. A whole life policy provides that upon accidental death as a passenger on an airplane a benefit of 1,000,000 will be paid. If death occurs from other accidental causes, a death benefit of 500,000 will be paid. If death occurs from a cause other than an accident, a death benefit of 250,000 will be paid.

You are given:

(i) Death benefits are payable at the moment of death.

(ii) $\mu^{(1)} = 1/2,000,000$ where (1) indicates accidental death as a passenger on an airplane.

(iii) $\mu^{(2)} = 1/250,000$ where (2) indicates death from other accidental causes.

(iv) $\mu^{(3)} = 1/10,000$ where (3) indicates non-accidental death.

(v) $\delta = 0.06$

Calculate the single benefit premium for this insurance.

(A) 450

(B) 460

(C) 470

(D) 480

(E) 490

6. For a special fully discrete whole life insurance of 1000 on (40):

(i) The level benefit premium for each of the first 20 years is π.

(ii) The benefit premium payable thereafter at age x is $1000 v q_x$, $x = 60, 61, 62, \ldots$

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

Calculate π.

(A) 4.79

(B) 5.11

(C) 5.34

(D) 5.75

(E) 6.07

7. For an annuity payable semiannually, you are given:

(i) Deaths are uniformly distributed over each year of age.

(ii) $q_{69} = 0.03$

(iii) $i = 0.06$

(iv) $1000\bar{A}_{70} = 530$

Calculate $\ddot{a}_{69}^{(2)}$.

(A) 8.35

(B) 8.47

(C) 8.59

(D) 8.72

(E) 8.85

8. For a sequence, $u(k)$ is defined by the following recursion formula

$$u(k) = \alpha(k) + \beta(k) \times u(k-1) \text{ for } k = 1, 2, 3, \ldots$$

(i) $\alpha(k) = -\left(\dfrac{q_{k-1}}{p_{k-1}}\right)$

(ii) $\beta(k) = \dfrac{1+i}{p_{k-1}}$

(iii) $u(70) = 1.0$

Which of the following is equal to $u(40)$?

(A) A_{30}

(B) A_{40}

(C) $A_{40:\overline{30|}}$

(D) $A^{1}_{40:\overline{30|}}$

(E) $A_{40:\overset{1}{\overline{30|}}}$

9. Subway trains arrive at a station at a Poisson rate of 20 per hour. 25% of the trains are express and 75% are local. The type of each train is independent of the types of preceding trains. An express gets you to the stop for work in 16 minutes and a local gets you there in 28 minutes. You always take the first train to arrive. Your co-worker always takes the first express. You both are waiting at the same station.

Calculate the probability that the train you take will arrive at the stop for work before the train your co-worker takes.

(A) 0.28

(B) 0.37

(C) 0.50

(D) 0.56

(E) 0.75

10. For a fully discrete whole life insurance of 1000 on (40), the contract premium is the level annual benefit premium based on the mortality assumption at issue. At time 10, the actuary decides to increase the mortality rates for ages 50 and higher.

You are given:

(i) $d = 0.05$

(ii) Mortality assumptions:

At issue	${}_{k\mid}q_{40} = 0.02,\ \ k = 0,1,2,...,49$
Revised prospectively at time 10	${}_{k\mid}q_{50} = 0.04,\ \ k = 0,1,2,...,24$

(iii) ${}_{10}L$ is the prospective loss random variable at time 10 using the contract premium.

Calculate $\mathrm{E}[{}_{10}L \mid K(40) \geq 10]$ using the revised mortality assumption.

(A) Less than 225

(B) At least 225, but less than 250

(C) At least 250, but less than 275

(D) At least 275, but less than 300

(E) At least 300

11. For a group of individuals all age x, of which 30% are smokers and 70% are non-smokers, you are given:

(i) $\delta = 0.10$

(ii) $\bar{A}_x^{\text{smoker}} = 0.444$

(iii) $\bar{A}_x^{\text{non-smoker}} = 0.286$

(iv) T is the future lifetime of (x).

(v) $\text{Var}\left[\bar{a}_{\overline{T|}}^{\text{smoker}}\right] = 8.818$

(vi) $\text{Var}\left[\bar{a}_{\overline{T|}}^{\text{non-smoker}}\right] = 8.503$

Calculate $\text{Var}\left[\bar{a}_{\overline{T|}}\right]$ for an individual chosen at random from this group.

(A) 8.5

(B) 8.6

(C) 8.8

(D) 9.0

(E) 9.1

12. T, the future lifetime of (0), has a spliced distribution.

(i) $f_1(t)$ follows the Illustrative Life Table.

(ii) $f_2(t)$ follows DeMoivre's law with $\omega = 100$.

(iii) $f_T(t) = \begin{cases} k\, f_1(t), & 0 \le t \le 50 \\ 1.2\, f_2(t), & 50 < t \end{cases}$

Calculate ${}_{10}p_{40}$.

(A) 0.81

(B) 0.85

(C) 0.88

(D) 0.92

(E) 0.96

13. A population has 30% who are smokers with a constant force of mortality 0.2 and 70% who are non-smokers with a constant force of mortality 0.1.

Calculate the 75th percentile of the distribution of the future lifetime of an individual selected at random from this population.

(A) 10.7

(B) 11.0

(C) 11.2

(D) 11.6

(E) 11.8

14. Aggregate losses for a portfolio of policies are modeled as follows:

(i) The number of losses before any coverage modifications follows a Poisson distribution with mean λ.

(ii) The severity of each loss before any coverage modifications is uniformly distributed between 0 and b.

The insurer would like to model the impact of imposing an ordinary deductible, $d\,(0 < d < b)$, on each loss and reimbursing only a percentage, $c\ (0 < c \le 1)$, of each loss in excess of the deductible.

It is assumed that the coverage modifications will not affect the loss distribution.
The insurer models its claims with modified frequency and severity distributions. The modified claim amount is uniformly distributed on the interval $\left[0, c(b-d)\right]$.

Determine the mean of the modified frequency distribution.

(A) λ

(B) λc

(C) $\lambda \frac{d}{b}$

(D) $\lambda \frac{b-d}{b}$

(E) $\lambda c \frac{b-d}{b}$

15. The RIP Life Insurance Company specializes in selling a fully discrete whole life insurance of 10,000 to 65 year olds by telephone. For each policy:

(i) The annual contract premium is 500.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$

The number of telephone inquiries RIP receives follows a Poisson process with mean 50 per day. 20% of the inquiries result in the sale of a policy.

The number of inquiries and the future lifetimes of all the insureds who purchase policies on a particular day are independent.

Using the normal approximation, calculate the probability that S, the total prospective loss at issue for all the policies sold on a particular day, will be less than zero.

(A) 0.33

(B) 0.50

(C) 0.67

(D) 0.84

(E) 0.99

16. For a special fully discrete whole life insurance on (40):

(i) The death benefit is 1000 for the first 20 years; 5000 for the next 5 years; 1000 thereafter.

(ii) The annual benefit premium is $1000P_{40}$ for the first 20 years; $5000P_{40}$ for the next 5 years; π thereafter.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

Calculate ${}_{21}V$, the benefit reserve at the end of year 21 for this insurance.

(A) 255

(B) 259

(C) 263

(D) 267

(E) 271

17. For a whole life insurance of 1 on (41) with death benefit payable at the end of year of death, you are given:

(i) $i = 0.05$

(ii) $p_{40} = 0.9972$

(iii) $A_{41} - A_{40} = 0.00822$

(iv) ${}^2A_{41} - {}^2A_{40} = 0.00433$

(v) Z is the present-value random variable for this insurance.

Calculate Var(Z).

(A) 0.023

(B) 0.024

(C) 0.025

(D) 0.026

(E) 0.027

18. For a perpetuity-immediate with annual payments of 1:

(i) The sequence of annual discount factors follows a Markov chain with the following three states:

State number	0	1	2
Annual discount factor, *v*	0.95	0.94	0.93

(ii) The transition matrix for the annual discount factors is:

$$\begin{bmatrix} 0.0 & 1.0 & 0.0 \\ 0.9 & 0.0 & 0.1 \\ 0.0 & 1.0 & 0.0 \end{bmatrix}$$

Y is the present value of the perpetuity payments when the initial state is 1.

Calculate E(Y).

(A) 15.67

(B) 15.71

(C) 15.75

(D) 16.82

(E) 16.86

19. A member of a high school math team is practicing for a contest. Her advisor has given her three practice problems: #1, #2, and #3.

She randomly chooses one of the problems, and works on it until she solves it. Then she randomly chooses one of the remaining unsolved problems, and works on it until solved. Then she works on the last unsolved problem.

She solves problems at a Poisson rate of 1 problem per 5 minutes.

Calculate the probability that she has solved problem #3 within 10 minutes of starting the problems.

(A) 0.18

(B) 0.34

(C) 0.45

(D) 0.51

(E) 0.59

20. For a double decrement table, you are given:

(i) $\mu_x^{(1)}(t) = 0.2\,\mu_x^{(\tau)}(t), \quad t > 0$

(ii) $\mu_x^{(\tau)}(t) = k\,t^2, \quad t > 0$

(iii) $q_x'^{(1)} = 0.04$

Calculate ${}_2q_x^{(2)}$.

(A) 0.45

(B) 0.53

(C) 0.58

(D) 0.64

(E) 0.73

21. For (x):

(i) K is the curtate future lifetime random variable.

(ii) $q_{x+k} = 0.1(k+1)$, $k = 0, 1, 2, \ldots, 9$

Calculate $\mathrm{Var}(K \wedge 3)$.

(A) 1.1

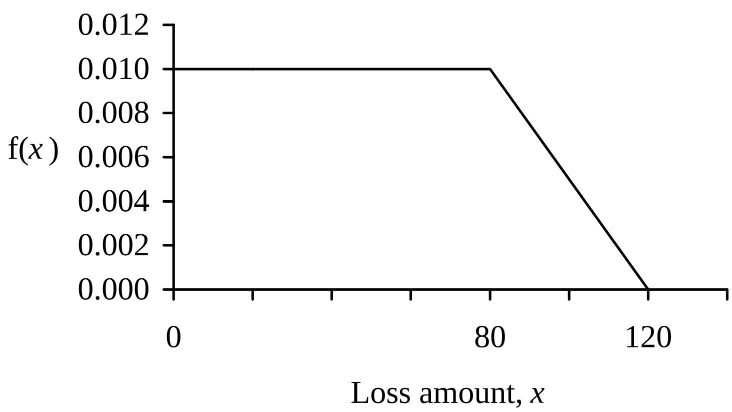
(B) 1.2

(C) 1.3

(D) 1.4

(E) 1.5

22. The graph of the density function for losses is:



Calculate the loss elimination ratio for an ordinary deductible of 20.

(A) 0.20

(B) 0.24

(C) 0.28

(D) 0.32

(E) 0.36

23. Michel, age 45, is expected to experience higher than standard mortality only at age 64. For a special fully discrete whole life insurance of 1 on Michel, you are given:

(i) The benefit premiums are not level.

(ii) The benefit premium for year 20, π_{19}, exceeds P_{45} for a standard risk by 0.010.

(iii) Benefit reserves on his insurance are the same as benefit reserves for a fully discrete whole life insurance of 1 on (45) with standard mortality and level benefit premiums.

(iv) $i = 0.03$

(v) ${}_{20}V_{45} = 0.427$

Calculate the excess of q_{64} for Michel over the standard q_{64}.

(A) 0.012

(B) 0.014

(C) 0.016

(D) 0.018

(E) 0.020

24. For a block of fully discrete whole life insurances of 1 on independent lives age x, you are given:

(i) $i = 0.06$

(ii) $A_x = 0.24905$

(iii) ${}^2A_x = 0.09476$

(iv) $\pi = 0.025$, where π is the contract premium for each policy.

(v) Losses are based on the contract premium.

Using the normal approximation, calculate the minimum number of policies the insurer must issue so that the probability of a positive total loss on the policies issued is less than or equal to 0.05.

(A) 25

(B) 27

(C) 29

(D) 31

(E) 33

25. Your company currently offers a whole life annuity product that pays the annuitant 12,000 at the beginning of each year. A member of your product development team suggests enhancing the product by adding a death benefit that will be paid at the end of the year of death.

Using a discount rate, d, of 8%, calculate the death benefit that minimizes the variance of the present value random variable of the new product.

(A) 0

(B) 50,000

(C) 100,000

(D) 150,000

(E) 200,000

26. A towing company provides all towing services to members of the City Automobile Club. You are given:

Towing Distance	Towing Cost	Frequency
0-9.99 miles	80	50%
10-29.99 miles	100	40%
30+ miles	160	10%

(i) The automobile owner must pay 10% of the cost and the remainder is paid by the City Automobile Club.

(ii) The number of towings has a Poisson distribution with mean of 1000 per year.

(iii) The number of towings and the costs of individual towings are all mutually independent.

Using the normal approximation for the distribution of aggregate towing costs, calculate the probability that the City Automobile Club pays more than 90,000 in any given year.

(A) 3%

(B) 10%

(C) 50%

(D) 90%

(E) 97%

27. You are given:

(i) Losses follow an exponential distribution with the same mean in all years.

(ii) The loss elimination ratio this year is 70%.

(iii) The ordinary deductible for the coming year is 4/3 of the current deductible.

Compute the loss elimination ratio for the coming year.

(A) 70%

(B) 75%

(C) 80%

(D) 85%

(E) 90%

28. For T, the future lifetime random variable for (0):

(i) $\omega > 70$

(ii) ${}_{40}p_0 = 0.6$

(iii) $E(T) = 62$

(iv) $E(T \wedge t) = t - 0.005t^2, \quad 0 < t < 60$

Calculate the complete expectation of life at 40.

(A) 30

(B) 35

(C) 40

(D) 45

(E) 50

29. Two actuaries use the same mortality table to price a fully discrete 2-year endowment insurance of 1000 on (x).

(i) Kevin calculates non-level benefit premiums of 608 for the first year and 350 for the second year.

(ii) Kira calculates level annual benefit premiums of π.

(iii) $d = 0.05$

Calculate π.

(A) 482

(B) 489

(C) 497

(D) 508

(E) 517

30. For a fully discrete 10-payment whole life insurance of 100,000 on (x), you are given:

(i) $i = 0.05$

(ii) $q_{x+9} = 0.011$

(iii) $q_{x+10} = 0.012$

(iv) $q_{x+11} = 0.014$

(v) The level annual benefit premium is 2078.

(vi) The benefit reserve at the end of year 9 is 32,535.

Calculate $100,000 A_{x+11}$.

(A) 34,100

(B) 34,300

(C) 35,500

(D) 36,500

(E) 36,700

31. You are given:

(i) Mortality follows DeMoivre's law with $\omega = 105$.

(ii) (45) and (65) have independent future lifetimes.

Calculate $\mathring{e}_{\overline{45:65}}$.

(A) 33

(B) 34

(C) 35

(D) 36

(E) 37

32. Given: The survival function $s(x)$, where

$$s(x) = 1, \qquad 0 \le x < 1$$

$$s(x) = 1 - \left\{ \left(e^{x}\right) / 100 \right\}, \qquad 1 \le x < 4.5$$

$$s(x) = 0, \qquad 4.5 \le x$$

Calculate $\mu(4)$.

(A) 0.45

(B) 0.55

(C) 0.80

(D) 1.00

(E) 1.20

33. For a triple decrement table, you are given:

(i) $\mu_x^{(1)}(t) = 0.3, \quad t > 0$

(ii) $\mu_x^{(2)}(t) = 0.5, \quad t > 0$

(iii) $\mu_x^{(3)}(t) = 0.7, \quad t > 0$

Calculate $q_x^{(2)}$.

(A) 0.26

(B) 0.30

(C) 0.33

(D) 0.36

(E) 0.39

34. You are given:

(i) the following select-and-ultimate mortality table with 3-year select period:

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
60	0.09	0.11	0.13	0.15	63
61	0.10	0.12	0.14	0.16	64
62	0.11	0.13	0.15	0.17	65
63	0.12	0.14	0.16	0.18	66
64	0.13	0.15	0.17	0.19	67

(ii) $i = 0.03$

Calculate ${}_{2|2}A_{[60]}$, the actuarial present value of a 2-year deferred 2-year term insurance on $[60]$.

(A) 0.156

(B) 0.160

(C) 0.186

(D) 0.190

(E) 0.195

35. You are given:

(i) $\mu_x(t) = 0.01, \qquad 0 \le t < 5$

(ii) $\mu_x(t) = 0.02, \qquad 5 \le t$

(iii) $\delta = 0.06$

Calculate $\bar{a}_x$.

(A) 12.5

(B) 13.0

(C) 13.4

(D) 13.9

(E) 14.3

36. Actuaries have modeled auto windshield claim frequencies. They have concluded that the number of windshield claims filed per year per driver follows the Poisson distribution with parameter λ, where λ follows the gamma distribution with mean 3 and variance 3.

Calculate the probability that a driver selected at random will file no more than 1 windshield claim next year.

(A) 0.15

(B) 0.19

(C) 0.20

(D) 0.24

(E) 0.31

37. The number of auto vandalism claims reported per month at Sunny Daze Insurance Company (SDIC) has mean 110 and variance 750. Individual losses have mean 1101 and standard deviation 70. The number of claims and the amounts of individual losses are independent.

Using the normal approximation, calculate the probability that SDIC's aggregate auto vandalism losses reported for a month will be less than 100,000.

(A) 0.24

(B) 0.31

(C) 0.36

(D) 0.39

(E) 0.49

38. For an allosaur with 10,000 calories stored at the start of a day:

(i) The allosaur uses calories uniformly at a rate of 5,000 per day. If his stored calories reach 0, he dies.

(ii) Each day, the allosaur eats 1 scientist (10,000 calories) with probability 0.45 and no scientist with probability 0.55.

(iii) The allosaur eats only scientists.

(iv) The allosaur can store calories without limit until needed.

Calculate the probability that the allosaur ever has 15,000 or more calories stored.

(A) 0.54

(B) 0.57

(C) 0.60

(D) 0.63

(E) 0.66

39. Lucky Tom finds coins on his way to work at a Poisson rate of 0.5 coins/minute. The denominations are randomly distributed:

(i) 60% of the coins are worth 1 each

(ii) 20% of the coins are worth 5 each

(iii) 20% of the coins are worth 10 each.

Calculate the probability that in the first ten minutes of his walk he finds at least 2 coins worth 10 each, and in the first twenty minutes finds at least 3 coins worth 10 each.

(A) 0.08

(B) 0.12

(C) 0.16

(D) 0.20

(E) 0.24

40. For a fully discrete whole life insurance of 1000 on (60), the annual benefit premium was calculated using the following:

(i) $i = 0.06$

(ii) $q_{60} = 0.01376$

(iii) $1000 A_{60} = 369.33$

(iv) $1000 A_{61} = 383.00$

A particular insured is expected to experience a first-year mortality rate ten times the rate used to calculate the annual benefit premium. The expected mortality rates for all other years are the ones originally used.

Calculate the expected loss at issue for this insured, based on the original benefit premium.

(A) 72

(B) 86

(C) 100

(D) 114

(E) 128

41. For a fully discrete whole life insurance of 1000 on (40), you are given:

(i) $i = 0.06$

(ii) Mortality follows the Illustrative Life Table.

(iii) $\ddot{a}_{40:\overline{10}|} = 7.70$

(iv) $\ddot{a}_{50:\overline{10}|} = 7.57$

(v) $1000A^{1}_{40:\overline{20}|} = 60.00$

At the end of the tenth year, the insured elects an option to retain the coverage of 1000 for life, but pay premiums for the next ten years only.

Calculate the revised annual benefit premium for the next 10 years.

(A) 11

(B) 15

(C) 17

(D) 19

(E) 21

42. For a double-decrement table where cause 1 is death and cause 2 is withdrawal, you are given:

(i) Deaths are uniformly distributed over each year of age in the single-decrement table.

(ii) Withdrawals occur only at the end of each year of age.

(iii) $l_x^{(\tau)} = 1000$

(iv) $q_x^{(2)} = 0.40$

(v) $d_x^{(1)} = 0.45\ d_x^{(2)}$

Calculate $p_x'^{(2)}$.

(A) 0.51

(B) 0.53

(C) 0.55

(D) 0.57

(E) 0.59

43. You intend to hire 200 employees for a new management-training program. To predict the number who will complete the program, you build a multiple decrement table. You decide that the following associated single decrement assumptions are appropriate:

(i) Of 40 hires, the number who fail to make adequate progress in each of the first three years is 10, 6, and 8, respectively.

(ii) Of 30 hires, the number who resign from the company in each of the first three years is 6, 8, and 2, respectively.

(iii) Of 20 hires, the number who leave the program for other reasons in each of the first three years is 2, 2, and 4, respectively.

(iv) You use the uniform distribution of decrements assumption in each year in the multiple decrement table.

Calculate the expected number who fail to make adequate progress in the third year.

(A) 4

(B) 8

(C) 12

(D) 14

(E) 17

44. Bob is an overworked underwriter. Applications arrive at his desk at a Poisson rate of 60 per day. Each application has a 1/3 chance of being a "bad" risk and a 2/3 chance of being a "good" risk.

Since Bob is overworked, each time he gets an application he flips a fair coin. If it comes up heads, he accepts the application without looking at it. If the coin comes up tails, he accepts the application if and only if it is a "good" risk. The expected profit on a "good" risk is 300 with variance 10,000. The expected profit on a "bad" risk is –100 with variance 90,000.

Calculate the variance of the profit on the applications he accepts today.

(A) 4,000,000

(B) 4,500,000

(C) 5,000,000

(D) 5,500,000

(E) 6,000,000

45. Prescription drug losses, S, are modeled assuming the number of claims has a geometric distribution with mean 4, and the amount of each prescription is 40.

Calculate $E\left[(S-100)_+\right]$.

(A) 60

(B) 82

(C) 92

(D) 114

(E) 146

46. For a temporary life annuity-immediate on independent lives (30) and (40):

(i) Mortality follows the Illustrative Life Table.

(ii) $i = 0.06$

Calculate $a_{30:40:\overline{10}|}$.

(A) 6.64

(B) 7.17

(C) 7.88

(D) 8.74

(E) 9.86

47. For a special whole life insurance on (35), you are given:

(i) The annual benefit premium is payable at the beginning of each year.

(ii) The death benefit is equal to 1000 plus the return of all benefit premiums paid in the past without interest.

(iii) The death benefit is paid at the end of the year of death.

(iv) $A_{35} = 0.42898$

(v) $(IA)_{35} = 6.16761$

(vi) $i = 0.05$

Calculate the annual benefit premium for this insurance.

(A) 73.66

(B) 75.28

(C) 77.42

(D) 78.95

(E) 81.66

48. Subway trains arrive at a station at a Poisson rate of 20 per hour. 25% of the trains are express and 75% are local. The types of each train are independent. An express gets you to work in 16 minutes and a local gets you there in 28 minutes. You always take the first train to arrive. Your co-worker always takes the first express. You both are waiting at the same station.

Which of the following is true?

(A) Your expected arrival time is 6 minutes earlier than your co-worker's.

(B) Your expected arrival time is 4.5 minutes earlier than your co-worker's.

(C) Your expected arrival times are the same.

(D) Your expected arrival time is 4.5 minutes later than your co-worker's.

(E) Your expected arrival time is 6 minutes later than your co-worker's.

49. For a special fully continuous whole life insurance of 1 on the last-survivor of (*x*) and (*y*), you are given:

(i) $T(x)$ and $T(y)$ are independent.

(ii) $\mu_x(t) = \mu_y(t) = 0.07, \quad t > 0$

(iii) $\delta = 0.05$

(iv) Premiums are payable until the first death.

Calculate the level annual benefit premium for this insurance.

(A) 0.04

(B) 0.07

(C) 0.08

(D) 0.10

(E) 0.14

50. For a fully discrete whole life insurance of 1000 on (20), you are given:

(i) $1000\, P_{20} = 10$

(ii) $1000\ {}_{20}V_{20} = 490$

(iii) $1000\ {}_{21}V_{20} = 545$

(iv) $1000\ {}_{22}V_{20} = 605$

(v) $q_{40} = 0.022$

Calculate q_{41}.

(A) 0.024

(B) 0.025

(C) 0.026

(D) 0.027

(E) 0.028

51. For a fully discrete whole life insurance of 1000 on (60), you are given:

(i) $i = 0.06$

(ii) Mortality follows the Illustrative Life Table, except that there are extra mortality risks at age 60 such that $q_{60} = 0.015$.

Calculate the annual benefit premium for this insurance.

(A) 31.5

(B) 32.0

(C) 32.1

(D) 33.1

(E) 33.2

52. At the beginning of each round of a game of chance the player pays 12.5. The player then rolls one die with outcome *N*. The player then rolls *N* dice and wins an amount equal to the total of the numbers showing on the *N* dice. All dice have 6 sides and are fair.

Using the normal approximation, calculate the probability that a player starting with 15,000 will have at least 15,000 after 1000 rounds.

(A) 0.01

(B) 0.04

(C) 0.06

(D) 0.09

(E) 0.12

53. *X* is a discrete random variable with a probability function which is a member of the (a,b,0) class of distributions.

You are given:

(i) $P(X=0)=P(X=1)=0.25$

(ii) $P(X=2)=0.1875$

Calculate $P(X=3)$.

(A) 0.120

(B) 0.125

(C) 0.130

(D) 0.135

(E) 0.140

54. Nancy reviews the interest rates each year for a 30-year fixed mortgage issued on July 1. She models interest rate behavior by a Markov model assuming:

(i) Interest rates always change between years.

(ii) The change in any given year is dependent on the change in prior years as follows:

from year $t-3$ to year $t-2$	from year $t-2$ to year $t-1$	Probability that year t will increase from year $t-1$
Increase	Increase	0.10
Decrease	Decrease	0.20
Increase	Decrease	0.40
Decrease	Increase	0.25

She notes that interest rates decreased from year 2000 to 2001 and from year 2001 to 2002.

Calculate the probability that interest rates will decrease from year 2003 to 2004.

(A) 0.76

(B) 0.79

(C) 0.82

(D) 0.84

(E) 0.87

55. For a 20-year deferred whole life annuity-due of 1 per year on (45), you are given:

(i) Mortality follows De Moivre's law with $\omega = 105$.

(ii) $i = 0$

Calculate the probability that the sum of the annuity payments actually made will exceed the actuarial present value at issue of the annuity.

(A) 0.425

(B) 0.450

(C) 0.475

(D) 0.500

(E) 0.525

56. For a continuously increasing whole life insurance on (x), you are given:

(i) The force of mortality is constant.

(ii) $\delta = 0.06$

(iii) ${}^2\overline{A}_x = 0.25$

Calculate $\left(\overline{IA}\right)_x$.

(A) 2.889

(B) 3.125

(C) 4.000

(D) 4.667

(E) 5.500

57. XYZ Co. has just purchased two new tools with independent future lifetimes. Each tool has its own distinct De Moivre survival pattern. One tool has a 10-year maximum lifetime and the other a 7-year maximum lifetime.

Calculate the expected time until both tools have failed.

(A) 5.0

(B) 5.2

(C) 5.4

(D) 5.6

(E) 5.8

58. XYZ Paper Mill purchases a 5-year special insurance paying a benefit in the event its machine breaks down. If the cause is "minor" (1), only a repair is needed. If the cause is "major" (2), the machine must be replaced.

Given:

(i) The benefit for cause (1) is 2000 payable at the moment of breakdown.

(ii) The benefit for cause (2) is 500,000 payable at the moment of breakdown.

(iii) Once a benefit is paid, the insurance contract is terminated.

(iv) $\mu^{(1)}(t) = 0.100$ and $\mu^{(2)}(t) = 0.004$, for $t > 0$

(v) $\delta = 0.04$

Calculate the actuarial present value of this insurance.

(A) 7840

(B) 7880

(C) 7920

(D) 7960

(E) 8000

59. You are given:

(i) $R = 1 - e^{-\int_0^1 \mu_x(t)dt}$

(ii) $S = 1 - e^{-\int_0^1 (\mu_x(t)+k)dt}$

(iii) k is a constant such that $S = 0.75R$

Determine an expression for k.

(A) $1\text{n}\left((1-q_x)/(1-0.75q_x)\right)$

(B) $1\text{n}\left((1-0.75q_x)/(1-p_x)\right)$

(C) $1\text{n}\left((1-0.75p_x)/(1-p_x)\right)$

(D) $1\text{n}\left((1-p_x)/(1-0.75q_x)\right)$

(E) $1\text{n}\left((1-0.75q_x)/(1-q_x)\right)$

60. The number of claims in a period has a geometric distribution with mean 4. The amount of each claim X follows $P(X = x) = 0.25$, $x = 1,2,3,4$. The number of claims and the claim amounts are independent. S is the aggregate claim amount in the period.

Calculate $F_s(3)$.

(A) 0.27

(B) 0.29

(C) 0.31

(D) 0.33

(E) 0.35

61. Insurance agent Hunt N. Quotum will receive no annual bonus if the ratio of incurred losses to earned premiums for his book of business is 60% or more for the year. If the ratio is less than 60%, Hunt's bonus will be a percentage of his earned premium equal to 15% of the difference between his ratio and 60%. Hunt's annual earned premium is 800,000.

Incurred losses are distributed according to the Pareto distribution, with $\theta = 500{,}000$ and $\alpha = 2$.
Calculate the expected value of Hunt's bonus.

(A) 13,000

(B) 17,000

(C) 24,000

(D) 29,000

(E) 35,000

62. A large machine in the ABC Paper Mill is 25 years old when ABC purchases a 5-year term insurance paying a benefit in the event the machine breaks down.

Given:
(i) Annual benefit premiums of 6643 are payable at the beginning of the year.

(ii) A benefit of 500,000 is payable at the moment of breakdown.

(iii) Once a benefit is paid, the insurance contract is terminated.

(iv) Machine breakdowns follow De Moivre's law with $l_x = 100 - x$.

(v) $i = 0.06$

Calculate the benefit reserve for this insurance at the end of the third year.

(A) –91

(B) 0

(C) 163

(D) 287

(E) 422

63. For a whole life insurance of 1 on (x), you are given:

(i) The force of mortality is $\mu_x(t)$.

(ii) The benefits are payable at the moment of death.

(iii) $\delta = 0.06$

(iv) $\bar{A}_x = 0.60$

Calculate the revised actuarial present value of this insurance assuming $\mu_x(t)$ is increased by 0.03 for all t and δ is decreased by 0.03.

(A) 0.5

(B) 0.6

(C) 0.7

(D) 0.8

(E) 0.9

64. A maintenance contract on a hotel promises to replace burned out light bulbs at the end of each year for three years. The hotel has 10,000 light bulbs. The light bulbs are all new. If a replacement bulb burns out, it too will be replaced with a new bulb.

You are given:

(i) For new light bulbs, $q_0 = 0.10$
$q_1 = 0.30$
$q_2 = 0.50$

(ii) Each light bulb costs 1.

(iii) $i = 0.05$

Calculate the actuarial present value of this contract.

(A) 6700

(B) 7000

(C) 7300

(D) 7600

(E) 8000

65. You are given:

$$\mu(x) = \begin{cases} 0.04, & 0 < x < 40 \\ 0.05, & x > 40 \end{cases}$$

Calculate $\mathring{e}_{25:\overline{25}|}$.

(A) 14.0

(B) 14.4

(C) 14.8

(D) 15.2

(E) 15.6

66. For a select-and-ultimate mortality table with a 3-year select period:

(i)

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
60	0.09	0.11	0.13	0.15	63
61	0.10	0.12	0.14	0.16	64
62	0.11	0.13	0.15	0.17	65
63	0.12	0.14	0.16	0.18	66
64	0.13	0.15	0.17	0.19	67

(ii) White was a newly selected life on 01/01/2000.

(iii) White's age on 01/01/2001 is 61.

(iv) P is the probability on 01/01/2001 that White will be alive on 01/01/2006.

Calculate P.

(A) $0 \le P < 0.43$

(B) $0.43 \le P < 0.45$

(C) $0.45 \le P < 0.47$

(D) $0.47 \le P < 0.49$

(E) $0.49 \le P \le 1.00$

67. For a continuous whole life annuity of 1 on (x):

(i) $T(x)$ is the future lifetime random variable for (x).

(ii) The force of interest and force of mortality are constant and equal.

(iii) $\bar{a}_x = 12.50$

Calculate the standard deviation of $\bar{a}_{\overline{T(x)}|}$.

(A) 1.67

(B) 2.50

(C) 2.89

(D) 6.25

(E) 7.22

68. For a special fully discrete whole life insurance on (x):

(i) The death benefit is 0 in the first year and 5000 thereafter.

(ii) Level benefit premiums are payable for life.

(iii) $q_x = 0.05$

(iv) $v = 0.90$

(v) $\ddot{a}_x = 5.00$

(vi) ${}_{10}V_x = 0.20$

(vii) ${}_{10}V$ is the benefit reserve at the end of year 10 for this insurance.

Calculate ${}_{10}V$.

(A) 795

(B) 1000

(C) 1090

(D) 1180

(E) 1225

69. For a fully discrete 2-year term insurance of 1 on (x):

(i) 0.95 is the lowest premium such that there is a 0% chance of loss in year 1.

(ii) $p_x = 0.75$

(iii) $p_{x+1} = 0.80$

(iv) Z is the random variable for the present value at issue of future benefits.

Calculate $\mathrm{Var}(Z)$.

(A) 0.15

(B) 0.17

(C) 0.19

(D) 0.21

(E) 0.23

70. A group dental policy has a negative binomial claim count distribution with mean 300 and variance 800.

Ground-up severity is given by the following table:

Severity	Probability
40	0.25
80	0.25
120	0.25
200	0.25

You expect severity to increase 50% with no change in frequency. You decide to impose a per claim deductible of 100.

Calculate the expected total claim payment after these changes.

(A) Less than 18,000

(B) At least 18,000, but less than 20,000

(C) At least 20,000, but less than 22,000

(D) At least 22,000, but less than 24,000

(E) At least 24,000

71. You own a fancy light bulb factory. Your workforce is a bit clumsy – they keep dropping boxes of light bulbs. The boxes have varying numbers of light bulbs in them, and when dropped, the entire box is destroyed.

You are given:

Expected number of boxes dropped per month:	50
Variance of the number of boxes dropped per month:	100
Expected value per box:	200
Variance of the value per box:	400

You pay your employees a bonus if the value of light bulbs destroyed in a month is less than 8000.

Assuming independence and using the normal approximation, calculate the probability that you will pay your employees a bonus next month.

(A) 0.16

(B) 0.19

(C) 0.23

(D) 0.27

(E) 0.31

72. Each of 100 independent lives purchase a single premium 5-year deferred whole life insurance of 10 payable at the moment of death. You are given:

(i) $\mu = 0.04$

(ii) $\delta = 0.06$

(iii) F is the aggregate amount the insurer receives from the 100 lives.

Using the normal approximation, calculate F such that the probability the insurer has sufficient funds to pay all claims is 0.95.

(A) 280

(B) 390

(C) 500

(D) 610

(E) 720

73. For a select-and-ultimate table with a 2-year select period:

x	$p_{[x]}$	$p_{[x]+1}$	p_{x+2}	$x+2$
48	0.9865	0.9841	0.9713	50
49	0.9858	0.9831	0.9698	51
50	0.9849	0.9819	0.9682	52
51	0.9838	0.9803	0.9664	53

Keith and Clive are independent lives, both age 50. Keith was selected at age 45 and Clive was selected at age 50.

Calculate the probability that exactly one will be alive at the end of three years.

(A) Less than 0.115

(B) At least 0.115, but less than 0.125

(C) At least 0.125, but less than 0.135

(D) At least 0.135, but less than 0.145

(E) At least 0.145

74-75. *Use the following information for questions* 74 *and* 75.

For a tyrannosaur with 10,000 calories stored:

(i) The tyrannosaur uses calories uniformly at a rate of 10,000 per day. If his stored calories reach 0, he dies.

(ii) The tyrannosaur eats scientists (10,000 calories each) at a Poisson rate of 1 per day.

(iii) The tyrannosaur eats only scientists.

(iv) The tyrannosaur can store calories without limit until needed.

74. Calculate the probability that the tyrannosaur dies within the next 2.5 days.

(A) 0.30

(B) 0.40

(C) 0.50

(D) 0.60

(E) 0.70

75. Calculate the expected calories eaten in the next 2.5 days.

(A) 17,800

(B) 18,800

(C) 19,800

(D) 20,800

(E) 21,800

76. A fund is established by collecting an amount P from each of 100 independent lives age 70. The fund will pay the following benefits:

- 10, payable at the end of the year of death, for those who die before age 72, or

- P, payable at age 72, to those who survive.

You are given:
Mortality follows the Illustrative Life Table.

(i) $i = 0.08$
Calculate P, using the equivalence principle.

(A) 2.33

(B) 2.38

(C) 3.02

(D) 3.07

(E) 3.55

77. You are given:

(i) $P_x = 0.090$

(ii) ${}_nV_x = 0.563$

(iii) $P_{x:\overline{n}|}^{\;\;\;1} = 0.00864$

Calculate $P^1_{x:\overline{n}|}$.

(A) 0.008

(B) 0.024

(C) 0.040

(D) 0.065

(E) 0.085

78. You are given:

(i) Mortality follows De Moivre's law with $\omega = 100$.

(ii) $i = 0.05$

(iii) The following annuity-certain values:

$$\bar{a}_{\overline{40}|} = 17.58$$
$$\bar{a}_{\overline{50}|} = 18.71$$
$$\bar{a}_{\overline{60}|} = 19.40$$

Calculate ${}_{10}\bar{V}\left(\bar{A}_{40}\right)$.

(A) 0.075

(B) 0.077

(C) 0.079

(D) 0.081

(E) 0.083

79. For a group of individuals all age *x*, you are given:

(i) 30% are smokers and 70% are non-smokers.

(ii) The constant force of mortality for smokers is 0.06.

(iii) The constant force of mortality for non-smokers is 0.03.

(iv) $\delta = 0.08$

Calculate $\mathrm{Var}\left(\bar{a}_{\overline{T(x)}|}\right)$ for an individual chosen at random from this group.

(A) 13.0

(B) 13.3

(C) 13.8

(D) 14.1

(E) 14.6

80. For a certain company, losses follow a Poisson frequency distribution with mean 2 per year, and the amount of a loss is 1, 2, or 3, each with probability 1/3. Loss amounts are independent of the number of losses, and of each other.

An insurance policy covers all losses in a year, subject to an annual aggregate deductible of 2.

Calculate the expected claim payments for this insurance policy.

(A) 2.00

(B) 2.36

(C) 2.45

(D) 2.81

(E) 2.96

81. A Poisson claims process has two types of claims, Type I and Type II.

(i) The expected number of claims is 3000.

(ii) The probability that a claim is Type I is 1/3.

(iii) Type I claim amounts are exactly 10 each.

(iv) The variance of aggregate claims is 2,100,000.

Calculate the variance of aggregate claims with Type I claims excluded.

(A) 1,700,000

(B) 1,800,000

(C) 1,900,000

(D) 2,000,000

(E) 2,100,000

82. Don, age 50, is an actuarial science professor. His career is subject to two decrements:

(i) Decrement 1 is mortality. The associated single decrement table follows De Moivre's law with $\omega = 100$.

(ii) Decrement 2 is leaving academic employment, with

$$\mu_{50}^{(2)}(t) = 0.05, \quad t \geq 0$$

Calculate the probability that Don remains an actuarial science professor for at least five but less than ten years.

(A) 0.22

(B) 0.25

(C) 0.28

(D) 0.31

(E) 0.34

83. For a double decrement model:

(i) In the single decrement table associated with cause (1), $q_{40}'^{(1)} = 0.100$ and decrements are uniformly distributed over the year.

(ii) In the single decrement table associated with cause (2), $q_{40}'^{(2)} = 0.125$ and all decrements occur at time 0.7.

Calculate $q_{40}^{(2)}$.

(A) 0.114

(B) 0.115

(C) 0.116

(D) 0.117

(E) 0.118

84. For a special 2-payment whole life insurance on (80):

(i) Premiums of π are paid at the beginning of years 1 and 3.

(ii) The death benefit is paid at the end of the year of death.

(iii) There is a partial refund of premium feature:

If (80) dies in either year 1 or year 3, the death benefit is $1000+\dfrac{\pi}{2}$.

Otherwise, the death benefit is 1000.

(iv) Mortality follows the Illustrative Life Table.

(v) $i = 0.06$

Calculate π, using the equivalence principle.

(A) 369

(B) 381

(C) 397

(D) 409

(E) 425

85. For a special fully continuous whole life insurance on (65):

(i) The death benefit at time t is $b_t = 1000\,e^{0.04t}, \quad t \ge 0$.

(ii) Level benefit premiums are payable for life.

(iii) $\mu_{65}(t) = 0.02, \quad t \ge 0$

(iv) $\delta = 0.04$

Calculate ${}_2\bar{V}$, the benefit reserve at the end of year 2.

(A) 0

(B) 29

(C) 37

(D) 61

(E) 83

86. You are given:

(i) $A_x = 0.28$

(ii) $A_{x+20} = 0.40$

(iii) $A_{x:\overline{20}|}^{\;\;1} = 0.25$

(iv) $i = 0.05$

Calculate $a_{x:\overline{20}|}$.

(A) 11.0

(B) 11.2

(C) 11.7

(D) 12.0

(E) 12.3

87. On his walk to work, Lucky Tom finds coins on the ground at a Poisson rate. The Poisson rate, expressed in coins per minute, is constant during any one day, but varies from day to day according to a gamma distribution with mean 2 and variance 4.

Calculate the probability that Lucky Tom finds exactly one coin during the sixth minute of today's walk.

(A) 0.22

(B) 0.24

(C) 0.26

(D) 0.28

(E) 0.30

88. The unlimited severity distribution for claim amounts under an auto liability insurance policy is given by the cumulative distribution:

$$F(x) = 1 - 0.8e^{-0.02x} - 0.2e^{-0.001x}, \qquad x \geq 0$$

The insurance policy pays amounts up to a limit of 1000 per claim.

Calculate the expected payment under this policy for one claim.

(A) 57

(B) 108

(C) 166

(D) 205

(E) 240

89. A machine is in one of four states (F, G, H, I) and migrates annually among them according to a Markov process with transition matrix:

	F	G	H	I
F	0.20	0.80	0.00	0.00
G	0.50	0.00	0.50	0.00
H	0.75	0.00	0.00	0.25
I	1.00	0.00	0.00	0.00

At time 0, the machine is in State F. A salvage company will pay 500 at the end of 3 years if the machine is in State F.

Assuming $v = 0.90$, calculate the actuarial present value at time 0 of this payment.

(A) 150

(B) 155

(C) 160

(D) 165

(E) 170

90. The claims department of an insurance company receives envelopes with claims for insurance coverage at a Poisson rate of $\lambda = 50$ envelopes per week. For any period of time, the number of envelopes and the numbers of claims in the envelopes are independent. The numbers of claims in the envelopes have the following distribution:

Number of Claims	*Probability*
1	0.20
2	0.25
3	0.40
4	0.15

Using the normal approximation, calculate the 90^{th} percentile of the number of claims received in 13 weeks.

(A) 1690

(B) 1710

(C) 1730

(D) 1750

(E) 1770

91. You are given:

(i) The survival function for males is $s(x) = 1 - \dfrac{x}{75}, \quad 0 < x < 75$.

(ii) Female mortality follows De Moivre's law.

(iii) At age 60, the female force of mortality is 60% of the male force of mortality.

For two independent lives, a male age 65 and a female age 60, calculate the expected time until the second death.

(A) 4.33

(B) 5.63

(C) 7.23

(D) 11.88

(E) 13.17

92. For a fully continuous whole life insurance of 1:

(i) $\mu = 0.04$

(ii) $\delta = 0.08$

(iii) L is the loss-at-issue random variable based on the benefit premium.

Calculate Var (L).

(A) $\frac{1}{10}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

93. The random variable for a loss, X, has the following characteristics:

x	$F(x)$	$\mathrm{E}(X \wedge x)$
0	0.0	0
100	0.2	91
200	0.6	153
1000	1.0	331

Calculate the mean excess loss for a deductible of 100.

(A) 250

(B) 300

(C) 350

(D) 400

(E) 450

94. WidgetsRUs owns two factories. It buys insurance to protect itself against major repair costs. Profit equals revenues, less the sum of insurance premiums, retained major repair costs, and all other expenses. WidgetsRUs will pay a dividend equal to the profit, if it is positive.

You are given:

(i) Combined revenue for the two factories is 3.

(ii) Major repair costs at the factories are independent.

(iii) The distribution of major repair costs for each factory is

k	Prob (k)
0	0.4
1	0.3
2	0.2
3	0.1

(iv) At each factory, the insurance policy pays the major repair costs in excess of that factory's ordinary deductible of 1. The insurance premium is 110% of the expected claims.

(v) All other expenses are 15% of revenues.

Calculate the expected dividend.

(A) 0.43

(B) 0.47

(C) 0.51

(D) 0.55

(E) 0.59

95. For watches produced by a certain manufacturer:

(i) Lifetimes follow a single-parameter Pareto distribution with $\alpha > 1$ and $\theta = 4$.

(ii) The expected lifetime of a watch is 8 years.

Calculate the probability that the lifetime of a watch is at least 6 years.

(A) 0.44

(B) 0.50

(C) 0.56

(D) 0.61

(E) 0.67

96. For a special 3-year deferred whole life annuity-due on (x):

(i) $i = 0.04$

(ii) The first annual payment is 1000.

(iii) Payments in the following years increase by 4% per year.

(iv) There is no death benefit during the three year deferral period.

(v) Level benefit premiums are payable at the beginning of each of the first three years.

(vi) $e_x = 11.05$ is the curtate expectation of life for (x).

(vii)

k	1	2	3
${}_k p_x$	0.99	0.98	0.97

Calculate the annual benefit premium.

(A) 2625

(B) 2825

(C) 3025

(D) 3225

(E) 3425

97. For a special fully discrete 10-payment whole life insurance on (30) with level annual benefit premium π:

(i) The death benefit is equal to 1000 plus the refund, without interest, of the benefit premiums paid.

(ii) $A_{30} = 0.102$

(iii) ${}_{10|}A_{30} = 0.088$

(iv) $(IA)^{1}_{30:\overline{10}|} = 0.078$

(v) $\ddot{a}_{30:\overline{10}|} = 7.747$

Calculate π.

(A) 14.9

(B) 15.0

(C) 15.1

(D) 15.2

(E) 15.3

98. For a given life age 30, it is estimated that an impact of a medical breakthrough will be an increase of 4 years in $\mathring{e}_{30}$, the complete expectation of life.

Prior to the medical breakthrough, s(x) followed de Moivre’s law with $\omega = 100$ as the limiting age.

Assuming de Moivre’s law still applies after the medical breakthrough, calculate the new limiting age.

(A) 104

(B) 105

(C) 106

(D) 107

(E) 108

99. On January 1, 2002, Pat, age 40, purchases a 5-payment, 10-year term insurance of 100,000:

(i) Death benefits are payable at the moment of death.

(ii) Contract premiums of 4000 are payable annually at the beginning of each year for 5 years.

(iii) $i = 0.05$

(iv) L is the loss random variable at time of issue.

Calculate the value of L if Pat dies on June 30, 2004.

(A) 77,100

(B) 80,700

(C) 82,700

(D) 85,900

(E) 88,000

100. Glen is practicing his simulation skills.
He generates 1000 values of the random variable X as follows:

(i) He generates the observed value λ from the gamma distribution with $\alpha = 2$ and $\theta = 1$ (hence with mean 2 and variance 2).

(ii) He then generates x from the Poisson distribution with mean λ.

(iii) He repeats the process 999 more times: first generating a value λ, then generating x from the Poisson distribution with mean λ.

(iv) The repetitions are mutually independent.

Calculate the expected number of times that his simulated value of X is 3.

(A) 75

(B) 100

(C) 125

(D) 150

(E) 175

101. Lucky Tom finds coins on his way to work at a Poisson rate of 0.5 coins per minute. The denominations are randomly distributed:

(i) 60% of the coins are worth 1;

(ii) 20% of the coins are worth 5;

(iii) 20% of the coins are worth 10.

Calculate the variance of the value of the coins Tom finds during his one-hour walk to work.

(A) 379

(B) 487

(C) 566

(D) 670

(E) 768

102. For a fully discrete 20-payment whole life insurance of 1000 on (x), you are given:

(i) $i = 0.06$

(ii) $q_{x+19} = 0.01254$

(iii) The level annual benefit premium is 13.72.

(iv) The benefit reserve at the end of year 19 is 342.03.

Calculate 1000 P_{x+20}, the level annual benefit premium for a fully discrete whole life insurance of 1000 on (x+20).

(A) 27

(B) 29

(C) 31

(D) 33

(E) 35

103. For a multiple decrement model on (60):

(i) $\mu_{60}^{(1)}(t), \quad t \geq 0,$ follows the Illustrative Life Table.

(ii) $\mu_{60}^{(\tau)}(t) = 2\mu_{60}^{(1)}(t), \quad t \geq 0$

Calculate ${}_{10|}q_{60}^{(\tau)}$, the probability that decrement occurs during the 11th year.

(A) 0.03

(B) 0.04

(C) 0.05

(D) 0.06

(E) 0.07

104. (x) and (y) are two lives with identical expected mortality.
You are given:

$P_x = P_y = 0.1$

$P_{\overline{xy}} = 0.06$, where $P_{\overline{xy}}$ is the annual benefit premium for a fully discrete insurance of 1 on $(\overline{xy})$.

$d = 0.06$

Calculate the premium P_{xy}, the annual benefit premium for a fully discrete insurance of 1 on (xy).

(A) 0.14

(B) 0.16

(C) 0.18

(D) 0.20

(E) 0.22

105. For students entering a college, you are given the following from a multiple decrement model:

(i) 1000 students enter the college at $t = 0$.

(ii) Students leave the college for failure (1) or all other reasons (2).

(iii) $\mu^{(1)}(t) = \mu \qquad 0 \le t \le 4$

$\mu^{(2)}(t) = 0.04 \qquad 0 \le t < 4$

(iv) 48 students are expected to leave the college during their first year due to all causes.

Calculate the expected number of students who will leave because of failure during their fourth year.

(A) 8

(B) 10

(C) 24

(D) 34

(E) 41

106. The following graph is related to current human mortality:

Which of the following functions of age does the graph most likely show?

(A) $\mu(x)$

(B) $l_x \mu(x)$

(C) $l_x p_x$

(D) l_x

(E) $l_x^{\,2}$

107. An actuary for an automobile insurance company determines that the distribution of the annual number of claims for an insured chosen at random is modeled by the negative binomial distribution with mean 0.2 and variance 0.4.

The number of claims for each individual insured has a Poisson distribution and the means of these Poisson distributions are gamma distributed over the population of insureds.

Calculate the variance of this gamma distribution.

(A) 0.20

(B) 0.25

(C) 0.30

(D) 0.35

(E) 0.40

108. A dam is proposed for a river which is currently used for salmon breeding. You have modeled:

(i) For each hour the dam is opened the number of salmon that will pass through and reach the breeding grounds has a distribution with mean 100 and variance 900.

(ii) The number of eggs released by each salmon has a distribution with mean of 5 and variance of 5.

(iii) The number of salmon going through the dam each hour it is open and the numbers of eggs released by the salmon are independent.

Using the normal approximation for the aggregate number of eggs released, determine the least number of whole hours the dam should be left open so the probability that 10,000 eggs will be released is greater than 95%.

(A) 20

(B) 23

(C) 26

(D) 29

(E) 32

109. For a special 3-year term insurance on (x), you are given:

(i) Z is the present-value random variable for the death benefits.

(ii) $q_{x+k} = 0.02(k+1) \qquad k = 0, 1, 2$

(iii) The following death benefits, payable at the end of the year of death:

k	b_{k+1}
0	300,000
1	350,000
2	400,000

(iv) $i = 0.06$

Calculate $\mathrm{E}(Z)$.

(A) 36,800

(B) 39,100

(C) 41,400

(D) 43,700

(E) 46,000

110. For a special fully discrete 20-year endowment insurance on (55):

(i) Death benefits in year k are given by $b_k = (21 - k)$, k = 1, 2, …, 20.

(ii) The maturity benefit is 1.

(iii) Annual benefit premiums are level.

(iv) ${}_kV$ denotes the benefit reserve at the end of year k, k = 1, 2,…, 20.

(v) ${}_{10}V = 5.0$

(vi) ${}_{19}V = 0.6$

(vii) $q_{65} = 0.10$

(viii) $i = 0.08$

Calculate ${}_{11}V$.

(A) 4.5

(B) 4.6

(C) 4.8

(D) 5.1

(E) 5.3

111. For a stop-loss insurance on a three person group:

(i) Loss amounts are independent.

(ii) The distribution of loss amount for each person is:

Loss Amount	Probability
0	0.4
1	0.3
2	0.2
3	0.1

(iii) The stop-loss insurance has a deductible of 1 for the group.

Calculate the net stop-loss premium.

(A) 2.00

(B) 2.03

(C) 2.06

(D) 2.09

(E) 2.12

112. A continuous two-life annuity pays:

100 while both (30) and (40) are alive;
70 while (30) is alive but (40) is dead; and
50 while (40) is alive but (30) is dead.

The actuarial present value of this annuity is 1180. Continuous single life annuities paying 100 per year are available for (30) and (40) with actuarial present values of 1200 and 1000, respectively.

Calculate the actuarial present value of a two-life continuous annuity that pays 100 while at least one of them is alive.

(A) 1400

(B) 1500

(C) 1600

(D) 1700

(E) 1800

113. For a disability insurance claim:

(i) The claimant will receive payments at the rate of 20,000 per year, payable continuously as long as she remains disabled.

(ii) The length of the payment period in years is a random variable with the gamma distribution with parameters $\alpha = 2$ and $\theta = 1$.

(iii) Payments begin immediately.

(iv) $\delta = 0.05$

Calculate the actuarial present value of the disability payments at the time of disability.

(A) 36,400

(B) 37,200

(C) 38,100

(D) 39,200

(E) 40,000

114. For a discrete probability distribution, you are given the recursion relation

$$p(k) = \frac{2}{k} * p(k-1), \quad k = 1, 2, \ldots.$$

Determine $p(4)$.

(A) 0.07

(B) 0.08

(C) 0.09

(D) 0.10

(E) 0.11

115. A company insures a fleet of vehicles. Aggregate losses have a compound Poisson distribution. The expected number of losses is 20. Loss amounts, regardless of vehicle type, have exponential distribution with $\theta = 200$.

In order to reduce the cost of the insurance, two modifications are to be made:

(i) a certain type of vehicle will not be insured. It is estimated that this will reduce loss frequency by 20%.

(ii) a deductible of 100 per loss will be imposed.

Calculate the expected aggregate amount paid by the insurer after the modifications.

(A) 1600

(B) 1940

(C) 2520

(D) 3200

(E) 3880

116. For a population of individuals, you are given:

(i) Each individual has a constant force of mortality.

(ii) The forces of mortality are uniformly distributed over the interval (0,2).

Calculate the probability that an individual drawn at random from this population dies within one year.

(A) 0.37

(B) 0.43

(C) 0.50

(D) 0.57

(E) 0.63

117. You are the producer of a television quiz show that gives cash prizes. The number of prizes, N, and prize amounts, X, have the following distributions:

n	$\Pr(N=n)$
1	0.8
2	0.2

x	$\Pr(X=x)$
0	0.2
100	0.7
1000	0.1

Your budget for prizes equals the expected prizes plus the standard deviation of prizes.

Calculate your budget.

(A) 306

(B) 316

(C) 416

(D) 510

(E) 518

118. For a special fully discrete 3-year term insurance on (x):

(i) Level benefit premiums are paid at the beginning of each year.

(ii)

k	b_{k+1}	q_{x+k}
0	200,000	0.03
1	150,000	0.06
2	100,000	0.09

(iii) $i = 0.06$

Calculate the initial benefit reserve for year 2.

(A) 6,500

(B) 7,500

(C) 8,100

(D) 9,400

(E) 10,300

119. For a special fully continuous whole life insurance on (x):

(i) The level premium is determined using the equivalence principle.

(ii) Death benefits are given by $b_t = (1+i)^t$ where i is the interest rate.

(iii) L is the loss random variable at $t = 0$ for the insurance.

(iv) T is the future lifetime random variable of (x).

Which of the following expressions is equal to L?

(A) $\dfrac{\left(v^T - \bar{A}_x\right)}{\left(1 - \bar{A}_x\right)}$

(B) $\left(v^T - \bar{A}_x\right)\left(1 + \bar{A}_x\right)$

(C) $\dfrac{\left(v^T - \bar{A}_x\right)}{\left(1 + \bar{A}_x\right)}$

(D) $\left(v^T - \bar{A}_x\right)\left(1 - \bar{A}_x\right)$

(E) $\dfrac{\left(v^T + \bar{A}_x\right)}{\left(1 + \bar{A}_x\right)}$

120. For a 4-year college, you are given the following probabilities for dropout from all causes:

$$q_0 = 0.15$$
$$q_1 = 0.10$$
$$q_2 = 0.05$$
$$q_3 = 0.01$$

Dropouts are uniformly distributed over each year.

Compute the temporary 1.5-year complete expected college lifetime of a student entering the second year, $\mathring{e}_{1:\overline{1.5}|}$.

(A) 1.25

(B) 1.30

(C) 1.35

(D) 1.40

(E) 1.45

121. Lee, age 63, considers the purchase of a single premium whole life insurance of 10,000 with death benefit payable at the end of the year of death.

The company calculates benefit premiums using:

(i) mortality based on the Illustrative Life Table,

(ii) $i = 0.05$

The company calculates contract premiums as 112% of benefit premiums.

The single contract premium at age 63 is 5233.

Lee decides to delay the purchase for two years and invests the 5233.

Calculate the minimum annual rate of return that the investment must earn to accumulate to an amount equal to the single contract premium at age 65.

(A) 0.030

(B) 0.035

(C) 0.040

(D) 0.045

(E) 0.050

122. You have calculated the actuarial present value of a last-survivor whole life insurance of 1 on (x) and (y). You assumed:

(i) The death benefit is payable at the moment of death.

(ii) The future lifetimes of (x) and (y) are independent, and each life has a constant force of mortality with $\mu = 0.06$.

(iii) $\delta = 0.05$

Your supervisor points out that these are not independent future lifetimes. Each mortality assumption is correct, but each includes a common shock component with constant force 0.02.

Calculate the increase in the actuarial present value over what you originally calculated.

(A) 0.020

(B) 0.039

(C) 0.093

(D) 0.109

(E) 0.163

123. The number of accidents follows a Poisson distribution with mean 12. Each accident generates 1, 2, or 3 claimants with probabilities $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, respectively.

Calculate the variance in the total number of claimants.

(A) 20

(B) 25

(C) 30

(D) 35

(E) 40

124. For a claims process, you are given:

(i) The number of claims $\{N(t),\ t \geq 0\}$ is a nonhomogeneous Poisson process with intensity function:

$$\lambda(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 2, & 1 \leq t < 2 \\ 3, & 2 \leq t \end{cases}$$

(ii) Claims amounts Y_i are independently and identically distributed random variables that are also independent of $N(t)$.

(iii) Each Y_i is uniformly distributed on [200,800].

(iv) The random variable P is the number of claims with claim amount less than 500 by time $t = 3$.

(v) The random variable Q is the number of claims with claim amount greater than 500 by time $t = 3$.

(vi) R is the conditional expected value of P, given $Q = 4$.

Calculate R.

(A) 2.0

(B) 2.5

(C) 3.0

(D) 3.5

(E) 4.0

125. Lottery Life issues a special fully discrete whole life insurance on (25):

(i) At the end of the year of death there is a random drawing. With probability 0.2, the death benefit is 1000. With probability 0.8, the death benefit is 0.

(ii) At the start of each year, including the first, while (25) is alive, there is a random drawing. With probability 0.8, the level premium π is paid. With probability 0.2, no premium is paid.

(iii) The random drawings are independent.

(iv) Mortality follows the Illustrative Life Table.

(v) $i = 0.06$

(vi) π is determined using the equivalence principle.

Calculate the benefit reserve at the end of year 10.

(A) 10.25

(B) 20.50

(C) 30.75

(D) 41.00

(E) 51.25

126. A government creates a fund to pay this year's lottery winners.

You are given:

(i) There are 100 winners each age 40.

(ii) Each winner receives payments of 10 per year for life, payable annually, beginning immediately.

(iii) Mortality follows the Illustrative Life Table.

(iv) The lifetimes are independent.

(v) $i = 0.06$

(vi) The amount of the fund is determined, using the normal approximation, such that the probability that the fund is sufficient to make all payments is 95%.

Calculate the initial amount of the fund.

(A) 14,800

(B) 14,900

(C) 15,050

(D) 15,150

(E) 15,250

127. For a special fully discrete 35-payment whole life insurance on (30):

(i) The death benefit is 1 for the first 20 years and is 5 thereafter.

(ii) The initial benefit premium paid during the each of the first 20 years is one fifth of the benefit premium paid during each of the 15 subsequent years.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) $A_{30:\overline{20|}} = 0.32307$

(vi) $\ddot{a}_{30:\overline{35|}} = 14.835$

Calculate the initial annual benefit premium.

(A) 0.010

(B) 0.015

(C) 0.020

(D) 0.025

(F) 0.030

128. For independent lives (x) and (y):

(i) $q_x = 0.05$

(ii) $q_y = 0.10$

(iii) Deaths are uniformly distributed over each year of age.

Calculate ${}_{0.75}q_{xy}$.

(A) 0.1088

(B) 0.1097

(C) 0.1106

(D) 0.1116

(E) 0.1125

129. In a clinic, physicians volunteer their time on a daily basis to provide care to those who are not eligible to obtain care otherwise. The number of physicians who volunteer in any day is uniformly distributed on the integers 1 through 5. The number of patients that can be served by a given physician has a Poisson distribution with mean 30.

Determine the probability that 120 or more patients can be served in a day at the clinic, using the normal approximation with continuity correction.

(A) $1-\Phi(0.68)$

(B) $1-\Phi(0.72)$

(C) $1-\Phi(0.93)$

(D) $1-\Phi(3.13)$

(E) $1-\Phi(3.16)$

130. A person age 40 wins 10,000 in the actuarial lottery. Rather than receiving the money at once, the winner is offered the actuarially equivalent option of receiving an annual payment of K (at the beginning of each year) guaranteed for 10 years and continuing thereafter for life.

You are given:

(i) $i = 0.04$

(ii) $A_{40} = 0.30$

(iii) $A_{50} = 0.35$

(iv) $A^{1}_{40:\overline{10}|} = 0.09$

Calculate K.

(A) 538

(B) 541

(C) 545

(D) 548

(E) 551

131. Mortality for Audra, age 25, follows De Moivre's law with $\omega = 100$. If she takes up hot air ballooning for the coming year, her assumed mortality will be adjusted so that for the coming year only, she will have a constant force of mortality of 0.1.

Calculate the decrease in the 11-year temporary complete life expectancy for Audra if she takes up hot air ballooning.

(A) 0.10

(B) 0.35

(C) 0.60

(D) 0.80

(E) 1.00

132. For a 5-year fully continuous term insurance on (x):

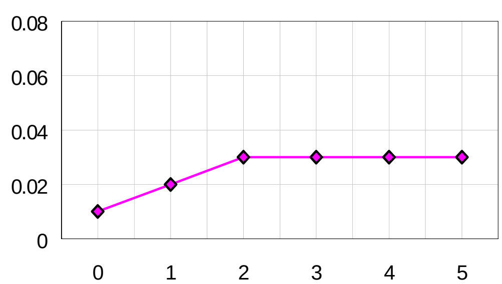
(i) $\delta = 0.10$

(ii) All the graphs below are to the same scale.

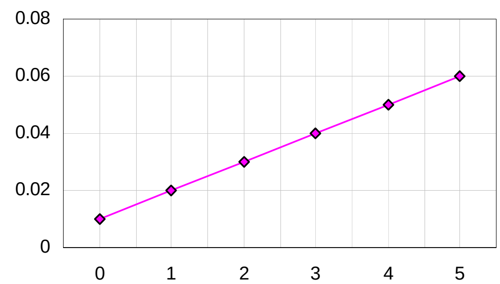
(iii) All the graphs show $\mu_x(t)$ on the vertical axis and t on the horizontal axis.

Which of the following mortality assumptions would produce the highest benefit reserve at the end of year 2?

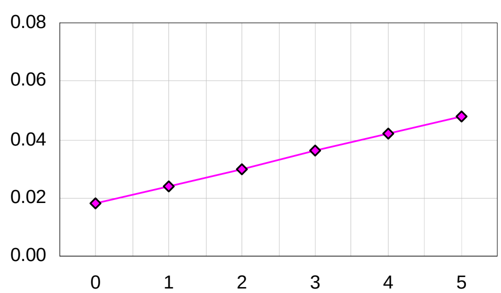
(A)



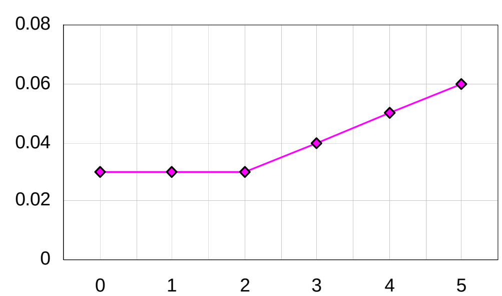
(B)



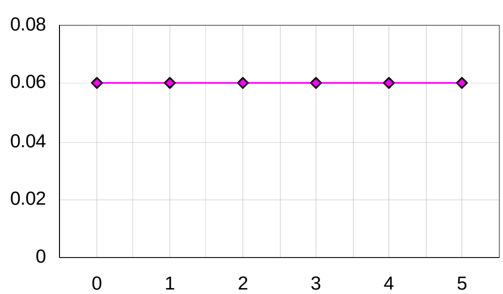
(C)



(D)



(E)



133. For a multiple decrement table, you are given:

(i) Decrement (1) is death, decrement (2) is disability, and decrement (3) is withdrawal.

(ii) $q'^{(1)}_{60} = 0.010$

(iii) $q'^{(2)}_{60} = 0.050$

(iv) $q'^{(3)}_{60} = 0.100$

(v) Withdrawals occur only at the end of the year.

(vi) Mortality and disability are uniformly distributed over each year of age in the associated single decrement tables.

Calculate $q^{(3)}_{60}$.

(A) 0.088

(B) 0.091

(C) 0.094

(D) 0.097

(E) 0.100

134. The number of claims, N, made on an insurance portfolio follows the following distribution:

n	$\Pr(N=n)$
0	0.7
2	0.2
3	0.1

If a claim occurs, the benefit is 0 or 10 with probability 0.8 and 0.2, respectively.

The number of claims and the benefit for each claim are independent.

Calculate the probability that aggregate benefits will exceed expected benefits by more than 2 standard deviations.

(A) 0.02

(B) 0.05

(C) 0.07

(D) 0.09

(E) 0.12

135. For a special whole life insurance of 100,000 on (x), you are given:

(i) $\delta = 0.06$

(ii) The death benefit is payable at the moment of death.

(iii) If death occurs by accident during the first 30 years, the death benefit is doubled.

(iv) $\mu_x^{(\tau)}(t) = 0.008,\ t \geq 0$

(v) $\mu_x^{(1)}(t) = 0.001, \quad t \geq 0,$ where $\mu_x^{(1)}$ is the force of decrement due to death by accident.

Calculate the single benefit premium for this insurance.

(A) 11,765

(B) 12,195

(C) 12,622

(D) 13,044

(E) 13,235

136. You are given the following extract from a select-and-ultimate mortality table with a 2-year select period:

x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
60	80,625	79,954	78,839	62
61	79,137	78,402	77,252	63
62	77,575	76,770	75,578	64

Assume that deaths are uniformly distributed between integral ages.

Calculate ${}_{0.9}q_{[60]+0.6}$.

(A) 0.0102

(B) 0.0103

(C) 0.0104

(D) 0.0105

(E) 0.0106

137. A claim count distribution can be expressed as a mixed Poisson distribution. The mean of the Poisson distribution is uniformly distributed over the interval [0,5].

Calculate the probability that there are 2 or more claims.

(A) 0.61

(B) 0.66

(C) 0.71

(D) 0.76

(E) 0.81

138. For a double decrement table with $l_{40}^{(\tau)} = 2000$:

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x'^{(1)}$	$q_x'^{(2)}$
40	0.24	0.10	0.25	y
41	--	--	0.20	$2y$

Calculate $l_{42}^{(\tau)}$.

(A) 800

(B) 820

(C) 840

(D) 860

(E) 880

139. For a fully discrete whole life insurance of 10,000 on (30):

(i) π denotes the annual premium and $\mathrm{L}(\pi)$ denotes the loss-at-issue random variable for this insurance.

(ii) Mortality follows the Illustrative Life Table.

(iii) i=0.06

Calculate the lowest premium, π', such that the probability is less than 0.5 that the loss $\mathrm{L}(\pi')$ is positive.

(A) 34.6

(B) 36.6

(C) 36.8

(D) 39.0

(E) 39.1

140. Y is the present-value random variable for a special 3-year temporary life annuity-due on (x). You are given:

(i) $_t p_x = 0.9^t, \quad t \geq 0$

(ii) K is the curtate-future-lifetime random variable for (x).

(iii) $$Y = \begin{cases} 1.00, & K = 0 \\ 1.87, & K = 1 \\ 2.72, & K = 2,3,\ldots \end{cases}$$

Calculate Var(Y).

(A) 0.19

(B) 0.30

(C) 0.37

(D) 0.46

(E) 0.55

141. A claim severity distribution is exponential with mean 1000. An insurance company will pay the amount of each claim in excess of a deductible of 100.

Calculate the variance of the amount paid by the insurance company for one claim, including the possibility that the amount paid is 0.

(A) 810,000

(B) 860,000

(C) 900,000

(D) 990,000

(E) 1,000,000

142. For a fully continuous whole life insurance of 1 on (x):

(i) π is the benefit premium.

(ii) L is the loss-at-issue random variable with the premium equal to π.

(iii) L^* is the loss-at-issue random variable with the premium equal to 1.25 π.

(iv) $\bar{a}_x = 5.0$

(v) $\delta = 0.08$

(vi) $\operatorname{Var}(L) = 0.5625$

Calculate the sum of the expected value and the standard deviation of L^*.

(A) 0.59

(B) 0.71

(C) 0.86

(D) 0.89

(E) 1.01

143. Workers' compensation claims are reported according to a Poisson process with mean 100 per month. The number of claims reported and the claim amounts are independently distributed. 2% of the claims exceed 30,000.

Calculate the number of complete months of data that must be gathered to have at least a 90% chance of observing at least 3 claims each exceeding 30,000.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

144. For students entering a three-year law school, you are given:

(i) The following double decrement table:

Academic Year	For a student at the beginning of that academic year, probability of		
	Academic Failure	Withdrawal for All Other Reasons	Survival Through Academic Year
1	0.40	0.20	--
2	--	0.30	--
3	--	--	0.60

(ii) Ten times as many students survive year 2 as fail during year 3.

(iii) The number of students who fail during year 2 is 40% of the number of students who survive year 2.

Calculate the probability that a student entering the school will withdraw for reasons other than academic failure before graduation.

(A) Less than 0.35

(B) At least 0.35, but less than 0.40

(C) At least 0.40, but less than 0.45

(D) At least 0.45, but less than 0.50

(E) At least 0.50

145. Given:

(i) Superscripts *M* and *N* identify two forces of mortality and the curtate expectations of life calculated from them.

(ii) $$\mu_{25}^{N}(t)=\begin{cases}\mu_{25}^{M}(t)+0.1*(1-t) & 0\le t\le 1\\ \mu_{25}^{M}(t) & t>1\end{cases}$$

(iii) $e_{25}^{M}=10.0$

Calculate e_{25}^{N}.

(A) 9.2

(B) 9.3

(C) 9.4

(D) 9.5

(E) 9.6

146. A fund is established to pay annuities to 100 independent lives age *x*. Each annuitant will receive 10,000 per year continuously until death. You are given:

(i) $\delta=0.06$

(ii) $\bar{A}_x=0.40$

(iii) ${}^2\bar{A}_x=0.25$

Calculate the amount (in millions) needed in the fund so that the probability, using the normal approximation, is 0.90 that the fund will be sufficient to provide the payments.

(A) 9.74

(B) 9.96

(C) 10.30

(D) 10.64

(E) 11.10

147. Total hospital claims for a health plan were previously modeled by a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 500$.

The health plan begins to provide financial incentives to physicians by paying a bonus of 50% of the amount by which total hospital claims are less than 500. No bonus is paid if total claims exceed 500.

Total hospital claims for the health plan are now modeled by a new Pareto distribution with $\alpha = 2$ and $\theta = K$. The expected claims plus the expected bonus under the revised model equals expected claims under the previous model.

Calculate *K*.

(A) 250

(B) 300

(C) 350

(D) 400

(E) 450

148. A decreasing term life insurance on (80) pays (20-*k*) at the end of the year of death if (80) dies in year *k*+1, for *k*=0,1,2,…,19.

You are given:
(i) i=0.06

(ii) For a certain mortality table with $q_{80} = 0.2$, the single benefit premium for this insurance is 13.

(iii) For this same mortality table, except that $q_{80} = 0.1$, the single benefit premium for this insurance is *P*.

Calculate *P*.

(A) 11.1

(B) 11.4

(C) 11.7

(D) 12.0

(E) 12.3

149. Job offers for a college graduate arrive according to a Poisson process with mean 2 per month. A job offer is acceptable if the wages are at least 28,000. Wages offered are mutually independent and follow a lognormal distribution with $\mu = 10.12$ and $\sigma = 0.12$.

Calculate the probability that it will take a college graduate more than 3 months to receive an acceptable job offer.

(A) 0.27

(B) 0.39

(C) 0.45

(D) 0.58

(E) 0.61

150. For independent lives (50) and (60):

$$\mu(x) = \frac{1}{100 - x}, \quad 0 \le x < 100$$

Calculate $\mathring{e}_{\overline{50:60}}$.

(A) 30

(B) 31

(C) 32

(D) 33

(E) 34

151. For an industry-wide study of patients admitted to hospitals for treatment of cardiovascular illness in 1998, you are given:

(i)

Duration In Days	Number of Patients Remaining Hospitalized
0	4,386,000
5	1,461,554
10	486,739
15	161,801
20	53,488
25	17,384
30	5,349
35	1,337
40	0

(ii) Discharges from the hospital are uniformly distributed between the durations shown in the table.

Calculate the mean residual time remaining hospitalized, in days, for a patient who has been hospitalized for 21 days.

(A) 4.4

(B) 4.9

(C) 5.3

(D) 5.8

(E) 6.3

152. For an individual over 65:

(i) The number of pharmacy claims is a Poisson random variable with mean 25.

(ii) The amount of each pharmacy claim is uniformly distributed between 5 and 95.

(iii) The amounts of the claims and the number of claims are mutually independent.

Determine the probability that aggregate claims for this individual will exceed 2000 using the normal approximation.

(A) $1-\Phi(1.33)$

(B) $1-\Phi(1.66)$

(C) $1-\Phi(2.33)$

(D) $1-\Phi(2.66)$

(E) $1-\Phi(3.33)$

153. For a fully discrete three-year endowment insurance of 10,000 on (50), you are given:

(i) $i = 0.03$

(ii) $1000q_{50} = 8.32$

(iii) $1000q_{51} = 9.11$

(iv) $10{,}000\ {}_1V_{50:\overline{3|}} = 3209$

(v) $10{,}000\ {}_2V_{50:\overline{3|}} = 6539$

(vi) ${}_0L$ is the prospective loss random variable at issue, based on the benefit premium.

Calculate the variance of ${}_0L$.

(A) 277,000

(B) 303,000

(C) 357,000

(D) 403,000

(E) 454,000

154. For a special 30-year deferred annual whole life annuity-due of 1 on (35):

(i) If death occurs during the deferral period, the single benefit premium is refunded without interest at the end of the year of death.

(ii) $\ddot{a}_{65} = 9.90$

(iii) $A_{35:\overline{30|}} = 0.21$

(iv) $A^{1}_{35:\overline{30|}} = 0.07$

Calculate the single benefit premium for this special deferred annuity.

(A) 1.3

(B) 1.4

(C) 1.5

(D) 1.6

(E) 1.7

155. Given:

(i) $\mu(x) = F + e^{2x}, \qquad x \geq 0$

(ii) ${}_{0.4}p_0 = 0.50$

Calculate F.

(A) -0.20

(B) -0.09

(C) 0.00

(D) 0.09

(E) 0.20

156-157 *Use the following information for questions 156 and 157.*

An insurer has excess-of-loss reinsurance on auto insurance. You are given:

(i) Total expected losses in the year 2001 are 10,000,000.

(ii) In the year 2001 individual losses have a Pareto distribution with

$$F(x) = 1 - \left(\frac{2000}{x+2000}\right)^2, \ x > 0.$$

(iii) Reinsurance will pay the excess of each loss over 3000.

(iv) Each year, the reinsurer is paid a ceded premium, C_{year}, equal to 110% of the expected losses covered by the reinsurance.

(v) Individual losses increase 5% each year due to inflation.

(vi) The frequency distribution does not change.

156. Calculate C_{2001}.

(A) 2,200,000

(B) 3,300,000

(C) 4,400,000

(D) 5,500,000

(E) 6,600,000

157. Calculate C_{2002} / C_{2001}.

(A) 1.04

(B) 1.05

(C) 1.06

(D) 1.07

(E) 1.08

SOCIETY OF ACTUARIES

EXAM M ACTUARIAL MODELS

EXAM M SAMPLE SOLUTIONS

Some of the questions in this study note are taken from past SOA examinations.

M-09-05 **PRINTED IN U.S.A.**

Question #1

Key: E

$${}_{2|}q_{\overline{30:34}} = {}_2p_{\overline{30:34}} - {}_3p_{\overline{30:34}}$$

$${}_2p_{30} = (0.9)(0.8) = 0.72$$

$${}_2p_{34} = (0.5)(0.4) = 0.20$$

$${}_2p_{30:34} = (0.72)(0.20) = 0.144$$

$${}_2p_{\overline{30:34}} = 0.72 + 0.20 - 0.144 = 0.776$$

$${}_3p_{30} = (0.72)(0.7) = 0.504$$

$${}_3p_{34} = (0.20)(0.3) = 0.06$$

$${}_3p_{30:34} = (0.504)(0.06) = 0.03024$$

$$\begin{aligned}{}_3p_{\overline{30:34}} &= 0.504 + 0.06 - 0.03024 \\ &= 0.53376\end{aligned}$$

$$\begin{aligned}{}_{2|}q_{\overline{30:34}} &= 0.776 - 0.53376 \\ &= 0.24224\end{aligned}$$

Alternatively,

$${}_{2|}q_{\overline{30:34}} = {}_{2|}q_{30} + {}_{2|}q_{34} - {}_{2|}q_{30:34}$$

$$= {}_2p_{30}q_{32} + {}_2p_{34}q_{36} - {}_2p_{30:34}(1 - p_{32:36})$$

= (0.9)(0.8)(0.3) + (0.5)(0.4)(0.7) – (0.9)(0.8)(0.5)(0.4) [1-(0.7)(0.3)]
= 0.216 + 0.140 – 0.144(0.79)
= 0.24224

Alternatively,

$${}_{2|}q_{\overline{30:34}} = {}_3q_{30} \times {}_3q_{34} - {}_2q_{30} \times {}_2q_{34}$$

$$= (1 - {}_3p_{30})(1 - {}_3p_{34}) - (1 - {}_2p_{30})(1 - {}_2p_{34})$$

$$= (1 - 0.504)(1 - 0.06) - (1 - 0.72)(1 - 0.20)$$

$$= 0.24224$$

(see first solution for ${}_2p_{30}$, ${}_2p_{34}$, ${}_3p_{30}$, ${}_3p_{34}$)

Question #2

Key: E

$$
\begin{aligned}
1000\bar{A}_x &= 1000\left[\bar{A}^{1}_{x:\overline{10}|} + {}_{10|}\bar{A}_x\right] \\
&= 1000\left[\int_0^{10} e^{-0.04t}e^{-0.06t}(0.06)dt + e^{-0.4}e^{-0.6}\int_0^{\infty} e^{-0.05t}e^{-0.07t}(0.07)dt\right] \\
&= 1000\left[0.06\int_0^{10} e^{-0.1t}dt + e^{-1}(0.07)\int_0^{\infty} e^{-0.12t}dt\right] \\
&= 1000\left[0.06\left[\tfrac{-e^{-0.10t}}{0.10}\right]_0^{10} + e^{-1}(0.07)\left[\tfrac{-e^{-0.12t}}{0.12}\right]_0^{\infty}\right] \\
&= 1000\left[\tfrac{0.06}{0.10}\left[1-e^{-1}\right] + \tfrac{0.07}{0.12}e^{-1}\left[1-e^{-1.2}\right]\right] \\
&= 1000\left(0.37927 + 0.21460\right) = 593.87
\end{aligned}
$$

Because this is a timed exam, many candidates will know common results for constant force
and constant interest without integration.

For example

$$
\begin{aligned}
\bar{A}^{1}_{x:\overline{10}|} &= \frac{\mu}{\mu+\delta}\left(1 - {}_{10}E_x\right) \\
{}_{10}E_x &= e^{-10(\mu+\delta)} \\
\bar{A}_x &= \frac{\mu}{\mu+\delta}
\end{aligned}
$$

With those relationships, the solution becomes

$$
\begin{aligned}
1000\bar{A}_x &= 1000\left[\bar{A}^{1}_{x:\overline{10}|} + {}_{10}E_x\ A_{x+10}\right] \\
&= 1000\left[\left(\frac{0.06}{0.06+0.04}\right)\left(1-e^{-(0.06+0.04)10}\right) + e^{-(0.06+0.04)10}\left(\frac{0.07}{0.07+0.05}\right)\right] \\
&= 1000\left[(0.60)\left(1-e^{-1}\right) + 0.5833e^{-1}\right] \\
&= 593.86
\end{aligned}
$$

Question #3
Key: A

$$B = \begin{cases} c(400-x) & x<400 \\ 0 & x \ge 400 \end{cases}$$

$$100 = E(B) = c \bullet 400 - cE(X \wedge 400)$$

$$= c \bullet 400 - c \bullet 300\left(1 - \frac{300}{300+400}\right)$$

$$= c\left(400 - 300 \bullet \frac{4}{7}\right)$$

$$c = \frac{100}{228.6} = 0.44$$

Question #4

Key: C

Let $N = \#$ of computers in department
Let *X* = cost of a maintenance call
Let *S* = aggregate cost

$$\text{Var}(X) = [\text{Standard Deviation } (X)]^2 = 200^2 = 40,000$$

$$E(X^2) = \text{Var}(X) + [E(X)]^2$$

$$= 40,000 + 80^2 = 46,400$$

$$E(S) = N \times \lambda \times E(X) = N \times 3 \times 80 = 240N$$

$$\text{Var}(S) = N \times \lambda \times E(X^2) = N \times 3 \times 46,400 = 139,200N$$

$$\text{We want} \quad 0.1 \ge \Pr(S > 1.2E(S))$$

$$\ge \Pr\left(\frac{S - E(S)}{\sqrt{139,200N}} > \frac{0.2E(S)}{\sqrt{139,200N}}\right) \Rightarrow \frac{0.2 \times 240N}{373.1\sqrt{N}} \ge 1.282 = \Phi(0.9)$$

$$N \ge \left(\frac{1.282 \times 373.1}{48}\right)^2 = 99.3$$

Question #5

Key: B

$$\mu_x^{(\tau)} = \mu_x^{(1)} + \mu_x^{(2)} + \mu_x^{(3)} = 0.0001045$$
$${}_t p_x^{(\tau)} = e^{-0.0001045t}$$

$$\text{APV Benefits} = \int_0^\infty e^{-\delta t} 1{,}000{,}000\, {}_t p_x^{(\tau)} \mu_x^{(1)} dt$$
$$+\int_0^\infty e^{-\delta t} 500{,}000\, {}_t p_x^{(\tau)} \mu_x^{(2)} dt$$
$$+\int_0^\infty e^{-\delta \tau} 200{,}000\, {}_t p_x^{(\tau)} \mu_x^{(3)} dt$$
$$= \frac{1{,}000{,}000}{2{,}000{,}000}\int_0^\infty e^{-0.0601045t} dt + \frac{500{,}000}{250{,}000}\int_0^\infty e^{-0.0601045t} dt + \frac{250{,}000}{10{,}000}\int_0^\infty e^{-0.0601045t} dt$$
$$= 27.5\left(16.6377\right) = 457.54$$

Question #6

Key: B

$$APV \text{ Benefits} = 1000 A^{1}_{40:\overline{20|}} + \sum_{k=20}^{\infty} {}_k E_{40} 1000 v q_{40+k}$$
$$APV \text{ Premiums} = \pi \ddot{a}_{40:\overline{20|}} + \sum_{k=20}^{\infty} {}_k E_{40} 1000 v q_{40+k}$$
$$\text{Benefit premiums } \Rightarrow \text{Equivalence principle } \Rightarrow$$
$$1000 A^{1}_{40:\overline{20|}} + \sum_{k=20}^{\infty} {}_k E_{40} 1000 v q_{40+k} = \pi \ddot{a}_{40:\overline{20|}} + \sum_{20}^{\infty} {}_k E_{40} 1000 v q_{40+k}$$
$$\pi = 1000 A^{1}_{40:\overline{20|}} / \ddot{a}_{40:\overline{20|}}$$
$$= \frac{161.32 - \left(0.27414\right)\left(369.13\right)}{14.8166 - \left(0.27414\right)\left(11.1454\right)}$$
$$= 5.11$$

While this solution above recognized that $\pi = 1000 P^{1}_{40:\overline{20|}}$ and was structured to take advantage of that, it wasn't necessary, nor would it save much time. Instead, you could do:

$$APV \text{ Benefits } = 1000 A_{40} = 161.32$$

$$
\begin{aligned}
APV \text{ Premiums } &= \pi \ddot{a}_{40:\overline{20}|} + {}_{20}E_{40} \sum_{k=0}^{\infty} {}_kE_{60} 1000 v q_{60+k} \\
&= \pi \ddot{a}_{40:\overline{20}|} + {}_{20}E_{40} 1000 A_{60} \\
&= \pi \left[14.8166 - (0.27414)(11.1454) \right] + (0.27414)(369.13) \\
&= 11.7612\pi + 101.19
\end{aligned}
$$

$$11.7612\pi + 101.19 = 161.32$$

$$\pi = \frac{161.32 - 101.19}{11.7612} = 5.11$$

Question #7

Key: C

$$A_{70} = \frac{\delta}{i} \bar{A}_{70} = \frac{\ln(1.06)}{0.06}(0.53) = 0.5147$$

$$\ddot{a}_{70} = \frac{1 - A_{70}}{d} = \frac{1 - 0.5147}{0.06/1.06} = 8.5736$$

$$\ddot{a}_{69} = 1 + v p_{69} \ddot{a}_{70} = 1 + \left(\frac{0.97}{1.06}\right)(8.5736) = 8.8457$$

$$
\begin{aligned}
\ddot{a}_{69}^{(2)} &= \alpha(2) \ddot{a}_{69} - \beta(2) = (1.00021)(8.8457) - 0.25739 \\
&= 8.5902
\end{aligned}
$$

Note that the approximation $\ddot{a}_x^{(m)} \cong \ddot{a}_x - \frac{(m-1)}{2m}$ works well (is closest to the exact answer, only off by less than 0.01). Since m = 2, this estimate becomes $8.8457 - \frac{1}{4} = 8.5957$

Question #8
Key: C

The following steps would do in this multiple-choice context:

1. From the answer choices, this is a recursion for an insurance or pure endowment.
2. Only C and E would satisfy $u(70) = 1.0$.
3. It is not E. The recursion for a pure endowment is simpler: $u(k) = \frac{1+i}{p_{k-1}} u(k-1)$
4. Thus, it must be C.

More rigorously, transform the recursion to its backward equivalent, $u(k-1)$ in terms of $u(k)$:

$$u(k) = -\left(\frac{q_{k-1}}{p_{k-1}}\right) + \left(\frac{1+i}{p_{k-1}}\right) u(k-1)$$
$$p_{k-1} u(k) = -q_{k-1} + (1+i)\, u(k-1)$$
$$u(k-1) = v q_{k-1} + v p_{k-1}\, u(k)$$

This is the form of (a), (b) and (c) on page 119 of Bowers with $x = k-1$. Thus, the recursion could be:

$$A_x = v q_x + v p_x A_{x+1}$$

or $$A^1_{x:\overline{y-x}|} = v q_x + v p_x \; A^{\;1}_{x+1:\overline{y-x-1}|}$$

or $$A_{x:\overline{y-x}|} = v q_x + v p_x \; A_{x+1:\overline{y-x-1}|}$$

Condition (iii) forces it to be answer choice C

$u(k-1) = A_x$ fails at $x = 69$ since it is not true that

$$A_{69} = v q_{69} + (v p_{69})(1)$$

$u(k-1) = A^1_{x:\overline{y-x}|}$ fails at $x = 69$ since it is not true that

$$A^1_{69:\overline{1}|} = v q_{69} + (v p_{69})(1)$$

$u(k-1) = A_{x:\overline{y-x}|}$ is OK at $x = 69$ since

$$A_{69:\overline{1}|} = v q_{69} + (v p_{69})(1)$$

Note: While writing recursion in backward form gave us something exactly like page 119 of Bowers, in its original forward form it is comparable to problem 8.7 on page 251. Reasoning from that formula, with $\pi_h = 0$ and $b_{h+1} = 1$, should also lead to the correct answer.

Question #9
Key: A

You arrive first if <u>both</u> (A) the first train to arrive is a local <u>and</u> (B) no express arrives in the 12 minutes after the local arrives.

$P(A) = 0.75$

Expresses arrive at Poisson rate of $(0.25)(20) = 5$ per hour, hence 1 per 12 minutes.

$$f(0) = \frac{e^{-1}1^0}{0!} = 0.368$$

A and B are independent, so

$P(A \text{ and } B) = (0.75)(0.368) = 0.276$

Question #10

Key: E

d = 0.05 $\rightarrow$ v = 0.095

At issue

$$A_{40} = \sum_{k=0}^{49} v^{k+1}{}_{k|}q_{40} = 0.02\left(v^1 + ... + v^{50}\right) = 0.02v\left(1 - v^{50}\right)/d = 0.35076$$

$$\text{and } \ddot{a}_{40} = \left(1 - A_{40}\right)/d = (1 - 0.35076)/0.05 = 12.9848$$

$$\text{so } P_{40} = \frac{1000A_{40}}{\ddot{a}_{40}} = \frac{350.76}{12.9848} = 27.013$$

$$E\left({}_{10}L \middle| K(40) \geq 10\right) = 1000A_{50}^{\text{Revised}} - P_{40}\ddot{a}_{50}^{\text{Revised}} = 549.18 - (27.013)(9.0164) = 305.62$$

where

$$A_{50}^{\text{Revised}} = \sum_{k=0}^{24} v^{k+1}{}_{k|}q_{50}^{\text{Revised}} = 0.04\left(v^1 + ... + v^{25}\right) = 0.04v\left(1 - v^{25}\right)/d = 0.54918$$

$$\text{and } \ddot{a}_{50}^{\text{Revised}} = \left(1 - A_{50}^{\text{Revised}}\right)/d = (1 - 0.54918)/0.05 = 9.0164$$

Question #11
Key: E

Let NS denote non-smokers and S denote smokers.

The shortest solution is based on the conditional variance formula

$$\mathrm{Var}(X) = E\left(\mathrm{Var}(X|Y)\right) + \mathrm{Var}\left(E(X|Y)\right)$$

Let Y = 1 if smoker; Y = 0 if non-smoker

$$E\left(\bar{a}_{\overline{T|}}|Y=1\right) = \bar{a}_x^S = \frac{1-\bar{A}_x^S}{\delta}$$

$$= \frac{1-0.444}{0.1} = 5.56$$

Similarly $E\left(\bar{a}_{\overline{T|}}|Y=0\right) = \dfrac{1-0.286}{0.1} = 7.14$

$$E\left(E\left(\bar{a}_{\overline{T|}}|Y\right)\right) = E\left(E\left(\bar{a}_{\overline{T|}}|0\right)\right)\times \mathrm{Prob}(\mathrm{Y{=}0}) + E\left(E\left(\bar{a}_{\overline{T|}}|1\right)\right)\times \mathrm{Prob}(\mathrm{Y{=}1})$$

$$= (7.14)(0.70) + (5.56)(0.30)$$

$$= 6.67$$

$$E\left[\left(E\left(\bar{a}_{\overline{T|}}|Y\right)\right)^2\right] = \left(7.14^2\right)(0.70) + \left(5.56^2\right)(0.30)$$

$$= 44.96$$

$$\mathrm{Var}\left(E\left(\bar{a}_{\overline{T|}}|Y\right)\right) = 44.96 - 6.67^2 = 0.47$$

$$E\left(\mathrm{Var}\left(\bar{a}_{\overline{T|}}|Y\right)\right) = (8.503)(0.70) + (8.818)(0.30)$$

$$= 8.60$$

$$\mathrm{Var}\left(\bar{a}_{\overline{T|}}\right) = 8.60 + 0.47 = 9.07$$

Alternatively, here is a solution based on
$\mathrm{Var}(Y) = E\left(Y^2\right) - \left[E(Y)\right]^2$, a formula for the variance of any random variable. This can be
transformed into $E\left(Y^2\right) = \mathrm{Var}(Y) + \left[E(Y)\right]^2$ which we will use in its conditional form

$$E\left(\left(\bar{a}_{\overline{T|}}\right)^2|\mathrm{NS}\right) = \mathrm{Var}\left(\bar{a}_{\overline{T|}}|\mathrm{NS}\right) + \left[E\left(\bar{a}_{\overline{T|}}|\mathrm{NS}\right)\right]^2$$

$$\mathrm{Var}\left[\bar{a}_{\overline{T|}}\right] = E\left[\left(\bar{a}_{\overline{T|}}\right)^2\right] - \left(E\left[\bar{a}_{\overline{T|}}\right]\right)^2$$

$$E\left[\bar{a}_{\overline{T|}}\right] = E\left[\bar{a}_{\overline{T|}}|\mathrm{S}\right]\times \mathrm{Prob}[\mathrm{S}] + E\left[\bar{a}_{\overline{T|}}|\mathrm{NS}\right]\times \mathrm{Prob}[\mathrm{NS}]$$

$$= 0.30\bar{a}_x^{\ \mathrm{S}} + 0.70\bar{a}_x^{\ \mathrm{NS}}$$

$$= \frac{0.30\left(1-\bar{A}_x^{\mathrm{S}}\right)}{0.1} + \frac{0.70\left(1-\bar{A}_x^{\mathrm{NS}}\right)}{0.1}$$

$$= \frac{0.30(1-0.444)+0.70(1-0.286)}{0.1} = (0.30)(5.56)+(0.70)(7.14)$$

$$= 1.67+5.00 = 6.67$$

$$E\left[\left(\bar{a}_{\overline{T|}}\right)^2\right] = E\left[\bar{a}_{\overline{T|}}^{\,2}\,\middle|\,\mathrm{S}\right]\times \mathrm{Prob}[\mathrm{S}] + E\left[\bar{a}_{\overline{T|}}^{\,2}\,\middle|\,\mathrm{NS}\right]\times \mathrm{Prob}[\mathrm{NS}]$$

$$= 0.30\left(\mathrm{Var}\left(\bar{a}_{\overline{T|}}\middle|\mathrm{S}\right) + \left(E\left[\bar{a}_{\overline{T|}}\middle|\mathrm{S}\right]\right)^2\right)$$

$$+0.70\left(\mathrm{Var}\left(\bar{a}_{\overline{T|}}\middle|\mathrm{NS}\right) + E\left(\bar{a}_{\overline{T|}}\middle|\mathrm{NS}\right)^2\right)$$

$$= 0.30\left[8.818+(5.56)^2\right] + 0.70\left[8.503+(7.14)^2\right]$$

11.919 + 41.638 = 53.557

$$\mathrm{Var}\left[\bar{a}_{\overline{T|}}\right] = 53.557-(6.67)^2 = 9.1$$

Alternatively, here is a solution based on $\bar{a}_{\overline{T|}} = \frac{1-v^T}{\delta}$

$$\mathrm{Var}\left(\bar{a}_{\overline{T|}}\right) = \mathrm{Var}\left(\frac{1}{\delta}-\frac{v^T}{\delta}\right)$$

$$= \mathrm{Var}\left(\frac{-v^T}{\delta}\right) \text{ since } \mathrm{Var}(X+\text{constant}) = \mathrm{Var}(X)$$

$$= \frac{\mathrm{Var}\left(v^T\right)}{\delta^2} \text{ since } \mathrm{Var}(\text{constant}\times X) = \text{constant}^2\times \mathrm{Var}(X)$$

$$= \frac{{}^2\bar{A}_x-\left(\bar{A}_x\right)^2}{\delta^2} \text{ which is Bowers formula 5.2.9}$$

This could be transformed into ${}^2A_x = \delta^2\,\mathrm{Var}\left(\bar{a}_{\overline{T|}}\right)+\bar{A}_x^2$, which we will use to get ${}^2A_x^{\ \mathrm{NS}}$ and ${}^2A_x^{\ \mathrm{S}}$.

$$
\begin{aligned}
{}^{2}\bar{A}_x &= E\left[v^{2T}\right] \\
&= E\left[v^{2T}\,|\mathrm{NS}\right]\times \mathrm{Prob}(\mathrm{NS}) + E\left[v^{2T}\,|\mathrm{S}\right]\times \mathrm{Prob}(\mathrm{S}) \\
&= \left[\delta^2 \mathrm{Var}\left(\bar{a}_{\overline{T|}}\,|\mathrm{NS}\right) + \left(\bar{A}_x^{\mathrm{NS}}\right)^2\right]\times \mathrm{Prob}(\mathrm{NS}) \\
&+ \left[\delta^2 \mathrm{Var}\left(\bar{a}_{\overline{T|}}\,|\mathrm{S}\right) + \left(\bar{A}_x^{\mathrm{S}}\right)^2\right]\times \mathrm{Prob}(\mathrm{S}) \\
&= \left[(0.01)(8.503) + 0.286^2\right]\times 0.70 \\
&+ \left[(0.01)(8.818) + 0.444^2\right]\times 0.30 \\
&= (0.16683)(0.70) + (0.28532)(0.30) \\
&= 0.20238
\end{aligned}
$$

$$
\begin{aligned}
\bar{A}_x &= E\left[v^{T}\right] \\
&= E\left[v^{T}\,|\mathrm{NS}\right]\times \mathrm{Prob}(\mathrm{NS}) + E\left[v^{T}\,|\mathrm{S}\right]\times \mathrm{Prob}(\mathrm{S}) \\
&= (0.286)(0.70) + (0.444)(0.30) \\
&= 0.3334
\end{aligned}
$$

$$
\begin{aligned}
\mathrm{Var}\left(\bar{a}_{\overline{T|}}\right) &= \frac{{}^{2}\bar{A}_x - \left(\bar{A}_x\right)^2}{\delta^2} \\
&= \frac{0.20238 - 0.3334^2}{0.01} = 9.12
\end{aligned}
$$

Question #12
Key: A

To be a density function, the integral of *f* must be 1 (i.e., everyone dies eventually). The solution is written for the general case, with upper limit ∞. Given the distribution of $f_2(t)$, we could have used upper limit 100 here.

Preliminary calculations from the Illustrative Life Table:

$$\frac{l_{50}}{l_0} = 0.8951$$

$$\frac{l_{40}}{l_0} = 0.9313$$

$$1 = \int_0^\infty f_T(t)\,dt = \int_0^{50} k\, f_1(t)\,dt + \int_{50}^\infty 1.2\, f_2(t)\,dt$$
$$= k\int_0^{50} f_1(t)\,dt + 1.2\int_{50}^\infty f_2(t)\,dt$$
$$= kF_1(50) + 1.2\left(F_2(\infty) - F_2(50)\right)$$
$$= k\left(1 - {}_{50}p_0\right) + 1.2(1 - 0.5)$$
$$= k(1 - 0.8951) + 0.6$$
$$k = \frac{1 - 0.6}{1 - 0.8951} = 3.813$$

For $x \le 50, F_T(x) = \int_0^x 3.813\, f_1(t)\,dt = 3.813F_1(x)$

$$F_T(40) = 3.813\left(1 - \frac{l_{40}}{l_0}\right) = 3.813(1 - 0.9313) = 0.262$$
$$F_T(50) = 3.813\left(1 - \frac{l_{50}}{l_0}\right) = 3.813(1 - 0.8951) = 0.400$$
$${}_{10}p_{40} = \frac{1 - F_T(50)}{1 - F_T(40)} = \frac{1 - 0.400}{1 - 0.262} = 0.813$$

Question #13
Key: D

Let NS denote non-smokers, S denote smokers.

$$\text{Prob}(T < t) = \text{Prob}(T < t \mid \text{NS}) \times \text{Prob}(\text{NS}) + \text{Prob}(T < t \mid \text{S}) \times \text{Prob}(\text{S})$$
$$= \left(1 - e^{-0.1t}\right) \times 0.7 + \left(1 - e^{-0.2t}\right) \times 0.3$$
$$= 1 - 0.7e^{-0.1t} - 0.3e^{-0.2t}$$

$S(t) = 0.3e^{-0.2t} + 0.7e^{-0.1t}$

Want $\hat{t}$ such that $0.75 = 1 - S(\hat{t})$ or $0.25 = S(\hat{t})$

$$0.25 = 0.3e^{-2\hat{t}} + 0.7e^{-0.1\hat{t}} = 0.3\left(e^{-0.1\hat{t}}\right)^2 + 0.7e^{-0.1\hat{t}}$$

Substitute: let $x = e^{-0.1\hat{t}}$

$0.3x^2 + 0.7x - 0.25 = 0$

This is quadratic, so $x = \dfrac{-0.7 \pm \sqrt{0.49 + (0.3)(0.25)4}}{2(0.3)}$

$$x = 0.3147$$

$$e^{-0.1\hat{t}} = 0.3147 \quad \text{so } \hat{t} = 11.56$$

Question #14
Key: D

The modified severity, X*, represents the conditional payment amount given that a payment occurs. Given that a payment is required (X > *d*), the payment must be uniformly distributed between 0 and $c \cdot (b - d)$.

The modified frequency, N*, represents the number of losses that result in a payment. The deductible eliminates payments for losses below *d*, so only $1 - F_x(d) = \dfrac{b-d}{b}$ of losses will require payments. Therefore, the Poisson parameter for the modified frequency distribution is $\lambda \cdot \dfrac{b-d}{b}$. (Reimbursing c% after the deductible affects only the payment amount and not the frequency of payments).

Question #15
Key: C

Let *N* = number of sales on that day
S = aggregate prospective loss at issue on those sales
K = curtate future lifetime

$$N \sim \text{Poisson}(0.2 * 50) \qquad \Rightarrow E[N] = \text{Var}[N] = 10$$

$${}_0L = 10{,}000 v^{K+1} - 500 \ddot{a}_{\overline{K+1}|} \qquad \Rightarrow E[{}_0L] = 10{,}000 A_{65} - 500 \ddot{a}_{65}$$

$${}_0L = \left(10{,}000 + \frac{500}{d}\right) v^{K+1} - \frac{500}{d} \qquad \Rightarrow Var[{}_0L] = \left(10{,}000 + \frac{500}{d}\right)^2 \left[{}^2A_{65} - (A_{65})^2 \right]$$

$$S = {}_0L_1 + {}_0L_2 + \ldots + {}_0L_N$$
$$E[S] = E[N] \cdot E[{}_0L]$$
$$Var[S] = Var[{}_0L] \cdot E[N] + (E[{}_0L])^2 \cdot \text{Var}[N]$$

$$\Pr(S<0)=\Pr\left(Z<\frac{0-E[S]}{\sqrt{Var[S]}}\right)$$

Substituting d = 0.06/(1+0.06), ${}^2A_{65}=0.23603$, $A_{65}=0.43980$ and $\ddot{a}_{65}=9.8969$ yields

$$E[{}_0L]=-550.45$$

$$\mathrm{Var}[{}_0L]=15{,}112{,}000$$

$$E[S]=-5504.5$$

$$\mathrm{Var}[S]=154{,}150{,}000$$

Std Dev (S) = 12,416

$$\Pr(S<0)=\Pr\left(\frac{\mathrm{S}+5504.5}{12{,}416}<\frac{5504.5}{12{,}416}\right)$$

$$=\Pr(Z<0.443)$$

$$=0.67$$

With the answer choices, it was sufficient to recognize that:

$$0.6554=\phi(0.4)<\phi(0.443)<\phi(0.5)=0.6915$$

By interpolation, $\phi(0.443)\approx(0.43)\phi(0.5)+(0.57)\phi(0.4)$

$$=(0.43)(0.6915)+(0.57)(0.6554)$$

$$=0.6709$$

Question #16
Key: A

$$1000P_{40} = \frac{A_{40}}{\ddot{a}_{40}} = \frac{161.32}{14.8166} = 10.89$$

$$1000\,{}_{20}V_{40} = 1000\left(1 - \frac{\ddot{a}_{60}}{\ddot{a}_{40}}\right) = 1000\left(1 - \frac{11.1454}{14.8166}\right) = 247.78$$

$$_{21}V = \frac{\left({}_{20}V + 5000P_{40}\right)(1+i) - 5000q_{60}}{P_{60}}$$

$$= \frac{\left(247.78 + (5)(10.89)\right)\times 1.06 - 5000\left(0.01376\right)}{1 - 0.01376} = 255$$

[Note: For this insurance, $_{20}V = 1000\,{}_{20}V_{40}$ because retrospectively, this is identical to whole life]

Though it would have taken <u>much</u> longer, you can do this as a prospective reserve. The prospective solution is included for educational purposes, not to suggest it would be suitable under exam time constraints.

$1000P_{40} = 10.89$ as above

$$1000A_{40} + 4000\,{}_{20}E_{40}A^{1}_{60:\overline{5}|} = 1000P_{40} + 5000P_{40} \times {}_{20}E_{40}\,\ddot{a}_{60:\overline{5}|} + \pi\,{}_{20}E_{40} \times {}_{5}E_{60}\,\ddot{a}_{65}$$

where $A^{1}_{60:\overline{5}|} = A_{60} - {}_{5}E_{60}\,A_{65} = 0.06674$

$$\ddot{a}_{40:\overline{20}|} = \ddot{a}_{40} - {}_{20}E_{40}\,\ddot{a}_{60} = 11.7612$$

$$\ddot{a}_{60:\overline{5}|} = \ddot{a}_{60} - {}_{5}E_{60}\,\ddot{a}_{65} = 4.3407$$

$$1000\left(0.16132\right) + \left(4000\right)\left(0.27414\right)\left(0.06674\right) =$$

$$= \left(10.89\right)\left(11.7612\right) + \left(5\right)\left(10.89\right)\left(0.27414\right)\left(4.3407\right) + \pi\left(0.27414\right)\left(0.68756\right)\left(9.8969\right)$$

$$\pi = \frac{161.32 + 73.18 - 128.08 - 64.79}{1.86544}$$

$$= 22.32$$

Having struggled to solve for π, you could calculate $_{20}V$ prospectively then (as above)
calculate $_{21}V$ recursively.

$$_{20}V = 4000A^{1}_{60:\overline{5}|} + 1000A_{60} - 5000P_{40}\,\ddot{a}_{60:\overline{5}|} - \pi\,{}_{5}E_{60}\,\ddot{a}_{65}$$

$$= \left(4000\right)\left(0.06674\right) + 369.13 - \left(5000\right)\left(0.01089\right)\left(4.3407\right) - \left(22.32\right)\left(0.68756\right)\left(9.8969\right)$$

$$= 247.86 \text{ (minor rounding difference from } 1000\,{}_{20}V_{40}\text{)}$$

Or we can continue to $_{21}V$ prospectively

$$_{21}V = 5000A^1_{61:\overline{4|}} + 1000\,_4E_{61}\,A_{65} - 5000P_{40}\,\ddot{a}_{61:\overline{4|}} - \pi\,_4E_{61}\,\ddot{a}_{65}$$

where $_4E_{61} = \frac{l_{65}}{l_{61}}v^4 = \left(\frac{7,533,964}{8,075,403}\right)(0.79209) = 0.73898$

$$A^1_{61:\overline{4|}} = A_{61} - \,_4E_{61}\,A_{65} = 0.38279 - 0.73898 \times 0.43980$$
$$= 0.05779$$
$$\ddot{a}_{61:\overline{4|}} = \ddot{a}_{61} - \,_4E_{61}\,\ddot{a}_{65} = 10.9041 - 0.73898 \times 9.8969$$
$$= 3.5905$$

$$_{21}V = (5000)(0.05779) + (1000)(0.73898)(0.43980)$$
$$-(5)(10.89)(3.5905) - 22.32(0.73898)(9.8969)$$
$$= 255$$

Finally. A moral victory. Under exam conditions since prospective benefit reserves must equal retrospective benefit reserves, calculate whichever is simpler.

Question #17
Key: C

$$\text{Var}(Z) = {}^2A_{41} - (A_{41})^2$$
$$A_{41} - A_{40} = 0.00822 = A_{41} - (vq_{40} + vp_{40}A_{41})$$
$$= A_{41} - (0.0028/1.05 + (0.9972/1.05)A_{41})$$
$$\Rightarrow A_{41} = 0.21650$$

$${}^2A_{41} - {}^2A_{40} = 0.00433 = {}^2A_{41} - \left(v^2q_{40} + v^2p_{40}\,{}^2A_{41}\right)$$
$$= {}^2A_{41} - (0.0028/1.05^2 + \left(0.9972/1.05^2\right)^2 A_{41})$$
$${}^2A_{41} = 0.07193$$

$$\text{Var}(Z) = 0.07193 - 0.21650^2$$
$$= 0.02544$$

Question #18
Key: D

This solution looks imposing because there is no standard notation. Try to focus on the big picture ideas rather than starting with the details of the formulas.

Big picture ideas:

1. We can express the present values of the perpetuity recursively.
2. Because the interest rates follow a Markov process, the present value (at time t) of the future payments at time t depends only on the state you are in at time t, not how you got there.
3. Because the interest rates follow a Markov process, the present value of the future payments at times t_1 and t_2 are equal if you are in the same state at times t_1 and t_2.

Method 1: Attack <u>without</u> considering the special characteristics of this transition matrix.

Let $s_k =$ state you are in at time k $\left(\text{thus } s_k = 0, 1 \text{ or } 2\right)$

Let $Y_k =$ present value, at time k, of the future payments.
Y_k is a random variable because its value depends on the pattern of discount factors, which are random. The expected value of Y_k is not constant; it depends on what state we are in at time *k*.

Recursively we can write

$Y_k = v \times \left(1 + Y_{k+1}\right)$, where it would be better to have notation that indicates the *v*'s are not constant, but are realizations of a random variable, where the random variable itself has different distributions depending on what state we're in. However, that would make the notation so complex as to mask the simplicity of the relationship.

Every time we are in state 0 we have

$$E\left[Y_k \middle| s_k = 0\right] = 0.95 \times \left(1 + E\left[Y_{k+1} \middle| s_k = 0\right]\right)$$

$$= 0.95 \times \left(1 + \left\{\left(E\left[Y_{k+1} \middle| s_{k+1} = 0\right]\right)\right) \times \mathrm{Prob}\left(\mathrm{s}_{\mathrm{k+1}} = 0 \middle| s_k = 0\right]\right)$$

$$+ \left(E\left[Y_{k+1} \middle| s_{k+1} = 1\right]\right) \times \mathrm{Prob}\left(s_{k+1} = 1 \middle| s_k = 0\right]\right)$$

$$= \left(E\left[Y_{k+1} \middle| s_{k+1} = 2\right]\right) \times \mathrm{Prob}\left(s_{k+1} = 2 \middle| s_k = 0\right]\right)\Big\}$$

$$= 0.95 \times \left(1 + E\left[Y_{k+1} \middle| s_{k+1} = 1\right]\right)$$

That last step follows because from the transition matrix if we are in state 0, we always move to state 1 one period later.

Similarly, every time we are in state 2 we have

$$E\left[Y_k \middle| s_k = 2\right] = 0.93 \times \left(1 + E\left[Y_{k+1} \middle| s_k = 2\right]\right)$$
$$= 0.93 \times \left(1 + E\left[Y_{k+1} \middle| s_{k+1} = 1\right]\right)$$

That last step follows because from the transition matrix if we are in state 2, we always move to state 1 one period later.

Finally, every time we are in state 1 we have

$$E\left[Y_k \middle| s_k = 1\right] = 0.94 \times \left(1 + E\left[Y_{k+1} \middle| s_k = 1\right]\right)$$
$$= 0.94 \times \left(1 + \left\{E\left[Y_{k+1} \middle| s_{k+1} = 0\right] \times \Pr\left[s_{k+1} = 0 \middle| s_k = 1\right] + E\left[Y_{k+1} \middle| s_{k+1} = 2\right] \times \Pr\left[s_{k+1} = 2 \middle| s_k = 1\right]\right\}\right)$$
$$= 0.94 \times \left(1 + \left\{E\left[Y_{k+1} \middle| s_{k+1} = 0\right] \times 0.9 + E\left[Y_{k+1} \middle| s_{k+1} = 2\right] \times 0.1\right\}\right).$$

Those last two steps follow from the fact that from state 1 we always go to either state 0 (with probability 0.9) or state 2 (with probability 0.1).

Now let's write those last three paragraphs using this shorter notation: $x_n = E\left[Y_k \middle| s_k = n\right]$. We can do this because (big picture idea #3), the conditional expected value is only a function of the state we are in, not when we are in it or how we got there.

$$x_0 = 0.95(1 + x_1)$$
$$x_1 = 0.94(1 + 0.9x_0 + 0.1x_2)$$
$$x_2 = 0.93(1 + x_1)$$

That's three equations in three unknowns. Solve (by substituting the first and third into the second) to get $x_1 = 16.82$.

That's the answer to the question, the expected present value of the future payments given in state 1.

The solution above is almost exactly what we would have to do with <u>any</u> 3×3 transition matrix. As we worked through, we put only the non-zero entries into our formulas. But if for example the top row of the transition matrix had been $\begin{pmatrix} 0.4 & 0.5 & 0.1 \end{pmatrix}$, then the first of our three equations would have become $x_0 = 0.95(1 + 0.4x_0 + 0.5x_1 + 0.1x_2)$, similar in structure to our actual equation for x_1. We would still have ended up with three linear equations in three unknowns, just more tedious ones to solve.

Method 2: Recognize the patterns of changes for <u>this particular transition matrix</u>.

This particular transition matrix has a recurring pattern that leads to a much quicker solution. We are starting in state 1 and are guaranteed to be back in state 1 two steps later, with the same prospective value then as we have now.
Thus,

$$E[Y] = E[Y|\text{first move is to } 0] \times \Pr[\text{first move is to } 0] + E[Y|\text{first move is to } 2] \times \Pr[\text{first move is to } 2]$$

$$= 0.94 \times \left[\left(1 + 0.95 \times \left(1 + E[Y]\right)\right)\right] \times 0.9 + \left[0.94 \times \left[\left(1 + 0.93 \times \left(1 + E[Y]\right)\right) \times 0.1\right]\right.$$

(Note that the equation above is exactly what you get when you substitute x_0 and x_2 into the formula for x_1 in Method 1.)

$$= 1.6497 + 0.8037 E[Y] + 0.1814 + 0.0874 E[Y]$$

$$E[Y] = \frac{1.6497 + 0.1814}{(1 - 0.8037 - 0.0874)}$$

$$= 16.82$$

Question #19
Key: E

The number of problems solved in 10 minutes is Poisson with mean 2.
If she solves exactly one, there is 1/3 probability that it is #3.
If she solves exactly two, there is a 2/3 probability that she solved #3.
If she solves #3 or more, she got #3.

$f(0) = 0.1353$
$f(1) = 0.2707$
$f(2) = 0.2707$

$$P = \left(\frac{1}{3}\right)(0.2707) + \left(\frac{2}{3}\right)(0.2707) + (1 - 0.1353 - 0.2707 - 0.2707) = 0.594$$

Question #20
Key: D

$$\mu_x^{(\tau)} = \mu_x^{(1)}(t) + \mu_x^{(2)}(t)$$

$$= 0.2\mu_x^{(\tau)}(t) + \mu_x^{(2)}(t)$$

$$\Rightarrow \mu_x^{(2)}(t) = 0.8\mu_x^{(\tau)}(t)$$

$$q_x^{'(1)} = 1 - p_x^{'(1)} = 1 - e^{-\int_0^1 0.2k\, t^2\, dt} = 1 - e^{-0.2\frac{k}{3}} = 0.04$$

$$k/3 \Rightarrow \ln(1-0.04)/(-0.2) = 0.2041$$
$$k = 0.6123$$

$${}_2q_x^{(2)} = \int_0^2 {}_tp_x^{(\tau)}\mu_x^{(2)}\ dt = 0.8\int_0^2\ {}_tp_x^{(\tau)}\ \ \mu_x^{(\tau)}(t)\ dt$$

$$= 0.8\ {}_2q_x^{(\tau)} = 0.8\left(1 - {}_2p_x^{(\tau)}\right)$$

$${}_2p_x^{(\tau)} = e^{-\int_0^2 \mu_x(t)dt}$$

$$= e^{-\int_0^2 kt^2\, dt}$$

$$= e^{\frac{-8k}{3}}$$

$$= e^{\frac{-(8)(0.6123)}{3}}$$

$$= 0.19538$$

$${}_2q_x^{(2)} = 0.8(1-0.19538) = 0.644$$

Question #21
Key: A

k	$k \wedge 3$	*f*(*k*)	$f(k)\times(k \wedge 3)$	$f(k)\times(k \wedge 3)^2$
0	0	0.1	0	0
1	1	(0.9)(0.2) = 0.18	0.18	0.18
2	2	(0.72)(0.3) = 0.216	0.432	0.864
3+	3	1-0.1-0.18-0.216 = 0.504	<u>1.512</u>	<u>4.536</u>
			2.124	5.580

$E(K \wedge 3) = 2.124$

$E\left((K \wedge 3)^2\right) = 5.580$

$\text{Var}(K \wedge 3) = 5.580 - 2.124^2 = 1.07$

Note that $E[K \wedge 3]$ is the temporary curtate life expectancy, $e_{x:\overline{3}|}$ if the life is age *x*. Problem 3.17 in Bowers, pages 86 and 87, gives an alternative formula for the variance, basing the calculation on ${}_k p_x$ rather than ${}_{k|}q_x$.

Question #22
Key: E

$$\begin{aligned} f(x) &= 0.01, \quad 0 \le x \le 80 \\ &= 0.01 - 0.00025(x-80) = 0.03 - 0.00025x, \quad 80 < x \le 120 \end{aligned}$$

$$\begin{aligned} E(x) &= \int_0^{80} 0.01x\,dx + \int_{80}^{120}\left(0.03x - 0.00025x^2\right)dx \\ &= \frac{0.01x^2}{2}\Big|_0^{80} + \frac{0.03x^2}{2}\Big|_{80}^{120} - \frac{0.00025x^3}{3}\Big|_{80}^{120} \\ &= 32 + 120 - 101.33 = 50.66667 \end{aligned}$$

$$\begin{aligned} E(X-20)_+ &= E(X) - \int_0^{20} x f(x)\,dx - 20(1 - \int_0^{20} f(x)dx) \\ &= 50.6667 - \frac{0.01x^2}{2}\Big|_0^{20} - 20\left(1 - 0.01x\Big|_0^{20}\right) \\ &= 50.6667 - 2 - 20(0.8) = 32.6667 \end{aligned}$$

Loss Elimination Ratio = $1 - \dfrac{32.6667}{50.6667} = 0.3553$

Question #23
Key: D

Let q_{64} for Michel equal the standard q_{64} plus *c*. We need to solve for *c*.
Recursion formula for a standard insurance:

$$ {}_{20}V_{45} = \left({}_{19}V_{45} + P_{45}\right)(1.03) - q_{64}\left(1 - {}_{20}V_{45}\right) $$

Recursion formula for Michel's insurance

$$ {}_{20}V_{45} = \left({}_{19}V_{45} + P_{45} + 0.01\right)(1.03) - \left(q_{64} + c\right)(1 - {}_{20}V_{45}) $$

The values of ${}_{19}V_{45}$ and ${}_{20}V_{45}$ are the same in the two equations because we are told Michel's benefit reserves are the same as for a standard insurance.

Subtract the second equation from the first to get:

$$ 0 = -(1.03)(0.01) + c(1 - {}_{20}V_{45}) $$

$$ c = \frac{(1.03)(0.01)}{\left(1 - {}_{20}V_{45}\right)} $$

$$ = \frac{0.0103}{1 - 0.427} $$

$$ = 0.018 $$

Question #24
Key: B

K is the curtate future lifetime for one insured.
L is the loss random variable for one insurance.
L_{AGG} is the aggregate loss random variables for the individual insurances.
σ_{AGG} is the standard deviation of L_{AGG}.
M is the number of policies.

$$ L = v^{K+1} - \pi \ddot{a}_{\overline{K+1}|} = \left(1 + \frac{\pi}{d}\right) v^{K+1} - \pi/d $$

$$ E[L] = \left(A_x - \pi \ddot{a}_x\right) = A_x - \pi \frac{\left(1 - A_x\right)}{d} $$

$$ = 0.24905 - 0.025\left(\frac{0.75095}{0.056604}\right) = -0.082618 $$

$$\text{Var}[L] = \left(1 + \frac{\pi}{d}\right)^2 \left({}^2A_x - A_x^2\right) = \left(1 + \frac{0.025}{0.056604}\right)^2 \left(0.09476 - (0.24905)^2\right) = 0.068034$$

$$E[L_{AGG}] = M\,E[L] = -0.082618M$$

$$\text{Var}[L_{AGG}] = M\ \text{Var}[L] = M\,(0.068034) \Rightarrow \sigma_{AGG} = 0.260833\sqrt{M}$$

$$\Pr[L_{AGG} > 0] = \left[\frac{L_{AGG} - E[L_{AGG}]}{\sigma_{AGG}} > \frac{-E(L_{AGG})}{\sigma_{AGG}}\right]$$

$$\approx \Pr\left(N(0,1) > \frac{0.082618M}{\sqrt{M}\,(0.260833)}\right)$$

$$\Rightarrow 1.645 = \frac{0.082618\sqrt{M}}{0.260833}$$

$$\Rightarrow M = 26.97$$

$\Rightarrow$ minimum number needed = 27

Question #25
Key: D

Annuity benefit: $Z_1 = 12{,}000\dfrac{1 - v^{K+1}}{d}$ for $K = 0,1,2,\ldots$

Death benefit: $Z_2 = Bv^{K+1}$ for $K = 0,1,2,\ldots$

New benefit:
$$Z = Z_1 + Z_2 = 12{,}000\frac{1 - v^{K+1}}{d} + Bv^{K+1}$$
$$= \frac{12{,}000}{d} + \left(B - \frac{12{,}000}{d}\right)v^{K+1}$$

$$\text{Var}(Z) = \left(B - \frac{12{,}000}{d}\right)^2 \text{Var}\left(v^{K+1}\right)$$

$$\text{Var}(Z) = 0 \text{ if } B = \frac{12{,}000}{0.08} = 150{,}000\,.$$

In the first formula for $\text{Var}(Z)$, we used the formula, valid for any constants *a* and *b* and random variable *X*,

$$\text{Var}(a + bX) = b^2\text{Var}(X)$$

Question #26
Key: B

First restate the table to be CAC's cost, after the 10% payment by the auto owner:

Towing Cost, *x*	*p*(*x*)
72	50%
90	40%
144	10%

Then $E(X) = 0.5*72 + 0.4*90 + 0.1*144 = 86.4$

$E(X^2) = 0.5*72^2 + 0.4*90^2 + 0.1*144^2 = 7905.6$

$\mathrm{Var}(X) = 7905.6 - 86.4^2 = 440.64$

Because Poisson, $E(N) = \mathrm{Var}(N) = 1000$

$E(S) = E(X)E(N) = 86.4*1000 = 86,400$

$\mathrm{Var}(S) = E(N)\mathrm{Var}(X) + E(X)^2\mathrm{Var}(N) = 1000*440.64 + 86.4^2*1000 = 7,905,600$

$$\Pr(S > 90,000) + \Pr\left(\frac{S - E(S)}{\sqrt{\mathrm{Var}(S)}} > \frac{90,000 - 86,400}{\sqrt{7,905,600}}\right) = \Pr(Z > 1.28) = 1 - \Phi(1.28) = 0.10$$

Since the frequency is Poisson, you could also have used

$\mathrm{Var}(S) = \lambda E(X^2) = (1000)(7905.6) = 7,905,600$

That way, you would not need to have calculated $\mathrm{Var}(X)$.

Question #27

Key: C

$$\mathrm{LER} = \frac{E(X \wedge d)}{E(X)} = \frac{\theta\left(1 - e^{-d/\theta}\right)}{\theta} = 1 - e^{-d/\theta}$$

Last year $\quad 0.70 = 1 - e^{-d/\theta} \Rightarrow -d = \theta \log\ 0.30$

Next year: $\quad -d_{new} = \theta \log(1 - \mathrm{LER}_{new})$

Hence $\theta \log\left(1 - \mathrm{LER}_{\mathrm{new}}\right) = -d_{\mathrm{new}} = \frac{4}{3}\theta \log\ 0.30$

$\log\left(1 - \mathrm{LER}_{new}\right) = -1.6053$

$\left(1 - \mathrm{LER}_{new}\right) = e^{-1.6053} = 0.20$

$\mathrm{LER}_{new} = 0.80$

Question #28
Key: E

$E(X) = e(d)S(d) + E(X \wedge d)$ [Klugman Study Note, formula 3.10]

$62 = \mathring{e}_{40} \times {}_{40}p_0 + E(T \wedge 40)$

$62 = (\mathring{e}_{40})(0.6) + 40 - (0.005)(40^2)$

$= 0.6\,\mathring{e}_{40} + 32$

$\mathring{e}_{40} = \dfrac{(62-32)}{0.6} = 50$

The first equation, in the notation of Bowers, is $\mathring{e}_0 = \mathring{e}_{40} \times {}_{40}p_0 + \mathring{e}_{0:\overline{40}|}$. The corresponding formula, with $i > 0$, is a very commonly used one:

$\bar{a}_x = \bar{a}_{x:\overline{n}|} + {}_nE_x\,\bar{a}_{x+n}$

Question #29
Key: B

$d = 0.05 \Rightarrow v = 0.95$

Step 1 Determine p_x from Kevin's work:

$$608 + 350vp_x = 1000vq_x + 1000v^2 p_x\left(p_{x+1} + q_{x+1}\right)$$
$$608 + 350(0.95)p_x = 1000(0.95)(1-p_x) + 1000(0.9025)p_x(1)$$
$$608 + 332.5p_x = 950(1-p_x) + 902.5p_x$$
$$p_x = 342/380 = 0.9$$

Step 2 Calculate $1000P_{x:\overline{2}|}$, as Kira did:

$$608 + 350(0.95)(0.9) = 1000P_{x:\overline{2}|}\left[1 + (0.95)(0.9)\right]$$
$$1000P_{x:\overline{2}|} = \frac{[299.25 + 608]}{1.855} = 489.08$$

The first line of Kira's solution is that the actuarial present value of Kevin's benefit premiums is equal to the actuarial present value of Kira's, since each must equal the actuarial present value of benefits. The actuarial present value of benefits would also have been easy to calculate as

$(1000)(0.95)(0.1) + (1000)(0.95^2)(0.9) = 907.25$

Question #30
Key: E

Because no premiums are paid after year 10 for (*x*), ${}_{11}V_x = A_{x+11}$

Rearranging 8.3.10 from Bowers, we get ${}_{h+1}V = \dfrac{\left({}_hV + \pi_h\right)(1+i) - b_{h+1}q_{x+h}}{p_{x+h}}$

$$_{10}V = \frac{(32,535 + 2,078)\times(1.05) - 100,000 \times 0.011}{0.989} = 35,635.642$$

$$_{11}V = \frac{(35,635.642 + 0)\times(1.05) - 100,000 \times 0.012}{0.988} = 36,657.31 = A_{x+11}$$

Question #31
Key: B

For De Moivre's law where $s(x) = \left(1 - \dfrac{x}{\omega}\right)$:

$$\mathring{e}_x = \frac{\omega - x}{2} \ and \ {}_t p_x = \left(1 - \frac{t}{\omega - x}\right)$$

$$\mathring{e}_{45} = \frac{105-45}{2} = 30$$

$$\mathring{e}_{65} = \frac{105-65}{2} = 20$$

$$\mathring{e}_{45:65} = \int_0^{40} {}_t p_{45:65} dt = \int_0^{40} \frac{60-t}{60} \times \frac{40-t}{40} dt$$

$$= \frac{1}{60 \times 40}\left(60 \times 40 \times t - \frac{60+40}{2}t^2 + \frac{1}{3}t^3\right)\Big|_0^{40}$$

$$= 15.56$$

$$\mathring{e}_{\overline{45:65}} = \mathring{e}_{45} + \mathring{e}_{65} - \mathring{e}_{45:65}$$

$$= 30 + 20 - 15.56 = 34$$

In the integral for $\mathring{e}_{45:65}$, the upper limit is 40 since 65 (and thus the joint status also) can survive a maximum of 40 years.

Question #32
Answer: E

$$\mu(4) = -s'(4)/s(4)$$

$$= \frac{-\left(-e^4/100\right)}{1-e^4/100}$$

$$= \frac{e^4/100}{1-e^4/100}$$

$$= \frac{e^4}{100-e^4}$$

$$= 1.202553$$

Question # 33
Answer: A

$$q_x^{(i)} = q_x^{(\tau)}\left[\frac{\ln\ p_x'^{(i)}}{\ln\ p_x^{(\tau)}}\right] = q_x^{(\tau)}\left[\frac{\ln\ e^{-\mu^{(i)}}}{\ln\ e^{-\mu^{(\tau)}}}\right]$$

$$= q_x^{(\tau)} \times \frac{\mu^{(i)}}{\mu^{(\tau)}}$$

$$\mu_x^{(\tau)} = \mu_x^{(1)} + \mu_x^{(2)} + \mu_x^{(3)} = 1.5$$

$$q_x^{(\tau)} = 1 - e^{-\mu(\tau)} = 1 - e^{-1.5}$$

=0.7769

$$q_x^{(2)} = \frac{(0.7769)\mu^{(2)}}{\mu^{(\tau)}} = \frac{(0.5)(0.7769)}{1.5}$$

$$= 0.2590$$

Question # 34
Answer: D

$$
\begin{array}{lcccccc}
{}_{2|2}A_{[60]} = v^3 & \times & {}_2p_{[60]} & \times & q_{[60]+2} & + \\
\downarrow & & \downarrow & & \downarrow & \\
\text{pay at end} & & \text{live} & & \text{then die} & \\
\text{of year 3} & & \text{2 years} & & \text{in year 3} &
\end{array}
$$

$$
\begin{array}{lcccc}
+v^4 & \times & {}_3p_{[60]} & \times & q_{60+3} \\
\text{pay at end} & & \text{live} & & \text{then die} \\
\text{of year 4} & & \text{3 years} & & \text{in year 4}
\end{array}
$$

$$= \frac{1}{(1.03)^3}(1-0.09)(1-0.11)(0.13) + \frac{1}{(1.03)^4}(1-0.09)(1-0.11)(1-0.13)(0.15)$$

$$= 0.19$$

Question # 35
Answer: B

$$\bar{a}_x = \bar{a}_{x:\overline{5}|} + {}_5E_x\ \bar{a}_{x+5}$$

$\bar{a}_{x:\overline{5}|} = \dfrac{1-e^{-0.07(5)}}{0.07} = 4.219$, where $0.07 = \mu + \delta$ for $t < 5$

$${}_5E_x = e^{-0.07(5)} = 0.705$$

$\bar{a}_{x+5} = \dfrac{1}{0.08} = 12.5$, where $0.08 = \mu + \delta$ for $t \geq 5$

$$\therefore \bar{a}_x = 4.219 + (0.705)(12.5) = 13.03$$

Question # 36
Answer: E

The distribution of claims (a gamma mixture of Poissons) is negative binomial.

$$E(N) = E_{\wedge}\left(E(N|\Lambda)\right) = E_{\wedge}(\Lambda) = 3$$
$$Var(N) = E_{\wedge}\left(Var(N|\Lambda)\right) + Var_{\wedge}\left(E(N|\Lambda)\right)$$
$$= E_{\wedge}(\Lambda) + Var_{\wedge}(\Lambda) = 6$$
$$r\beta = 3$$
$$r\beta(1+\beta) = 6$$
$$(1+\beta) = 6/3 = 2\ ; \quad \beta = 1$$
$$r\beta = 3$$
$$r = 3$$

$$p_0 = (1+\beta)^{-r} = 0.125$$
$$p_1 = \frac{r\beta}{(1+\beta)^{r+1}} = 0.1875$$
$$\text{Prob}(\text{at most } 1) = p_0 + p_1$$
$$= 0.3125$$

Question # 37
Answer: A

$$E(S) = E(N) \times E(X) = 110 \times 1{,}101 = 121{,}110$$

$$Var(S) = E(N) \times Var(X) + E(X)^2 \times Var(N)$$
$$= 110 \times 70^2 + 1101^2 \times 750$$
$$= 909{,}689{,}750$$

$$\text{Std Dev } (S) = 30{,}161$$

$\Pr(S < 100{,}000) = \Pr\left(Z < (100{,}000 - 121{,}110)/30{,}161\right)$ where Z has standard normal distribution

$$= \Pr(Z < -0.70) = 0.242$$

Question # 38
Answer: C

This is just the Gambler's Ruin problem, in units of 5,000 calories.
Each day, up one with $p = 0.45$; down 1 with $q = 0.55$
Will Allosaur ever be up 1 before being down 2?

$$P_2 = \frac{\left(1-(0.55/0.45)^{\wedge}2\right)}{\left(1-(0.55/0.45)^{\wedge}3\right)} = 0.598$$

Or, by general principles instead of applying a memorized formula:
Let $P_1 =$ probability of ever reaching 3 (15,000 calories) if at 1 (5,000 calories).
Let $P_2 =$ probability of ever reaching 3 (15,000 calories) if at 2 (10,000 calories).

From either, we go up with $p = 0.45$, down with $q = 0.55$

P(reaching 3) = P(up) $\times$ P(reaching 3 after up) + P(down) $\times$ P(reaching 3 after down)

$$P_2 = 0.45 \times 1 + 0.55 \times P_1$$
$$P_1 = 0.45 \times P_2 + 0.55 \times 0 = 0.45 \times P_2$$
$$P_2 = 0.45 + 0.55 \times P_1 = 0.45 + 0.55 \times 0.45 \times P_2 = 0.45 + 0.2475 P_2$$
$$P_2 = 0.45/\left(1-0.2475\right) = 0.598$$

Here is another approach, feasible since the number of states is small.

Let states 0,1,2,3 correspond to 0; 5,000; 10,000; ever reached 15,000 calories. For purposes of this problem, state 3 is absorbing, since once the allosaur reaches 15,000 we don't care what happens thereafter.

The transition matrix is $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.55 & 0 & 0.45 & 0 \\ 0 & 0.55 & 0 & 0.45 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Starting with the allosaur in state 2;

[0	0	1	0]	at inception
[0	0.55	0	0.45]	after 1
[0.3025	0	0.2475	0.45]	after 2
[0.3025	0.1361	0	0.5614]	after 3
[0.3774	0	0.0612	0.5614]	after 4
[0.3774	0.0337	0	0.5889]	after 5
[0.3959	0	0.0152	0.5889]	after 6

By this step, if not before, Prob(state 3) must be converging to 0.60. It's already closer to 0.60 than 0.57, and its maximum is 0.5889 + 0.0152

Question # 39
Answer: D

Per 10 minutes, find coins worth exactly 10 at Poisson rate $(0.5)(0.2)(10)=1$

Per 10 minutes, $f(0)=0.3679 \quad F(0)=0.3679$
$f(1)=0.3679 \quad F(1)=0.7358$
$f(2)=0.1839 \quad F(2)=0.9197$
$f(3)=0.0613 \quad F(3)=0.9810$

Let Period 1 = first 10 minutes; period 2 = next 10.

Method 1, succeed with 3 or more in period 1; or exactly 2, then one or more in period 2

$$P=(1-F(2))+f(2)(1-F(0))=(1-0.9197)+(0.1839)(1-0.3679)$$
$$=0.1965$$

Method 2: fail in period 1 if < 2; $\Pr\text{ob}=F(1)=0.7358$

fail in period 2 if exactly 2 in period 1, then 0; $\Pr\text{ob}=f(2)f(0)$
$$=(0.1839)(0.3679)=0.0677$$

Succeed if fail neither period; $\Pr\text{ob}=1-0.7358-0.0677$
$$=0.1965$$

(Method 1 is attacking the problem as a stochastic process model; method 2 attacks it as a ruin model.)

Question # 40
Answer: D

Use Mod to designate values unique to this insured.

$$\ddot{a}_{60}=(1-A_{60})/d=(1-0.36933)/[(0.06)/(1.06)]=11.1418$$

$$1000P_{60}=1000A_{60}/\ddot{a}_{60}=1000(0.36933/11.1418)=33.15$$

$$A_{60}^{Mod}=v\left(q_{60}^{Mod}+p_{60}^{Mod}A_{61}\right)=\frac{1}{1.06}[0.1376+(0.8624)(0.383)]=0.44141$$

$$\ddot{a}^{Mod} = \left(1 - A_{60}^{Mod}\right) / d = (1 - 0.44141) / [0.06 / 1.06] = 9.8684$$

$$E\left[{}_0L^{Mod}\right] = 1000\left(A_{60}^{Mod} - P_{60}\ddot{a}_{60}^{Mod}\right)$$

$$= 1000[0.44141 - 0.03315(9.8684)]$$

$$= 114.27$$

Question # 41
Answer: D

The prospective reserve at age 60 per 1 of insurance is A_{60}, since there will be no future premiums. Equating that to the retrospective reserve per 1 of coverage, we have:

$$A_{60} = P_{40}\frac{\ddot{s}_{40:\overline{10}|}}{{}_{10}E_{50}} + P_{50}^{Mod}\ \ddot{s}_{50:\overline{10}|} - {}_{20}k_{40}$$

$$A_{60} = \frac{A_{40}}{\ddot{a}_{40}} \times \frac{\ddot{a}_{40:\overline{10}|}}{{}_{10}E_{40}\ {}_{10}E_{50}} + P_{50}^{Mod}\frac{\ddot{a}_{50:\overline{10}|}}{{}_{10}E_{50}} - \frac{A^{1}_{40:\overline{20}|}}{{}_{20}E_{40}}$$

$$0.36913 = \frac{0.16132}{14.8166} \times \frac{7.70}{(0.53667)(0.51081)} + P_{50}^{Mod}\frac{7.57}{0.51081} - \frac{0.06}{0.27414}$$

$$0.36913 = 0.30582 + 14.8196\ P_{50}^{Mod} - 0.21887$$

$$1000\ P_{50}^{Mod} = 19.04$$

Alternatively, you could equate the retrospective and prospective reserves at age 50. Your equation would be:

$$A_{50} - P_{50}^{Mod}\ \ddot{a}_{50:\overline{10}|} = \frac{A_{40}}{\ddot{a}_{40}} \times \frac{\ddot{a}_{40:\overline{10}|}}{{}_{10}E_{40}} - \frac{A^{1}_{40:\overline{10}|}}{{}_{10}E_{40}}$$

where $A^{1}_{40:\overline{10}|} = A_{40} - {}_{10}E_{40}\ A_{50}$

$$= 0.16132 - (0.53667)(0.24905)$$
$$= 0.02766$$

$$0.24905-\left(P_{50}^{Mod}\right)(7.57)=\frac{0.16132}{14.8166}\times\frac{7.70}{0.53667}-\frac{0.02766}{0.53667}$$

$$1000P_{50}^{Mod}=\frac{(1000)(0.14437)}{7.57}=19.07$$

Alternatively, you could set the actuarial present value of benefits at age 40 to the actuarial present value of benefit premiums. The change at age 50 did not change the benefits, only the pattern of paying for them.

$$A_{40}=P_{40}\,\ddot{a}_{40:\overline{10}|}+P_{50}^{Mod}\;{}_{10}E_{40}\;\ddot{a}_{50:\overline{10}|}$$

$$0.16132=\left(\frac{0.16132}{14.8166}\right)(7.70)+\left(P_{50}^{Mod}\right)(0.53667)(7.57)$$

$$1000P_{50}^{Mod}=\frac{(1000)(0.07748)}{4.0626}=19.07$$

Question # 42
Answer: A

$$d_x^{(2)}=q_x^{(2)}\times l_x^{(\tau)}=400$$

$$d_x^{(1)}=0.45(400)=180$$

$$q_x'^{(2)}=\frac{d_x^{(2)}}{l_x^{(\tau)}-d_x^{(1)}}=\frac{400}{1000-180}=0.488$$

$$p_x'^{(2)}=1-0.488=0.512$$

Note: The UDD assumption was not critical except to have all deaths during the year so that 1000 - 180 lives are subject to decrement 2.

Question #43
Answer: D

Use "age" subscripts for years completed in program. E.g., p_0 applies to a person newly hired ("age" 0).

Let decrement 1 = fail, 2 = resign, 3 = other.

Then $q_0'^{(1)} = 1/4,\ q_1'^{(1)} = 1/5,\ q_2'^{(1)} = 1/3$

$q_0'^{(2)} = 1/5,\ q_1'^{(2)} = 1/3,\ q_2'^{(2)} = 1/8$

$q_0'^{(3)} = 1/10,\ q_1'^{(3)} = 1/9,\ q_2'^{(3)} = 1/4$

This gives $p_0^{(\tau)} = (1-1/4)(1-1/5)(1-1/10) = 0.54$

$p_1^{(\tau)} = (1-1/5)(1-1/3)(1-1/9) = 0.474$

$p_2^{(\tau)} = (1-1/3)(1-1/8)(1-1/4) = 0.438$

So $1_0^{(\tau)} = 200,\ 1_1^{(\tau)} = 200\,(0.54) = 108$, and $1_2^{(\tau)} = 108\,(0.474) = 51.2$

$$q_2^{(1)} = \left[\log p_2'^{(1)} / \log p_2^{(\tau)}\right] q_2^{(\tau)}$$

$$q_2^{(1)} = \left[\log\left(\tfrac{2}{3}\right) / \log(0.438)\right]\left[1 - 0.438\right]$$

$$= (0.405 / 0.826)(0.562)$$

$$= 0.276$$

$$d_2^{(1)} = l_2^{(\tau)}\ q_2^{(1)}$$
$$= (51.2)(0.276) = 14$$

Question #44
Answer: C

Let: N = number
X = profit
S = aggregate profit
subscripts G = "good", B = "bad", AB = "accepted bad"

$\lambda_G = \left(\tfrac{2}{3}\right)(60) = 40$

$\lambda_{AB} = \left(\tfrac{1}{2}\right)\left(\tfrac{1}{3}\right)(60) = 10$ (If you have trouble accepting this, think instead of a heads-tails rule, that the application is accepted if the applicant's government-issued identification number, e.g. U.S. Social Security Number, is odd. It is <u>not</u> the same as saying he automatically alternates accepting and rejecting.)

$$Var(S_G) = E(N_G) \times Var(X_G) + Var(N_G) \times E(X_G)^2$$

$$= (40)(10{,}000) + (40)\left(300^2\right) = 4{,}000{,}000$$

$$Var(S_{AB}) = E(N_{AB}) \times Var(X_{AB}) + Var(N_{AB}) \times E(X_{AB})^2$$

$$= (10)(90{,}000) + (10)(-100)^2 = 1{,}000{,}000$$

S_G and S_{AB} are independent, so

$$Var(S) = Var(S_G) + Var(S_{AB}) = 4{,}000{,}000 + 1{,}000{,}000$$

$$= 5{,}000{,}000$$

If you don't treat it as three streams ("goods", "accepted bads", "rejected bads"), you can compute the mean and variance of the profit <u>per "bad" received</u>.

$\lambda_B = \left(\frac{1}{3}\right)(60) = 20$

If all "bads" were accepted, we would have $E\left(X^2{}_B\right) = Var(X_B) + E(X_B)^2$

$$= 90{,}000 + (-100)^2 = 100{,}000$$

Since the probability a "bad" will be accepted is only 50%,

$$E(X_B) = \text{Prob}(\text{accepted}) \times E(X_B|\text{accepted}) + \text{Prob}(\text{not accepted}) \times E(X_B|\text{not accepted})$$

$$= (0.5)(-100) + (0.5)(0) = -50$$

$$E\left(X_B^2\right) = (0.5)(100{,}000) + (0.5)(0) = 50{,}000$$

Likewise,

Now $Var(S_B) = E(N_B) \times Var(X_B) + Var(N_B) \times E(X_B)^2$

$$= (20)(47{,}500) + (20)\left(50^2\right) = 1{,}000{,}000$$

S_G and S_B are independent, so

$$Var(S) = Var(S_G) + Var(S_B) = 4{,}000{,}000 + 1{,}000{,}000$$

$$= 5{,}000{,}000$$

Question # 45
Answer: C

Let *N* = number of prescriptions then $S = N \times 40$

n	$f_N(n)$	$F_N(n)$	$1-F_N(n)$
0	0.2000	0.2000	0.8000
1	0.1600	0.3600	0.6400
2	0.1280	0.4880	0.5120
3	0.1024	0.5904	0.4096

$$E(N) = 4 = \sum_{j=0}^{\infty}(1-F(j))$$

$$\begin{aligned} E\left[(S-80)_+\right] &= 40 \times E\left[(N-2)_+\right] = 40 \times \sum_{j=2}^{\infty}(1-F(j)) \\ &= 40 \times \left[\sum_{j=0}^{\infty}(1-F(j)) - \sum_{j=0}^{1}(1-F(j))\right] \\ &= 40(4-1.44) = 40 \times 2.56 = 102.40 \end{aligned}$$

$$\begin{aligned} E\left[(S-120)_+\right] &= 40 \times E\left[(N-3)_+\right] = 40 \times \sum_{j=3}^{\infty}(1-F(j)) \\ &= 40 \times \left[\sum_{j=0}^{\infty}(1-F(j)) - \sum_{j=0}^{2}(1-F(j))\right] \\ &= 40(4-1.952) = 40 \times 2.048 = 81.92 \end{aligned}$$

Since no values of S between 80 and 120 are possible,

$$E\left[(S-100)_+\right] = \frac{(120-100) \times E\left[(S-80)_+\right] + (100-80) \times E\left[(S-120)_+\right]}{120} = 92.16$$

Alternatively,

$$E\left[(S-100)_+\right] = \sum_{j=0}^{\infty}(40j-100)f_N(j) + 100 f_N(0) + 60 f_N(1) + 20 f_N(2)$$

(The correction terms are needed because $(40j-100)$ would be negative for j = 0, 1, 2; we need to add back the amount those terms would be negative)

$$\begin{aligned} &= 40\sum_{j=0}^{\infty} j \times f_N(j) - 100\sum_{j=0}^{\infty} f_N(j) + (100)(0.200) + (0.16)(60) + (0.128)(20) \\ &= 40\,E(N) - 100 + 20 + 9.6 + 2.56 \\ &= 160 - 67.84 = 92.16 \end{aligned}$$

Question #46
Answer: B

$$ {}_{10}E_{30:40} = {}_{10}p_{30}\ {}_{10}p_{40}\ v^{10} = \left({}_{10}p_{30}v^{10}\right)\left({}_{10}p_{40}v^{10}\right)(1+i)^{10} $$
$$ = \left({}_{10}E_{30}\right)\left({}_{10}E_{40}\right)(1+i)^{10} $$
$$ = (0.54733)(0.53667)(1.79085) $$
$$ = 0.52604 $$

The above is only one of many possible ways to evaluate ${}_{10}p_{30}\ {}_{10}p_{40}\ v^{10}$, all of which should give 0.52604

$$ a_{30:40:\overline{10}|} = a_{30:40} - {}_{10}E_{30:40}\ a_{30+10:40+10} $$
$$ = \left(\ddot{a}_{30:40} - 1\right) - (0.52604)\left(\ddot{a}_{40:50} - 1\right) $$
$$ = (13.2068) - (0.52604)(11.4784) $$
$$ = 7.1687 $$

Question #47
Answer: A

Equivalence Principle, where π is annual benefit premium, gives

$$ 1000\left(A_{35} + (IA)_{35} \times \pi\right) = \ddot{a}_x \pi $$

$$ \pi = \frac{1000A_{35}}{\left(\ddot{a}_{35} - (IA)_{35}\right)} = \frac{1000 \times 0.42898}{(11.99143 - 6.16761)} $$
$$ = \frac{428.98}{5.82382} $$
$$ = 73.66 $$

We obtained $\ddot{a}_{35}$ from

$$ \ddot{a}_{35} = \frac{1 - A_{35}}{d} = \frac{1 - 0.42898}{0.047619} = 11.99143 $$

Question #48
Answer: C

Time until arrival = waiting time plus travel time.

Waiting time is exponentially distributed with mean $1/\lambda$. The time you may already have been waiting is irrelevant: exponential is memoryless.

You: E (wait) = $\frac{1}{20}$ hour = 3 minutes
E (travel) = $(0.25)(16)+(0.75)(28)=25$ minutes
E (total) = 28 minutes

Co-worker: E (wait) = $\frac{1}{5}$ hour = 12 minutes
E (travel) = 16 minutes
E (total) = 28 minutes

Question #49
Answer: C

$$\mu_{xy} = \mu_x + \mu_y = 0.14$$

$$\overline{A}_x = \overline{A}_y = \frac{\mu}{\mu+\delta} = \frac{0.07}{0.07+0.05} = 0.5833$$

$$\overline{A}_{xy} = \frac{\mu_{xy}}{\mu_{xy}+\delta} = \frac{0.14}{0.14+0.05} = \frac{0.14}{0.19} = 0.7368 \text{ and } \overline{a}_{xy} = \frac{1}{\mu_{xy}+\delta} = \frac{1}{0.14+0.05} = 5.2632$$

$$P = \frac{\overline{A}_{\overline{xy}}}{\overline{a}_{xy}} = \frac{\overline{A}_x + \overline{A}_y - \overline{A}_{xy}}{\overline{a}_{xy}} = \frac{2(0.5833)-0.7368}{5.2632} = 0.0817$$

Question #50
Answer: E

$$\left({}_{20}V_{20} + P_{20}\right)(1+i) - q_{40}\left(1 - {}_{21}V_{20}\right) = {}_{21}V_{20}$$

$$(0.49 + 0.01)(1+i) - 0.022(1 - 0.545) = 0.545$$

$$(1+i) = \frac{(0.545)(1 - 0.022) + 0.022}{0.50}$$

$$= 1.11$$

$$\left({}_{21}V_{20} + P_{20}\right)(1+i) - q_{41}\left(1 - {}_{22}V_{20}\right) = {}_{22}V_{20}$$

$$(0.545 + .01)(1.11) - q_{41}(1 - 0.605) = 0.605$$

$$q_{41} = \frac{0.61605 - 0.605}{0.395}$$

$$= 0.028$$

Question #51
Answer: E

$$1000\ P_{60} = 1000\ A_{60} / \ddot{a}_{60}$$

$$= 1000\ v\left(q_{60} + p_{60} A_{61}\right) / \left(1 + p_{60}\ v\ \ddot{a}_{61}\right)$$

$$= 1000\left(q_{60} + p_{60}\ A_{61}\right) / \left(1.06 + p_{60}\ \ddot{a}_{61}\right)$$

$$= \left(15 + (0.985)(382.79)\right) / \left(1.06 + (0.985)(10.9041)\right) = 33.22$$

Question #52
Answer: E

Method 1:

In each round,
N = result of first roll, to see how many dice you will roll
X = result of for one of the *N* dice you roll
S = sum of *X* for the *N* dice

$$E(X) = E(N) = 3.5$$
$$Var(X) = Var(N) = 2.9167$$

$$E(S) = E(N) * E(X) = 12.25$$
$$Var(S) = E(N)Var(X) + Var(N)E(X)^2$$
$$= (3.5)(2.9167) + (2.9167)(3.5)^2$$
$$= 45.938$$

Let $S_{1000} =$ the sum of the winnings after 1000 rounds

$$E(S_{1000}) = 1000 * 12.25 = 12,250$$
$$Stddev(S_{1000}) = sqrt(1000 * 45.938) = 214.33$$

After 1000 rounds, you have your initial 15,000, less payments of 12,500, plus winnings of S_{1000}.

Since actual possible outcomes are discrete, the solution tests for continuous outcomes greater than 15000-0.5. In this problem, that continuity correction has negligible impact.

$$\Pr(15000 - 12500 + S_{1000} > 14999.5) =$$
$$= \Pr((S_{1000} - 12250)/214.33 > (14999.5 - 2500 - 12250)/214.33) =$$
$$= 1 - \Phi(1.17) = 0.12$$

Method 2

Realize that you are going to determine *N* 1000 times and roll the sum of those 1000 *N*'s dice, adding the numbers showing.

Let $N_{1000} =$ sum of those *N*'s

$$E\left(N_{1000}\right) = 1000E\left(N\right) = (1000)(3.5) = 3500$$

$$Var\left(N_{1000}\right) = 1000Var(N) = 2916.7$$

$$E\left(S_{1000}\right) = E\left(N_{1000}\right)E\left(X\right) = (3500)(3.5) = 12.250$$

$$\begin{aligned} Var\left(S_{1000}\right) &= E\left(N_{1000}\right)Var\left(X\right) + Var\left(N_{1000}\right)E\left(X\right)^2 \\ &= (3500)(2.9167) + (2916.7)(3.5)^2 = 45.938 \end{aligned}$$

$$Stddev\left(S_{1000}\right) = 214.33$$

Now that you have the mean and standard deviation of S_{1000} (same values as method 1), use the normal approximation as shown with method 1.

Question #53
Answer: B

$$p_k = \left(a + \frac{b}{k}\right)p_{k-1}$$

$$0.25 = (a+b) \times 0.25 \Rightarrow a + b = 1$$

$$0.1875 = \left(a + \frac{b}{2}\right) \times 0.25 \Rightarrow \left(1 - \frac{b}{2}\right) \times 0.25 = 0.1875$$

$$b = 0.5$$

$$a = 0.5$$

$$p_3 = \left(0.5 + \frac{0.5}{3}\right) \times 0.1875 = 0.125$$

Question #54
Answer: B

Transform these scenarios into a four-state Markov chain, where the final disposition of rates in any scenario is that they decrease, rather than if rates increase, as what is given.

State	from year *t* – 3 to year *t* – 2	from year *t* – 2 to year *t* – 1	Probability that year *t* will decrease from year *t* - 1
0	Decrease	Decrease	0.8
1	Increase	Decrease	0.6
2	Decrease	Increase	0.75
3	Increase	Increase	0.9

Transition matrix is $\begin{bmatrix} 0.80 & 0.00 & 0.20 & 0.00 \\ 0.60 & 0.00 & 0.40 & 0.00 \\ 0.00 & 0.75 & 0.00 & 0.25 \\ 0.00 & 0.90 & 0.00 & 0.10 \end{bmatrix}$

$$P_{00}^2 + P_{01}^2 = 0.8 * 0.8 + 0.2 * 0.75 = 0.79$$

For this problem, you don't need the full transition matrix. There are two cases to consider. Case 1: decrease in 2003, then decrease in 2004; Case 2: increase in 2003, then decrease in 2004.

For Case 1: decrease in 2003 (following 2 decreases) is 0.8; decrease in 2004 (following 2 decreases is 0.8. Prob(both) = 0.8×0.8 = 0.64
For Case 2: increase in 2003 (following 2 decreases) is 0.2; decrease in 2004 (following a decrease, then increase) is 0.75. Prob(both) = 0.2×0.75 = 0.15
Combined probability of Case 1 and Case 2 is 0.64 + 0.15 = 0.79

Question #55
Answer: B

$$l_x = \omega - x = 105 - x$$
$$\Rightarrow {}_tP_{45} = l_{45+t} / l_{45} = 60 - t / 60$$

Let *K* be the curtate future lifetime of (45). Then the sum of the payments is 0 if $K \le 19$ and is *K* – 19 if $K \ge 20$.

$$_{20|}\ddot{a}_{45} = \sum_{K=20}^{60} 1 \times \left(\frac{60-K}{60}\right) \times 1$$

$$= \frac{(40+39+\ldots+1)}{60} = \frac{(40)(41)}{2(60)} = 13.6\overline{6}$$

Hence,

$$\text{Prob}\left(K - 19 > 13.6\overline{6}\right) = \text{Prob}\left(K > 32.6\overline{6}\right)$$

$$= \text{Prob}(K \ge 33) \text{ since } K \text{ is an integer}$$

$$= \text{Prob}(T \ge 33)$$

$$= {}_{33}p_{45} = \frac{l_{78}}{l_{45}} = \frac{27}{60}$$

$$= 0.450$$

Question #56
Answer: C

$$^2\bar{A}_x = \frac{\mu}{\mu + 2\delta} = 0.25 \rightarrow \mu = 0.04$$

$$\bar{A}_x = \frac{\mu}{\mu + \delta} = 0.4$$

$$\left(I\bar{A}\right)_x = \int_0^\infty {}_{s|}\bar{A}_x \, ds$$

$$\int_0^\infty {}_sE_x \, \bar{A}_x \, ds$$

$$= \int_0^\infty \left(e^{-0.1s}\right)(0.4)\,ds$$

$$= (0.4)\left(\frac{-e^{-0.1s}}{0.1}\right)\Bigg|_0^\infty = \frac{0.4}{0.1} = 4$$

Alternatively, using a more fundamental formula but requiring more difficult integration.

$$\left(I\bar{A}\right)_x = \int_0^\infty t \; {}_tp_x \, \mu_x(t) e^{-\delta t} dt$$

$$= \int_0^\infty t \, e^{-0.04t}(0.04)e^{-0.06t} dt$$

$$= 0.04 \int_0^\infty t \, e^{-0.1t} dt$$

(integration by parts, not shown)

$$= 0.04\left(\frac{-t}{0.1} - \frac{1}{0.01}\right)e^{-0.1t}\Big|_0^\infty$$

$$= \frac{0.04}{0.01} = 4$$

Question #57
Answer: E

Subscripts A and B here just distinguish between the tools and do not represent ages.

We have to find $\overset{o}{e}_{\overline{AB}}$

$$\overset{o}{e}_A = \int_0^{10}\left(1-\frac{t}{10}\right)dt = t-\frac{t^2}{20}\bigg|_0^{10} = 10-5=5$$

$$\overset{o}{e}_B = \int_0^{7}\left(1-\frac{t}{7}\right)dt = t-\frac{t^2}{14}\bigg|_0^{7} = 49-\frac{49}{14}=3.5$$

$$\overset{o}{e}_{AB} = \int_o^{7}\left(1-\frac{t}{7}\right)\left(1-\frac{t}{10}\right)dt = \int_0^{7}\left(1-\frac{t}{10}-\frac{t}{7}+\frac{t^2}{70}\right)dt$$

$$= t-\frac{t^2}{20}-\frac{t^2}{14}+\frac{t^3}{210}\bigg|_0^{7}$$

$$= 7-\frac{49}{20}-\frac{49}{14}+\frac{343}{210}=2.683$$

$$\overset{o}{e}_{\overline{AB}} = \overset{o}{e}_A+\overset{o}{e}_B-\overset{o}{e}_{AB}$$

$$=5+3.5-2.683=5.817$$

Question #58
Answer: A

$$\mu_x^{(\tau)}(t) = 0.100 + 0.004 = 0.104$$

$$_t p_x^{(\tau)} = e^{-0.104t}$$

Actuarial present value (APV) = APV for cause 1 + APV for cause 2.

$$2000\int_0^5 e^{-0.04t} e^{-0.104t}(0.100)dt + 500{,}000\int_0^5 e^{-0.04t}\, e^{-0.104t}(0.400)dt$$

$$= \left(2000(0.10) + 500{,}000(0.004)\right)\int_0^5 e^{-0.144t}dt$$

$$= \frac{2200}{0.144}\left(1 - e^{-0.144(5)}\right) = 7841$$

Question #59
Answer: A

$$R = 1 - p_x = q_x$$

$$S = 1 - p_x \times e^{(-k)} \text{ since } e^{-\int_0^1(\mu_x(t)+k)dt} = e^{-\int_0^1\mu_x(t)dt - \int_0^1 k\,dt}$$

$$= e^{-\int_0^1\mu_x(t)dt}\, e^{-\int_0^1 k\,dt}$$

So $S = 0.75R \Rightarrow 1 - p_x \times e^{-k} = 0.75q_x$

$$e^{-k} = \frac{1 - 0.75q_x}{p_x}$$

$$e^k = \frac{p_x}{1 - 0.75q_x} = \frac{1 - q_x}{1 - 0.75q_x}$$

$$k = \ln\left[\frac{1 - q_x}{1 - 0.75q_x}\right]$$

Question #60
Answer: E

$\beta =$ mean $= 4;\qquad p_k = \beta^k / (1+\beta)^{k+1}$

n	$P(N=n)$
0	0.2
1	0.16
2	0.128
3	0.1024

x	$f^{(1)}(x)$	$f^{(2)}(x)$	$f^{(3)}(x)$
0	0	0	0
1	0.25	0	0
2	0.25	0.0625	0
3	0.25	0.125	0.0156

$f^{(k)}(x) =$ probability that, given exactly *k* claims occur, that the aggregate amount is *x*.

$f^{(1)}(x) = f(x);$ the claim amount distribution for a single claim

$f^{(k)}(x) = \sum_{j=0}^{x} \left(f^{(k-1)}(j) \right) x\, f(x-j)$

$f_s(x) = \sum_{k=0}^{x} P(N=k) \times f^{(k)}(x)$; upper limit of sum is really ∞, but here with smallest possible claim size 1, $f^{(k)}(x) = 0$ for $k > x$

$f_s(0) = 0.2$
$f_s(1) = 0.16 * 0.25 = 0.04$
$f_s(2) = 0.16 * 0.25 + 0.128 * 0.0625 = 0.048$
$f_s(3) = 0.16 * 0.25 + 0.128 * 0.125 + 0.1024 * 0.0156 = 0.0576$

$F_s(3) = 0.2 + 0.04 + 0.048 + 0.0576 = 0.346$

Question #61
Answer: E

Let L = incurred losses; P = earned premium = 800,000

Bonus $= 0.15 \times \left(0.60 - \frac{L}{P}\right) \times P$ if positive

$= 0.15 \times (0.60P - L)$ if positive

$= 0.15 \times (480{,}000 - L)$ if positive

$= 0.15 \times \left(480{,}000 - (L \wedge 480{,}000)\right)$

E (Bonus) = 0.15 (480,000– $E\left(L \wedge 480{,}000\right)$)

From Appendix A.2.3.1

= 0.15{480,000 – [500,000 $\times$ (1 – (500,000 / (480,000+500,000)))]}

= 35,265

Question #62
Answer: D

$$\bar{A}^{\,1}_{28:\overline{2|}} = \int_0^2 e^{-\delta t}\, {}^{1}\!/_{72}\, dt$$

$$= \frac{1}{72\delta}\left(1 - e^{-2\delta}\right) = 0.02622 \text{ since } \delta = \ln(1.06) = 0.05827$$

$$\ddot{a}_{28:\overline{2|}} = 1 + v\left(\frac{71}{72}\right) = 1.9303$$

$${}_3V = 500{,}000\bar{A}^{\,1}_{28:\overline{2|}} - 6643\,\ddot{a}_{28:\overline{2|}}$$

= 287

Question #63
Answer: D

Let $\bar{A}_x$ and $\bar{a}_x$ be calculated with $\mu_x(t)$ and $\delta = 0.06$

Let $\bar{A}_x^{\,*}$ and $\bar{a}_x^{\,*}$ be the corresponding values with $\mu_x(t)$ increased by 0.03 and δ decreased by 0.03

$$\bar{a}_x = \frac{1-\bar{A}_x}{\delta} = \frac{0.4}{0.06} = 6.667$$

$$\bar{a}_x{}^* = \bar{a}_x$$

$$\left[\text{Proof: } \bar{a}_x{}^* = \int_0^\infty e^{-\int_0^t (\mu_x(s)+0.03)ds} e^{-0.03t} dt\right.$$

$$= \int_0^\infty e^{-\int_0^t \mu_x(s)ds} e^{-0.03t} e^{-0.03t} dt$$

$$= \int_0^\infty e^{-\int_0^t \mu_x(s)ds} e^{-0.06t} dt$$

$$\left. = \bar{a}_x \right]$$

$$\bar{A}_x{}^* = 1 - 0.03\bar{a}_x{}^* = 1 - 0.03\bar{a}_x$$

$$= 1 - (0.03)(6.667)$$

$$= 0.8$$

Question #64
Answer: A

	bulb ages				
Year	0	1	2	3	# replaced
0	10000	0	0	0	-
1	1000	9000	0	0	1000
2	100+2700	900	6300	0	2800
3	280+270+3150				3700

The diagonals represent bulbs that don’t burn out.
E.g., of the initial 10,000, (10,000) (1-0.1) = 9000 reach year 1.
(9000) (1-0.3) = 6300 of those reach year 2.

Replacement bulbs are new, so they start at age 0.
At the end of year 1, that’s (10,000) (0.1) = 1000
At the end of 2, it’s (9000) (0.3) + (1000) (0.1) = 2700 + 100
At the end of 3, it’s (2800) (0.1) + (900) (0.3) + (6300) (0.5) = 3700

Actuarial present value $= \frac{1000}{1.05} + \frac{2800}{1.05^2} + \frac{3700}{1.05^3}$

$= 6688$

Question #65
Key: E

Model Solution:

$$\begin{aligned}\mathring{e}_{25:\overline{25}|} &= \int_0^{15} {}_t p_{25}\,dt + {}_{15}p_{25}\int_0^{10} {}_t p_{40}\,dt \\ &= \int_0^{15} e^{-.04t}dt + \left(e^{-\int_0^{15}.04\,ds}\right)\int_0^{10} e^{-.05t}dt \\ &= \frac{1}{.04}\left(1-e^{-.60}\right) + e^{-.60}\left[\frac{1}{.05}\left(1-e^{-.50}\right)\right] \\ &= 11.2797 + 4.3187 \\ &= 15.60\end{aligned}$$

Question #66
Key: C

Model Solution:

$$\begin{aligned}&{}_5 p_{[60]+1} = \\ &\left(1-q_{[60]+1}\right)\left(1-q_{[60]+2}\right)\left(1-q_{63}\right)\left(1-q_{64}\right)\left(1-q_{65}\right) \\ &= (0.89)(0.87)(0.85)(0.84)(0.83) \\ &= 0.4589\end{aligned}$$

Question # 67
Key: E

Model Solution:

$$12.50 = \bar{a}_x = \frac{1}{\mu+\delta} \Rightarrow \mu+\delta = 0.08 \Rightarrow \mu = \delta = 0.04$$

$$\bar{A}_x = \frac{\mu}{\mu+\delta} = 0.5$$

$${}^2\bar{A}_x = \frac{\mu}{\mu+2\delta} = \frac{1}{3}$$

$$\mathrm{Var}\left(\bar{a}_{\overline{T|}}\right) = \frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2}$$

$$= \frac{\frac{1}{3} - \frac{1}{4}}{0.0016} = 52.083$$

S.D. $= \sqrt{52.083} = 7.217$

Question # 68
Key: D

Model Solution:

$$v = 0.90 \Rightarrow d = 0.10$$
$$A_x = 1 - d\ddot{a}_x = 1 - (0.10)(5) = 0.5$$

Benefit premium $\pi = \dfrac{5000A_x - 5000vq_x}{\ddot{a}_x}$

$$= \frac{(5000)(0.5) - 5000(0.90)(0.05)}{5} = 455$$

$${}_{10}V_x = 1 - \frac{\ddot{a}_{x+10}}{\ddot{a}_x}$$

$$0.2 = 1 - \frac{\ddot{a}_{x+10}}{5} \Rightarrow \ddot{a}_{x+10} = 4$$

$$A_{x+10} = 1 - d\ddot{a}_{x+10} = 1 - (0.10)(4) = 0.6$$
$${}_{10}V = 5000A_{x+10} - \pi\ddot{a}_{x+10} = (5000)(0.6) - (455)(4) = 1180$$

Question #68
Key: D

Model Solution:

v is the lowest premium to ensure a zero % chance of loss in year 1 (The present value of the payment upon death is *v*, so you must collect at least *v* to avoid a loss should death occur). Thus *v* = 0.95.

$$\mathrm{E}(\mathrm{Z}) = vq_x + v^2 p_x q_{x+1} = 0.95 \times 0.25 + (0.95)^2 \times 0.75 \times 0.2$$
$$= 0.3729$$
$$\mathrm{E}(\mathrm{Z}^2) = v^2 q_x + v^4 p_x q_{x+1} = (0.95)^2 \times 0.25 + (0.95)^4 \times 0.75 \times 0.2$$
$$= 0.3478$$

$$\mathrm{Var}(\mathrm{Z}) = \mathrm{E}(\mathrm{Z}^2) - (\mathrm{E}(\mathrm{Z}))^2 = 0.3478 - (0.3729)^2 = 0.21$$

Question # 70
Key: D

Model Solution:

Severity after increase	Severity after increase and deductible
60	0
120	20
180	80
300	200

Expected payment per loss $= 0.25 \times 0 + 0.25 \times 20 + 0.25 \times 80 + 0.25 \times 200$
= 75

Expected payments = Expected number of losses $\times$ Expected payment per loss
= 75×300
= 22,500

Question # 71
Key: A

Model Solution:

E (S) = E (N) E (X) = 50 $\times$ 200 = 10,000
$$\mathrm{Var}(\mathrm{S}) = \mathrm{E}(\mathrm{N})\mathrm{Var}(X) + \mathrm{E}(X)^2\,\mathrm{Var}(\mathrm{N})$$
$$= (50)(400) + (200^2)(100)$$
$$= 4{,}020{,}000$$

$$\Pr(S < 8{,}000) = \Pr\left(Z < \frac{8{,}000 - 10{,}000}{\sqrt{4{,}020{,}000}}\right)$$
$$= \Pr(Z < -0.998) \cong 16\%$$

Question #72
Key: A

Model Solution:

Let Z be the present value random variable for one life.
Let S be the present value random variable for the 100 lives.

$$E(Z) = 10\int_5^\infty e^{\delta t} e^{\mu t} \mu \, dt$$
$$= 10\frac{\mu}{\delta + \mu} e^{-(\delta+\mu)5}$$
$$= 2.426$$
$$E(Z^2) = 10^2\left(\frac{\mu}{2\delta + \mu}\right) e^{-(2\delta+\mu)5}$$
$$= 10^2\left(\frac{0.04}{0.16}\right)\left(e^{-0.8}\right) = 11.233$$
$$Var(Z) = E(Z^2) - (E(Z))^2$$
$$= 11.233 - 2.426^2$$
$$= 5.348$$

$$\mathrm{E}(\mathrm{S}) = 100\,\mathrm{E}(\mathrm{Z}) = 242.6$$
$$\mathrm{Var}(\mathrm{S}) = 100\ \mathrm{Var}(\mathrm{Z}) = 534.8$$
$$\frac{F - 242.6}{\sqrt{534.8}} = 1.645 \rightarrow F = 281$$

Question #73
Key: D

Model Solution:

Prob{only 1 survives} = 1-Prob{both survive}-Prob{neither survives}

$$
\begin{aligned}
&= 1 - {}_3p_{50} \times {}_3p_{[50]} - \left(1 - {}_3p_{50}\right)\left(1 - {}_3p_{[50]}\right) \\
&= 1 - \underbrace{(0.9713)(0.9698)(0.9682)}_{=0.912012}\underbrace{(0.9849)(0.9819)(0.9682)}_{0.936320} - (1 - 0.912012)(1 - 0.93632) \\
&= 0.140461
\end{aligned}
$$

Question # 74
Key: C

Model Solution:

The tyrannosaur dies at the end of the first day if it eats no scientists that day. It dies at the end of the second day if it eats exactly one the first day and none the second day. If it does not die by the end of the second day, it will have at least 10,000 calories then, and will survive beyond 2.5.

Prob (ruin) $= f(0) + f(1)f(0)$
$= 0.368 + (0.368)(0.368)$
$= 0.503$

since $f(0) = \dfrac{e^{-1}1^0}{0!} = 0.368$

$f(1) = \dfrac{e^{-1}1^1}{1!} = 0.368$

Question #75
Key: B

Model Solution:

Let X = expected scientists eaten.
For each period, $E[X] = E[X|\text{dead}] \times \text{Prob}(\text{already dead}) + E[X|\text{alive}] \times \text{Prob}(\text{alive})$
$= 0 \times \text{Prob}(\text{dead}) + E[X|\text{alive}] \times \text{Prob}(\text{alive})$

Day 1, $\text{E}[X_1] = 1$

$\text{Prob}(\text{dead at end of day 1}) = f(0) = \dfrac{e^{-1}0^1}{0!} = 0.368$

Day 2, $\text{E}[X_2] = 0 \times 0.368 + 1 \times (1 - 0.368) = 0.632$

Prob (dead at end of day 2) = 0.503
[per problem 10]

Day 2.5, $\mathrm{E}\left[X_{2.5}\right]=0\times 0.503+0.5\times(1-0.503)=0.249$
where $E\left[X_{2.5}|\text{alive}\right]=0.5$ since only $\frac{1}{2}$ day in period.

$$\mathrm{E}[X]=\mathrm{E}[X_1]+\mathrm{E}[X_2]+\mathrm{E}[X_{2.5}]=1+0.632+0.249=1.881$$
$$\mathrm{E}[10{,}000X]=18{,}810$$

Question # 76
Key: C

Model Solution:

This solution applies the equivalence principle to each life. Applying the equivalence principle to the 100 life group just multiplies both sides of the first equation by 100, producing the same result for *P*.

$$APV(\text{Prems})=P=APV(\text{Benefits})=10q_{70}v+10\,p_{70}q_{71}v^2+Pp_{70}p_{71}v^2$$
$$P=\frac{(10)(0.03318)}{1.08}+\frac{(10)(1-0.03318)(0.03626)}{1.08^2}+\frac{P(1-0.03318)(1-0.03626)}{1.08^2}$$
$$=0.3072+0.3006+0.7988P$$
$$P=\frac{0.6078}{0.2012}=3.02$$

(*APV* above means Actuarial Present Value).

Question #77
Key: E

Model Solution:

One approach is to recognize an interpretation of formula 7.4.11 or exercise 7.17a:

> Level benefit premiums can be split into two pieces: one piece to provide term insurance
> for *n* years; one to fund the reserve for those who survive.

If you think along those lines, you can derive formula 7.4.11:

$$P_x=P^1_{x:\overline{n}|}+P_{x:\overline{n}|}^{\ \ 1}\ {}_nV_x$$

And plug in to get

$$0.090 = P^{1}_{x:\overline{n}|} + (0.00864)(0.563)$$

$$P^{1}_{x:\overline{n}|} = 0.0851$$

Another approach is to think in terms of retrospective reserves. Here is one such solution:

$$\begin{aligned}
{}_nV_x &= \left(P_x - P^{1}_{x:\overline{n}|}\right)\ddot{s}_{x:\overline{n}|} \\
&= \left(P_x - P^{1}_{x:\overline{n}|}\right)\frac{\ddot{a}_{x:\overline{n}|}}{{}_nE_x} \\
&= \left(P_x - P^{1}_{x:\overline{n}|}\right)\frac{\ddot{a}_{x:\overline{n}|}}{P_{x:\overline{n}|}^{\ \ 1}\ddot{a}_{x:\overline{n}|}} \\
&= \frac{\left(P_x - P^{1}_{x:\overline{n}|}\right)}{\left(P_{x:\overline{n}|}^{\ \ 1}\right)}
\end{aligned}$$

$$0.563 = \left(0.090 - P^{1}_{x:\overline{n}|}\right) / \ 0.00864$$

$$\begin{aligned}
P^{1}_{x:\overline{n}|} &= 0.090 - (0.00864)(0.563) \\
&= 0.0851
\end{aligned}$$

Question #78
Key: A

Model Solution:

$$\delta = \ln(1.05) = 0.04879$$

$$\begin{aligned}
\bar{A}_x &= \int_0^{\omega - x} {}_tp_x\mu_x(t)e^{-\delta t}dt \\
&= \int_0^{\omega - x} \frac{1}{\omega - x}e^{-\delta t}dt \ \ \text{for DeMoivre} \\
&= \frac{1}{\omega - x}\bar{a}_{\overline{\omega - x}|}
\end{aligned}$$

From here, many formulas for ${}_{10}\bar{V}\left(\bar{A}_{40}\right)$ could be used. One approach is:

Since

$$\bar{A}_{50} = \frac{\bar{a}_{\overline{50}|}}{50} = \frac{18.71}{50} = 0.3742 \text{ so } \bar{a}_{50} = \left(\frac{1-\bar{A}_{50}}{\delta}\right) = 12.83$$

$$\bar{A}_{40} = \frac{\bar{a}_{\overline{60}|}}{60} = \frac{19.40}{60} = 0.3233 \text{ so } \bar{a}_{40} = \left(\frac{1-\bar{A}_{40}}{\delta}\right) = 13.87$$

$$\text{so } \bar{P}\left(\bar{A}_{40}\right) = \frac{0.3233}{13.87} = 0.02331$$

$${}_{10}\bar{V}\left(\bar{A}_{40}\right) = \left[\bar{A}_{50} - \bar{P}\left(\bar{A}_{40}\right)\bar{a}_{50}\right] = \left[0.3742 - (0.02331)(12.83)\right] = 0.0751.$$

Question #79
Key: D

Model Solution:

$$\bar{A}_x = E\left[v^{T(x)}\right] = E\left[v^{T(x)}|NS\right] \times \text{Prob}(\text{NS}) + E\left[v^{T(x)}|S\right] \times \text{Prob}(\text{S})$$
$$= \left(\frac{0.03}{0.03+0.08}\right) \times 0.70 + \left(\frac{0.6}{0.06+0.08}\right) \times 0.30$$
$$= 0.3195$$

Similarly, ${}^2\bar{A}_x = \left(\frac{0.03}{0.03+0.16}\right) \times 0.70 + \left(\frac{0.06}{0.06+0.16}\right) \times 0.30 = 0.1923.$

$$\text{Var}\left(\bar{a}_{\overline{T(x)}|}\right) = \frac{{}^2\bar{A}_x - \bar{A}_x^2}{\delta^2} = \frac{0.1923 - 0.3195^2}{0.08^2} = 14.1.$$

Question #80
Key: B

Model Solution:

Let *S* denote aggregate losses before deductible.

$E[S] = 2 \times 2 = 4$, since mean severity is 2.

$f_S(0) = \frac{e^{-2}2^0}{0!} = 0.1353$, since must have 0 number to get aggregate losses = 0.

$f_S(1) = \left(\frac{e^{-2}2}{1!}\right)\left(\frac{1}{3}\right) = 0.0902$, since must have 1 loss whose size is 1 to get aggregate losses = 1.

$$\begin{aligned} E(S \wedge 2) &= 0 \times f_S(0) + 1 \times f_S + 2 \times (1 - f_S(0) - f_S(1)) \\ &= 0 \times 0.1353 + 1 \times 0.0902 + 2 \times (1 - 0.1353 - 0.0902) \\ &= 1.6392 \end{aligned}$$

$$\begin{aligned} E\left[(S-2)_+\right] &= E[S] - E[S \wedge 2] \\ &= 4 - 1.6392 \\ &= 2.3608 \end{aligned}$$

Question #81
Key: D

Model Solution:

Poisson processes are separable. The aggregate claims process is therefore equivalent to two independent processes, one for Type I claims with expected frequency $\left(\frac{1}{3}\right)(3000) = 1000$ and one for Type II claims.

Let $S_I =$ aggregate Type I claims.
$N_I =$ number of Type I claims.
$X_I =$ severity of a Type I claim (here = 10).

Since $X_I = 10, \text{ a constant}, E(X_I) = 10; \text{ Var}(X_I) = 0.$

$$\begin{aligned} \text{Var}(S_I) &= E(N_I)\ \text{Var}(X_I) + \text{Var}(N_I)\left[E(X_I)\right]^2 \\ &= (1000)(0) + (1000)(10)^2 \\ &= 100{,}000 \end{aligned}$$

$$\text{Var}(S) = \text{Var}(S_I) + \text{Var}(S_{II}) \text{ since independent}$$
$$2{,}100{,}000 = 100{,}000 + \text{Var}(S_{II})$$
$$Var(S_{II}) = 2{,}000{,}000$$

Question #82
Key: A

Model Solution:

$$\begin{aligned} {}_5 p_{50}^{(\tau)} &= {}_5 p_{50}'^{(1)}\ {}_5 p_{50}'^{(2)} \\ &= \left(\frac{100-55}{100-50}\right) e^{-(0.05)(5)} \\ &= (0.9)(0.7788) = 0.7009 \end{aligned}$$

Similarly

$$\begin{aligned} {}_{10} p_{50}^{(\tau)} &= \left(\frac{100-60}{100-50}\right) e^{-(0.05)(10)} \\ &= (0.8)(0.6065) = 0.4852 \end{aligned}$$

$$\begin{aligned} {}_{5|5} q_{50}^{(\tau)} = {}_5 p_{50}^{(\tau)} - {}_{10} p_{50}^{(\tau)} &= 0.7009 - 0.4852 \\ &= 0.2157 \end{aligned}$$

Question #83
Key: C

Model Solution:

Only decrement 1 operates before $t = 0.7$

${}_{0.7} q_{40}'^{(1)} = (0.7)\, q_{40}'^{(1)} = (0.7)(0.10) = 0.07$ since UDD

Probability of reaching $t = 0.7$ is 1-0.07 = 0.93

Decrement 2 operates only at $t = 0.7$, eliminating 0.125 of those who reached 0.7

$$q_{40}^{(2)} = (0.93)(0.125) = 0.11625$$

Question #84
Key: C

Model Solution:

$$\pi\left(1+{}_2p_{80}v^2\right)=1000A_{80}+\frac{\pi v q_{80}}{2}+\frac{\pi v^3\,{}_2p_{80}q_{82}}{2}$$

$$\pi\left(1+\frac{0.83910}{1.06^2}\right)=665.75+\pi\left(\frac{0.08030}{2(1.06)}+\frac{0.83910\times 0.09561}{2(1.06)^3}\right)$$

$$\pi(1.74680)=665.75+\pi(0.07156)$$
$$\pi(1.67524)=665.75$$
$$\pi=397.41$$

Where ${}_2p_{80}=\dfrac{3{,}284{,}542}{3{,}914{,}365}=0.83910$

Or ${}_2p_{80}=(1-0.08030)(1-0.08764)=0.83910$

Question #85
Key: E

Model Solution:

At issue, actuarial present value (APV) of benefits

$$=\int_0^\infty b_t v^t\,{}_tp_{65}\,\mu_{65}(t)dt$$
$$=\int_0^\infty 1000\left(e^{0.04t}\right)\left(e^{-0.04t}\right){}_tp_{65}\,\mu_{65}(t)dt$$
$$=1000\int_0^\infty {}_tp_{65}\,\mu_{65}(t)dt=1000\,{}_\infty q_{65}=1000$$

APV of premiums $=\pi\,\bar{a}_{65}=\pi\left(\dfrac{1}{0.04+0.02}\right)=16.667\pi$

Benefit premium $\pi=1000/16.667=60$

$${}_2\bar{V}=\int_0^\infty b_{2+u}\,v^u\,{}_up_{67}\,\mu_{65}(2+u)du-\pi\,\bar{a}_{67}$$
$$=\int_0^\infty 1000\,e^{0.04(2+u)}e^{-0.04u}\,{}_up_{67}\,\mu_{65}(2+u)du-(60)(16.667)$$
$$=1000e^{0.08}\int_0^\infty {}_up_{67}\mu_{65}(2+u)\,du-1000$$
$$=1083.29\,{}_\infty q_{67}-1000=1083.29-1000=83.29$$

Question #86
Key: B

Model Solution:

(1) $a_{x:\overline{20}|} = \ddot{a}_{x:\overline{20}|} - 1 + {}_{20}E_x$

(2) $\ddot{a}_{x:\overline{20}|} = \dfrac{1 - A_{x:\overline{20}|}}{d}$

(3) $A_{x:\overline{20}|} = A^1_{x:\overline{20}|} + A_{x:\overline{20}|}^{\;\;\;1}$

(4) $A_x = A^1_{x:\overline{20}|} + {}_{20}E_x \; A_{x+20}$

$$0.28 = A^1_{x:\overline{20}|} + (0.25)(0.40)$$
$$A^1_{x:\overline{20}|} = 0.18$$

Now plug into (3): $A_{x:\overline{20}|} = 0.18 + 0.25 = 0.43$

Now plug into (2): $\ddot{a}_{x:\overline{20}|} = \dfrac{1 - 0.43}{(0.05/1.05)} = 11.97$

Now plug into (1): $a_{x:\overline{20}|} = 11.97 - 1 + 0.25 = 11.22$

Question #87
Key: A

Model Solution:

$$E[N] = E_\Lambda\big[E[N|\Lambda]\big] = E_\Lambda[\Lambda] = 2$$

$$\begin{aligned}\mathrm{Var}[N] &= E_\Lambda\big[\mathrm{Var}[N|\Lambda]\big] + \mathrm{Var}_\Lambda\big[E[N|\Lambda]\big] \\ &= E_\Lambda[\Lambda] + Var_\Lambda[\Lambda] = 2 + 4 = 6\end{aligned}$$

Distribution is negative binomial.

Per supplied tables:

$$r\beta = 2 \text{ and } r\beta(1+\beta) = 6$$
$$1+\beta = 3$$
$$\beta = 2$$
$$r\beta = 2 \quad r = 1$$

$$p_1 = \frac{r\beta^1}{1!(1+\beta)^{r+1}} = \frac{(1)(2)}{(1)(3)^2} = 0.22$$

Alternatively, if you don't recognize that *N* will have a negative binomial distribution, derive gamma density from moments (hoping α is an integer).

$$\text{Mean} = \theta\alpha = 2$$

$$Var = E\left[\Lambda^2\right] - E[\Lambda]^2 = \theta^2\left(\alpha^2+\alpha\right) - \theta^2\alpha^2$$
$$= \theta^2\alpha = 4$$

$$\theta = \frac{\theta^2\alpha}{\theta\alpha} = \frac{4}{2} = 2$$
$$\alpha = \frac{\theta\alpha}{\theta} = 1$$

$$p_1 = \int_0^\infty \left(p_1|\lambda\right) f(\lambda)d\lambda = \int_0^\infty \frac{e^{-\lambda}\lambda^1}{1!} \frac{(\lambda/2)e^{-(\lambda/2)}}{\lambda\Gamma(1)} d\lambda$$
$$= \frac{1}{2}\int_0^\infty \lambda e^{-\frac{\alpha}{2}} d\lambda$$

$\left[\text{Integrate by parts; not shown}\right]$

$$= \frac{1}{2}\left(-\frac{2}{3}\lambda e^{-\frac{3}{2}\lambda} - \frac{4}{9}e^{-\frac{3}{2}\lambda}\right)\Bigg|_0^\infty$$

$$= \frac{2}{9} = 0.22$$

Question #88
Key: C

Model Solution:

Limited expected value =

$$\int_0^{1000}\left(1-F(x)\right)dx=\int_0^{1000}\left(0.8e^{-0.02x}+0.2e^{-0.001x}\right)dx=\left(-40e^{-0.02x}-200e^{-0.001x}\right)\Big|_0^{1000}=40+126.4$$

= 166.4

Question #89
Key: E

Model Solution:

$$M=\text{Initial state matrix }=\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix}$$

$$T=\text{One year transition matrix }=\begin{bmatrix}0.20 & 0.80 & 0 & 0\\ 0.50 & 0 & 0.50 & 0\\ 0.75 & 0 & 0 & 0.25\\ 1.00 & 0 & 0 & 0\end{bmatrix}$$

$$M\times T=\begin{bmatrix}0.20 & 0.80 & 0 & 0\end{bmatrix}$$
$$(M\times T)\times T=\begin{bmatrix}0.44 & 0.16 & 0.40 & 0\end{bmatrix}$$
$$((M\times T)\times T)\times T=\begin{bmatrix}0.468 & 0.352 & 0.08 & 0.10\end{bmatrix}$$

Probability of being in state *F* after three years = 0.468.

$$\text{Actuarial present value}=\left(0.468v^3\right)(500)=171$$

Notes:
1. Only the first entry of the last matrix need be calculated (verifying that the four sum to 1 is useful "quality control.")
2. Compare this with solution 23. It would be valid to calculate T^3 here, but advancing *M* one year at a time seems easier.

Question #90
Key: B

Model Solution:

Let Y_i be the number of claims in the ith envelope.

Let $X(13)$ be the aggregate number of claims received in 13 weeks.

$$E[Y_i] = (1\times 0.2) + (2\times 0.25) + (3\times 0.4) + (4\times 0.15) = 2.5$$
$$E[Y_i^2] = (1\times 0.2) + (4\times 0.25) + (9\times 0.4) + (16\times 0.15) = 7.2$$
$$E[X(13)] = 50\times 13\times 2.5 = 1625$$
$$\text{Var}[X(13)] = 50\times 13\times 7.2 = 4680$$
$$\text{Prob}\{\text{X}(13) \le Z\} = 0.90 = \Phi(1.282)$$
$$\Rightarrow \text{Prob}\left\{\frac{X(13)-1625}{\sqrt{4680}} \le 1.282\right\}$$
$$X(13) \le 1712.7$$

Note: The formula for $\text{Var}[X(13)]$ took advantage of the frequency’s being Poisson. The more general formula for the variance of a compound distribution, $\text{Var}(S) = E(N)\ \text{Var}(X) + \text{Var}(N)E(X)^2$, would give the same result.

Question #91
Key: E

Model Solution:

$$\mu^M(60) = \frac{1}{\omega-60} = \frac{1}{75-60} = \frac{1}{15}$$
$$\mu^F(60) = \frac{1}{\omega'-60} = \frac{1}{15}\times\frac{3}{5} = \frac{1}{25} \Rightarrow \omega' = 85$$
$${}_t p_{65}^M = 1-\frac{t}{10}$$
$${}_t p_{60}^F = 1-\frac{t}{25}$$

Let *x* denote the male and *y* denote the female.

$\overset{\circ}{e}_x = 5$ (mean for uniform distribution over $(0,10)$)

$\overset{\circ}{e}_y = 12.5$ (mean for uniform distribution over $(0,25)$)

$$\begin{aligned}
\overset{\circ}{e}_{xy} &= \int_0^{10}\left(1-\frac{t}{10}\right)\left(1-\frac{t}{25}\right)\cdot dt \\
&= \int_0^{10}\left(1-\frac{7}{50}t+\frac{t^2}{250}\right)\cdot dt \\
&= \left(t-\frac{7}{100}t^2+\frac{t^3}{750}\right)\Bigg|_0^{10} = 10-\frac{7}{100}\times 100+\frac{1000}{750} \\
&= 10-7+\frac{4}{3}=\frac{13}{3}
\end{aligned}$$

$$\overset{\circ}{e}_{\overline{xy}} = \overset{\circ}{e}_x+\overset{\circ}{e}_y-\overset{\circ}{e}_{xy} = 5+\frac{25}{2}-\frac{13}{3}=\frac{30+75-26}{6}=13.17$$

Question #92
Key: B

Model Solution:

$$\bar{A}_x = \frac{\mu}{\mu+\delta}=\frac{1}{3}$$

$$^2\bar{A}_x = \frac{\mu}{\mu+2\delta}=\frac{1}{5}$$

$$\bar{P}(\bar{A}_x)=\mu=0.04$$

$$\begin{aligned}
\mathrm{Var}(L) &= \left(1+\frac{\bar{P}(\bar{A}_x)}{\delta}\right)^2\left({}^2\bar{A}_x-\bar{A}_x^2\right) \\
&= \left(1+\frac{0.04}{0.08}\right)^2\left(\frac{1}{5}-\left(\frac{1}{3}\right)^2\right) \\
&= \left(\frac{3}{2}\right)^2\left(\frac{4}{45}\right) \\
&= \frac{1}{5}
\end{aligned}$$

Question #93
Key: B

Model Solution:

$$\text{Mean excess loss} = \frac{E(X) - E(X \wedge 100)}{1 - F(100)}$$
$$= \frac{331 - 91}{0.8} = 300$$

$E(X) = E(X \wedge 1000)$ since $F(1000) = 1.0$

Question #94
Key: E

Model Solution:

Expected insurance benefits per factory $= E\left[(X-1)_+\right]$
$= 0.2 \times 1 + 0.1 \times 2 = 0.4.$

Insurance premium = (1.1) (2 factories) (0.4 per factory) = 0.88.

Let *R* = retained major repair costs, then

$$f_R(0) = 0.4^2 = 0.16$$
$$f_R(1) = 2 \times 0.4 \times 0.6 = 0.48$$
$$f_R(2) = 0.6^2 = 0.36$$

Dividend $= 3 - 0.88 - R - (0.15)(3)$, if positive
$= 1.67 - R$, if positive

$$E(\text{Dividend}) = (0.16)(1.67 - 0) + (0.48)(1.67 - 1) + (0.36)(0) = 0.5888$$

[The (0.36)(0) term in the last line represents that with probability 0.36, $(1.67 - R)$ is negative so the dividend is 0.]

Question #95
Key: A

Model Solution:

$$E[X] = \frac{\alpha\theta}{\alpha - 1} = \frac{4\alpha}{\alpha - 1} = 8 \Rightarrow 4\alpha = 8\alpha - 8$$
$$\alpha = 2$$

$$F(6) = 1 - \left(\frac{\theta}{6}\right)^{\alpha} = 1 - \left(\frac{4}{6}\right)^{2}$$

$$= 0.555$$

$$s(6) = 1 - F(6) = 0.444$$

Question #96
Key: B

Model Solution:

$$e_x = p_x + {}_2p_x + {}_3p_x + \ldots = 11.05$$

$$\begin{aligned}
\text{Annuity} &= v^3\ {}_3p_x 1000 + v^4\ {}_4p_x \times 1000 \times (1.04) + \ldots \\
&= \sum_{k=3}^{\infty} 1000(1.04)^{k-3} v^k\ {}_kp_x \\
&= 1000v^3 \sum_{k=3}^{\infty} {}_k p_x \\
&= 1000v^3(e_x - 0.99 - 0.98) = 1000\left(\frac{1}{1.04}\right)^3 \times 9.08 = 8072
\end{aligned}$$

Let π = benefit premium.

$$\begin{aligned}
\pi\left(1 + 0.99v + 0.98v^2\right) &= 8072 \\
2.8580\pi &= 8072 \\
\pi &= 2824
\end{aligned}$$

Question #97
Key B

Model Solution:

$$\pi\, \ddot{a}_{30:\overline{10}|} = 1000 A_{30} + P(IA)^{1}_{30:\overline{10}|} + (10\pi)\left({}_{10|}A_{30}\right)$$

$$\pi = \frac{1000 A_{30}}{\ddot{a}_{30:\overline{10}|} - (IA)^{1}_{30:\overline{10}|} - 10\,{}_{10|}A_{30}}$$

$$= \frac{1000(0.102)}{7.747 - 0.078 - 10(0.088)}$$

$$= \frac{102}{6.789}$$

$$= 15.024$$

Test Question: 98 Key: E

For de Moivre's law,

$$\mathring{e}_{30} = \int_0^{\omega-30}\left(1 - \frac{t}{\omega-30}\right)dt$$

$$= \left[t - \frac{t^2}{2(\omega-30)}\right]_0^{\omega-30}$$

$$= \frac{\omega-30}{2}$$

Prior to medical breakthrough $\omega = 100 \Rightarrow \mathring{e}_{30} = \dfrac{100-30}{2} = 35$

After medical breakthrough $\mathring{e}'_{30} = \mathring{e}_{30} + 4 = 39$

so $\mathring{e}'_{30} = 39 = \dfrac{\omega'-30}{2} \Rightarrow \omega' = 108$

Test Question: 99 Key: A

$_0L = 100{,}000v^{2.5} - 4000\ddot{a}_{\overline{3}|}$ @5%

= 77,079

Test Question: 100 Key: C

$$E[N] = E_\Lambda\left[E[N|\Lambda]\right] = E_\Lambda[\Lambda] = 2$$

$$Var[N] = E_\Lambda\left[Var[N|\Lambda]\right] + Var_\Lambda\left[E[N|\Lambda]\right]$$

$$= E_\Lambda[\Lambda] + Var_\Lambda[\Lambda] = 2 + 2 = 4$$

Distribution is negative binomial (<u>Loss Models</u>, 3.3.2)

Per supplied tables

$$mean = r\beta = 2$$

$$Var = r\beta(1+\beta) = 4$$

$$(1+\beta) = 2$$

$$\beta = 1$$

$$r\beta = 2$$

$$r = 2$$

From tables

$$p_3 = \frac{r(r+1)(r+2)\beta^3}{3!(1+\beta)^{r+3}} = \frac{(2)(3)(4)1^3}{3!2^5} = \frac{4}{32} = 0.125$$

1000 $p_3 = 125$

Test Question: 101 Key: E

$\mathrm{E}[N] = Var[N] = (60)(0.5) = 30$
$\mathrm{E}[X] = (0.6)(1) + (0.2)(5) + (0.2)(10) = 3.6$
$\mathrm{E}\left[X^2\right] = (0.6)(1) + (0.2)(25) + (0.2)(100) = 25.6$
$Var\ [X] = 25.6 - 3.6^2 = 12.64$

For any compound distribution, per <u>Loss Models</u>
$Var\ [S] = E[N]Var[X] + Var[N](E[X])^2$

= (30) (12.64) + (30) $\left(3.6^2\right)$

= 768

For specifically Compound Poisson, per <u>Probability Models</u>
$Var[S] = \lambda t\, E\left[X^2\right] =$ (60) (0.5) (25.6) = 768

Alternatively, consider this as 3 Compound Poisson processes (coins worth 1; worth 5; worth 10), where for each $Var(X) = 0$, thus for each $Var(S) = Var(N)E[X]^2$.
Processes are independent, so total *Var* is

$$Var = (60)(0.5)(0.6)1^2 + (60)(0.5)(0.2)5^2 + (60)(0.5)(0.2)(10)^2$$
$$= 768$$

Test Question: 102 Key: D

$$1000\ {}_{20}^{20}V_x = 1000A_{x+20} = \frac{1000\left({}_{19}^{20}V_x + {}_{20}P_x\right)(1.06) - q_{x+19}(1000)}{p_{x+19}}$$

$$= \frac{(342.03 + 13.72)(1.06) - 0.01254(1000)}{0.98746} = 369.18$$

$$\ddot{a}_{x+20} = \frac{1 - 0.36918}{(0.06/1.06)} = 11.1445$$

$$so \quad 1000P_{x+20} = 1000\frac{A_{x+20}}{\ddot{a}_{x+20}} = \frac{369.18}{11.1445} = 33.1$$

Test Question: 103 Key: B

$$ {}_k p_x^{(\tau)} = e^{-\int_0^k \mu_x^{(\tau)}(t)dt} = e^{-\int_0^k 2\mu_x^{(1)}(t)dt} $$

$$ = \left(e^{-\int_0^k \mu_x^{(1)}(t)dt} \right)^2 $$

$$ = \left({}_k p_x \right)^2 \text{ where } {}_k p_x \text{ is from Illustrative Life Table, since } \mu^{(1)} \text{ follows I.L.T.} $$

$$ {}_{10} p_{60} = \frac{6,616,155}{8,188,074} = 0.80802 $$

$$ {}_{11} p_{60} = \frac{6,396,609}{8,188,074} = 0.78121 $$

$$ {}_{10|}q_{60}^{(\tau)} = {}_{10}p_{60}^{(\tau)} - {}_{11}p_{60}^{(\tau)} $$

$$ = \left({}_{10} p_{60} \right)^2 - \left({}_{11} p_{60} \right)^2 \text{ from I.L.T.} $$

$$ = 0.80802^2 - 0.78121^2 = 0.0426 $$

Test Question: 104 Key: C

$P_s = \dfrac{1}{\ddot{a}_s} - d$, where s can stand for any of the statuses under consideration.

$$ \ddot{a}_s = \frac{1}{P_s + d} $$

$$ \ddot{a}_x = \ddot{a}_y = \frac{1}{0.1 + 0.06} = 6.25 $$

$$ \ddot{a}_{\overline{xy}} = \frac{1}{0.06 + 0.06} = 8.333 $$

$$ \ddot{a}_{\overline{xy}} + \ddot{a}_{xy} = \ddot{a}_x + \ddot{a}_y $$

$$ \ddot{a}_{xy} = 6.25 + 6.25 - 8.333 = 4.167 $$

$$ P_{xy} = \frac{1}{4.167} - 0.06 = 0.18 $$

Test Question: 105 **Key: A**

$$d_0^{(\tau)} = 1000\int_0^1 e^{-(\mu+0.04)t}(\mu+0.04)dt$$
$$= 1000\left(1-e^{-(\mu+0.04)}\right) = 48$$

$$e^{-(\mu+0.04)} = 0.952$$
$$\mu + 0.04 = -\ln(0.952)$$
$$= 0.049$$
$$\mu = 0.009$$

$$d_3^{(1)} = 1000\int_3^4 e^{-0.049t}(0.009)dt$$
$$= 1000\frac{0.009}{0.049}\left(e^{-(0.049)(3)} - e^{-(0.049)(4)}\right) = 7.6$$

Test Question: 106 **Key: B**

This is a graph of $l_x\mu(x)$.
$\mu(x)$ would be increasing in the interval $(80,100)$.
The graphs of $l_x p_x$, l_x and l_x^2 would be decreasing everywhere.
The graph shown is comparable to Figure 3.3.2 on page 65 of Actuarial Mathematics

Test Question: 107 **Key: A**

Using the conditional mean and variance formulas:

$$E[N] = E_\Lambda(N|\Lambda)$$

$$Var[N] = Var_\Lambda(E(N|\Lambda)) + E_\Lambda(Var(N|\Lambda))$$

Since *N*, given lambda, is just a Poisson distribution, this simplifies to:

$$E[N] = E_\Lambda(\Lambda)$$
$$Var[N] = Var_\Lambda(\Lambda) + E_\Lambda(\Lambda)$$

We are given that $E[N] = 0.2$ and $Var[N] = 0.4$, subtraction gives *Var* (Λ) = 0.2

Test Question: 108 **Key: B**

N = number of salmon
X = eggs from one salmon
S = total eggs.
E(N) = 100*t*
Var(N) = 900*t*

$$E(S) = E(N)E(X) = 500t$$

$$Var(S) = E(N)Var(X) + E^2(X)Var(N) = 100t \cdot 5 + 25 \cdot 900t = 23{,}000t$$

$$P(S > 10{,}000) = P\left(\frac{S - 500t}{\sqrt{23{,}000t}} > \frac{10{,}000 - 500t}{\sqrt{23{,}000t}}\right) = .95 \Rightarrow$$

$$10{,}000 - 500t = -1.645 \cdot \sqrt{23000}\sqrt{t} = -250\sqrt{t}$$

$$40 - 2t = -\sqrt{t}$$

$$2\left(\sqrt{t}\right)^2 - \sqrt{t} - 40 = 0$$

$$\sqrt{t} = \frac{1 \pm \sqrt{1 + 320}}{4} = 4.73$$

$$t = 22.4$$

round up to 23

Test Question: 109 Key: A

$$A\ P\ V\ (x\text{'s benefits}) = \sum_{k=0}^{2} v^{k+1} b_{k+1}\ {}_k p_x\ q_{x+k}$$

$$= 1000\left[300v(0.02) + 350v^2(0.98)(0.04) + 400v^3(0.98)(0.96)(0.06)\right]$$

$$= 36{,}829$$

Test Question: 110 Key: E

π denotes benefit premium

${}_{19}V = APV$ future benefits - APV future premiums

$$0.6 = \frac{1}{1.08} - \pi \Rightarrow \pi = 0.326$$

$$_{11}V = \frac{({}_{10}V + \pi)(1.08) - (q_{65})(10)}{p_{65}}$$

$$= \frac{(5.0 + 0.326)(1.08) - (0.10)(10)}{1 - 0.10}$$

=5.28

Test Question: 111 Key: C

X = losses on one life

$$E[X] = (0.3)(1) + (0.2)(2) + (0.1)(3)$$
$$= 1$$

S = total losses

$$E[S] = 3E[X] = 3$$

$$E\left[(S-1)_+\right] = E[S] - 1(1 - F_s(0))$$
$$= E[S] - (1)(1 - f_s(0))$$
$$= 3 - (1)(1 - 0.4^3)$$
$$= 3 - 0.936$$
$$= 2.064$$

Test Question: 112 Key: A

$$1180 = 70\bar{a}_{30} + 50\bar{a}_{40} - 20\bar{a}_{30:40}$$
$$1180 = (70)(12) + (50)(10) - 20\bar{a}_{30:40}$$
$$\bar{a}_{30:40} = 8$$
$$\bar{a}_{\overline{30:40}} = \bar{a}_{30} + \bar{a}_{40} - \bar{a}_{30:40} = 12 + 10 - 8 = 14$$
$$100\bar{a}_{\overline{30:40}} = 1400$$

Test Question: 113 Key: B

$$\bar{a} = \int_{\circ}^{\infty} \bar{a}_{\overline{t|}}\, f(t)dt \;= \int_{\circ}^{\infty} \frac{1-e^{-0.05t}}{0.05} \frac{1}{\Gamma(2)} te^{-t}dt$$

$$= \frac{1}{0.05}\int_{\circ}^{\infty}\left(te^{-t} - te^{-1.05t}\right)dt$$
$$= \frac{1}{0.05}\left[-(t+1)e^{-t} + \left(\frac{t}{1.05} + \frac{1}{1.05^2}\right)e^{-1.05t}\right]\Bigg|_0^{\infty}$$
$$= \frac{1}{0.05}\left[1 - \left(\frac{1}{1.05}\right)^2\right] = 1.85941$$

$$20{,}000 \times 1.85941 = 37{,}188$$

Test Question: 114 Key: C

$$p(k) = \frac{2}{k}\, p(k-1)$$
$$= \left[0 + \frac{2}{k}\right] p(k-1)$$

Thus an (*a*, *b*, 0) distribution with *a* = 0, *b* = 2.

Thus Poisson with $\lambda = 2$.

$$p(4) = \frac{e^{-2}2^4}{4!}$$
$$= 0.09$$

Test Question: 115 Key: B

By the memoryless property, the distribution of amounts paid in excess of 100 is still exponential with mean 200.

With the deductible, the probability that the amount paid is 0 is $F(100) = 1 - e^{-100/200} = 0.393.$

Thus the average amount paid per loss is (0.393) (0) + (0.607) (200) = 121.4

The expected number of losses is (20) (0.8) = 16.

The expected amount paid is (16) (121.4) = 1942.

Test Question: 116 Key: D

Let M = the force of mortality of an individual drawn at random; and $T =$ future lifetime of the individual.

$$
\begin{aligned}
\Pr[T \le 1] \quad &= E\{\Pr[T \le 1 | M]\} \\
&= \int_0^\infty \Pr[T \le 1 | M = \mu]\, f_M(\mu) d\mu \\
&= \int_0^2 \int_0^1 \mu e^{-\mu t} dt \frac{1}{2} d\mu \\
&= \int_0^2 \left(1 - e^{-\mu}\right)\frac{1}{2} du = \frac{1}{2}\left(2 + e^{-2} - 1\right) = \frac{1}{2}\left(1 + e^{-2}\right) \\
&= 0.56767
\end{aligned}
$$

Test Question: 117 Key: E

$$E[N] = (0.8)(1) + (0.2)(2) = 1.2$$
$$E\left[N^2\right] = (0.8)1 + (0.2)(4) = 1.6$$
$$\text{Var}(\text{N}) = 1.6 - 1.2^2 = 0.16$$
$$E[X] = 70 + 100 = 170$$
$$\text{Var}(\text{X}) = E\left[X^2\right] - E[X]^2 = (7000 + 100{,}000) - 170^2 = 78{,}100$$
$$E[S] = E[N]E[X] = 1.2(170) = 204$$
$$\text{Var}(\text{S}) = E[N]\text{Var}(\text{X}) + E[X]^2\,\text{Var}(\text{N}) = 1.2(78{,}100) + 170^2(0.16) = 98{,}344$$

Std dev $(S) = \sqrt{98{,}344} = 313.6$
So B = 204 + 314 = 518

Test Question: 118 Key: D

Let $\pi =$ benefit premium

Actuarial present value of benefits =
$$= (0.03)(200{,}000)v + (0.97)(0.06)(150{,}000)v^2 + (0.97)(0.94)(0.09)(100{,}000)v^3$$
$$= 5660.38 + 7769.67 + 6890.08$$
$$= 20{,}320.13$$

Actuarial present value of benefit premiums
$$= \ddot{a}_{x:\overline{3}|}\,\pi$$
$$= \left[1 + 0.97v + (0.97)(0.94)v^2\right]\pi$$
$$= 2.7266\,\pi$$
$$\pi = \frac{20{,}320.13}{2.7266} = 7452.55$$

$${}_1V = \frac{(7452.55)(1.06) - (200{,}000)(0.03)}{1 - 0.03}$$
$$= 1958.46$$

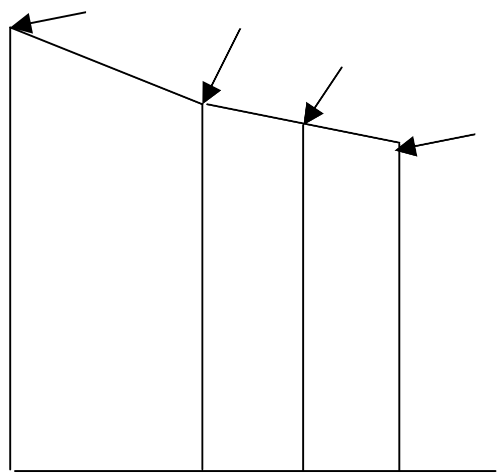
Initial reserve, year 2 = ${}_1V + \pi$
= 1958.56 + 7452.55
= 9411.01

Test Question: 119 Key: A

Let π denote the premium.

$$L = b_T v^T - \pi \bar{a}_{\overline{T|}} = (1+i)^T \times v^T - \pi \bar{a}_{\overline{T|}}$$
$$= 1 - \pi \bar{a}_{\overline{T|}}$$
$$E[L] = 1 - \pi \bar{a}_x = 0 \quad \Rightarrow \pi = 1/\bar{a}_x$$
$$\Rightarrow L = 1 - \pi \bar{a}_{\overline{T|}} = 1 - \frac{\bar{a}_{\overline{T|}}}{\bar{a}_x} = \frac{\delta \bar{a}_x - (1 - v^T)}{\delta \bar{a}_x}$$
$$= \frac{v^T - (1 - \delta \bar{a}_x)}{\delta \bar{a}_x} = \frac{v^T - \bar{A}_x}{1 - \bar{A}_x}$$

Test Question: 120 **Key: D**



$${}_1p_1 = (1 - 0.1) = 0.9$$
$${}_2p_1 = (0.9)(1 - 0.05) = 0.855$$

since uniform, ${}_{1.5}p_1 = (0.9 + 0.855)/2$
$$= 0.8775$$

$\overset{\circ}{e}_{1:\overline{1.5}|}$ = Area between $t = 0$ and $t = 1.5$

$$= \left(\frac{1+0.9}{2}\right)(1) + \left(\frac{0.9+0.8775}{2}\right)(0.5)$$
$$= 0.95 + 0.444$$
$$= 1.394$$

Alternatively,

$$\overset{\circ}{e}_{1:\overline{1.5}|} = \int_0^{1.5} {}_t p_1 dt$$
$$= \int_0^1 {}_t p_1 dt + {}_1p_1 \int_0^{0.5} {}_x p_2 dx$$
$$= \int_0^1 (1-0.1t)dt + 0.9\int_0^{0.5}(1-0.05x)dx$$
$$= \left[t - \tfrac{0.1t^2}{2}\right]\Big|_0^1 + 0.9\left[x - \tfrac{0.05x^2}{2}\right]\Big|_0^{0.5}$$
$$= 0.95 + 0.444 = 1.394$$

Test Question: 121 Key: A

$$10{,}000 A_{63}(1.12) = 5233$$
$$A_{63} = 0.4672$$
$$A_{x+1} = \frac{A_x(1+i) - q_x}{p_x}$$
$$A_{64} = \frac{(0.4672)(1.05) - 0.01788}{1-0.01788}$$
$$= 0.4813$$
$$A_{65} = \frac{(0.4813)(1.05) - 0.01952}{1-0.01952}$$
$$= 0.4955$$

Single contract premium at 65 = (1.12) (10,000) (0.4955)
= 5550

$$(1+i)^2 = \frac{5550}{5233} \qquad i = \sqrt{\frac{5550}{5233}} - 1 = 0.02984$$

Test Question: 122 Key: B

Original Calculation (assuming independence):

$$\mu_x = 0.06$$
$$\mu_y = 0.06$$
$$\mu_{xy} = 0.06 + 0.06 = 0.12$$
$$\bar{A}_x = \frac{\mu_x}{\mu_x + \delta} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\bar{A}_y = \frac{\mu_y}{\mu_y + \delta} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\bar{A}_{xy} = \frac{\mu_{xy}}{\mu_{xy} + \delta} = \frac{0.12}{0.12 + 0.05} = 0.70588$$
$$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.54545 + 0.54545 - 0.70588 = 0.38502$$

Revised Calculation (common shock model):

$$\mu_x = 0.06,\ \mu_x^{T^*(x)} = 0.04$$
$$\mu_y = 0.06,\ \mu_y^{T^*(y)} = 0.04$$
$$\mu_{xy} = \mu_x^{T^*(x)} + \mu_y^{T^*(y)} + \mu^Z + 0.04 + 0.04 + 0.02 = 0.10$$
$$\bar{A}_x = \frac{\mu_x}{\mu_x + \delta} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\bar{A}_y = \frac{\mu_y}{\mu_{y+\delta}} = \frac{0.06}{0.06 + 0.05} = 0.54545$$
$$\bar{A}_{xy} = \frac{\mu_{xy}}{\mu_{xy} + \delta} = \frac{0.10}{0.10 + 0.05} = 0.66667$$
$$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy} = 0.54545 + 0.54545 - 0.66667 = 0.42423$$

Difference $= 0.42423 - 0.38502 = 0.03921$

Test Question: 123 **Key: E**

Treat as three independent Poisson variables, corresponding to 1, 2 or 3 claimants.

$$\text{rate}_1 = 6 \quad \left[= \frac{1}{2} \times 12\right]$$
$$\text{rate}_2 = 4$$
$$\text{rate}_3 = 2$$

$$\text{Var}_1 = 6$$
$$\text{Var}_2 = 16 \quad \left[= 4 \times 2^2\right]$$
$$\text{Var}_3 = 18$$

total Var = $6+16+18=40$, since independent.

Alternatively,

$$E\left(X^2\right) = \frac{1^2}{2} + \frac{2^2}{3} + \frac{3^2}{6} = \frac{10}{3}$$

For compound Poisson, $\text{Var}[S] = E[N]E\left[X^2\right]$

$$= (12)\left(\frac{10}{3}\right) = 40$$

Test Question: 124 **Key: C**

$\int_0^3 \lambda(t)dt = 6$ so $N(3)$ is Poisson with $\lambda = 6$.

P is Poisson with mean 3 (with mean 3 since $\text{Prob}(y_i < 500) = 0.5$)

P and *Q* are independent, so the mean of P is 3, no matter what the value of *Q* is.

Test Question: 125 Key: A

At age *x*:

Actuarial Present value (APV) of future benefits = $\left(\frac{1}{5}A_x\right)1000$

APV of future premiums = $\left(\frac{4}{5}\ddot{a}_x\right)\pi$

$\frac{1000}{5}A_{25} = \frac{4}{5}\pi\,\ddot{a}_{25}$ by equivalence principle

$\frac{1000}{4}\frac{A_{25}}{\ddot{a}_{25}} = \pi \Rightarrow \pi = \frac{1}{4}\times\frac{81.65}{16.2242} = 1.258$

${}_{10}V =$ APV (Future benefits) – APV (Future benefit premiums)

$= \frac{1000}{5}A_{35} - \frac{4}{5}\pi\,\ddot{a}_{35}$

$= \frac{1}{5}(128.72) - \frac{4}{5}(1.258)(15.3926)$

$= 10.25$

Test Question: 126 Key: E

Let Y = present value random variable for payments on one life

$S = \sum Y =$ present value random variable for all payments

$E[Y] = 10\ddot{a}_{40} = 148.166$

$\mathrm{Var}[Y] = 10^2\,\frac{\left({}^2A_{40} - A_{40}^2\right)}{d^2}$

$= 100\left(0.04863 - 0.16132^2\right)(1.06/0.06)^2$

$= 705.55$

$E[S] = 100E[Y] = 14{,}816.6$

$\mathrm{Var}[S] = 100\,\mathrm{Var}[Y] = 70{,}555$

Standard deviation $[S] = \sqrt{70{,}555} = 265.62$

By normal approximation, need

E [S] + 1.645 Standard deviations = 14,816.6 + (1.645) (265.62)

= 15,254

Test Question: 127 Key: B

Initial Benefit Prem $= \dfrac{5A_{30} - 4\left(A^{1}_{30:\overline{20|}}\right)}{5\ddot{a}_{30:\overline{35|}} - 4\ddot{a}_{30:\overline{20|}}}$

$$= \frac{5(0.10248) - 4(0.02933)}{5(14.835) - 4(11.959)}$$

$$= \frac{0.5124 - 0.11732}{74.175 - 47.836} = \frac{0.39508}{26.339} = 0.015$$

Where

$A^{1}_{30:\overline{20|}} = \left(A_{30:\overline{20|}} - A_{30:\overline{20|}}^{\ \ \ 1}\right) = 0.32307 - 0.29374 = 0.02933$

and

$\ddot{a}_{30:\overline{20|}} = \dfrac{1 - A_{30:\overline{20|}}}{d} = \dfrac{1 - 0.32307}{\left(\frac{0.06}{1.06}\right)} = 11.959$

Comment: the numerator could equally well have been calculated as $A_{30} + 4\ {}_{20}E_{30}\ \ A_{50}$
= 0.10248 + (4) (0.29374) (0.24905)
= 0.39510

Test Question: 128 Key: B

$${}_{0.75}p_x = 1 - (0.75)(0.05)$$
$$= 0.9625$$
$${}_{0.75}p_y = 1 - (0.75)(.10)$$
$$= 0.925$$
$${}_{0.75}q_{xy} = 1 - {}_{0.75}p_{xy}$$
$$= 1 - \left({}_{0.75}p_x\right)\left({}_{0.75}p_y\right) \text{ since independent}$$
$$= 1 - (0.9625)(0.925)$$
$$= 0.1097$$

Test Question: 129 Key: A

N = number of physicians E(N) = 3 Var (N) = 2
X = visits per physician E(X) = 30 Var (X) = 30

S = total visits

E(S) = E (N) E(X) = 90

Var(S) = E(N) Var(X) + $E^2(X)$ Var(N) =

= $3 \cdot 30 + 900 \cdot 2 = 1890$

Standard deviation (S) = 43.5

Pr(S>119.5) = Pr$\left(\frac{S-90}{43.5} > \frac{119.5-90}{43.5}\right) = 1 - \Phi(0.68)$ Course 3: November 2000

Test Question: 130 Key: A

The person receives *K* per year guaranteed for 10 years $\Rightarrow K\ddot{a}_{\overline{10|}} = 8.4353K$

The person receives *K* per years alive starting 10 years from now $\Rightarrow {}_{10|}\ddot{a}_{40}K$

*Hence we have $10000 = (8.4353 + {}_{10}E_{40}\ddot{a}_{50})K$

Derive ${}_{10}E_{40}$:

$$A_{40} = A^1_{40:\overline{10|}} + ({}_{10}E_{40})A_{50}$$

$${}_{10}E_{40} = \frac{A_{40} - A^1_{40:\overline{10|}}}{A_{50}} = \frac{0.30 - 0.09}{0.35} = 0.60$$

Derive $\ddot{a}_{50} = \frac{1 - A_{50}}{d} = \frac{1 - 0.35}{\frac{.04}{1.04}} = 16.90$

Plug in values:

$$10{,}000 = (8.4353 + (0.60)(16.90))K$$

$$= 18.5753K$$

$$K = 538.35$$

Test Question: 131 Key: D

STANDARD: $\mathring{e}_{25:\overline{11|}} = \int_0^{11}\left(1 - \frac{t}{75}\right)dt = t - \frac{t^2}{2 \times 75}\Big|_0^{11} = 10.1933$

MODIFIED: $p_{25} = e^{-\int_0^1 0.1 ds} = e^{-1} = 0.90484$

$$\overset{\circ}{e}_{25:\overline{11}|} = \int_0^1 {}_t p_{25} dt + p_{25} \int_0^{10} \left(1 - \frac{t}{74}\right) dt$$

$$= \int_0^1 e^{-0.1t} dt + e^{-0.1} \int_0^{10} \left(1 - \frac{t}{74}\right) dt$$

$$= \frac{1 - e^{-0.1}}{0.1} + e^{-0.1} \left(t - \frac{t^2}{2 \times 74} \right) \Big|_0^{10}$$

$$= 0.95163 + 0.90484(9.32432) = 9.3886$$

Difference =0.8047

Test Question: 132 Key: B

Comparing B & D: Prospectively at time 2, they have the same future benefits. At issue, B has the lower benefit premium. Thus, by formula 7.2.2, B has the higher reserve.

Comparing A to B: use formula 7.3.5. At issue, B has the higher benefit premium. Until time 2, they have had the same benefits, so B has the higher reserve.

Comparing B to C: Visualize a graph C* that matches graph B on one side of t=2 and matches graph C on the other side. By using the logic of the two preceding paragraphs, C's reserve is lower than C*'s which is lower than B's.

Comparing B to E: Reserves on E are constant at 0.

Test Question: 133 Key: C

Since only decrements (1) and (2) occur during the year, probability of reaching the end of the year is

$$p_{60}'^{(1)} \times p_{60}'^{(2)} = (1 - 0.01)(1 - 0.05) = 0.9405$$

Probability of remaining through the year is

$$p_{60}'^{(1)} \times p_{60}'^{(2)} \times p_{60}'^{(3)} = (1 - 0.01)(1 - 0.05)(1 - 0.10) = 0.84645$$

Probability of exiting at the end of the year is

$$q_{60}^{(3)} = 0.9405 - 0.84645 = 0.09405$$

Test Question: 134 Key: E

$\mathrm{E}(N) = .7$

$\mathrm{Var}(N) = 4 \times .2 + 9 \times .1 - .49 = 1.21$

$\mathrm{E}(X) = 2$

$\mathrm{Var}(X) = 100 \times .2 - 4 = 16$

$\mathrm{E}(S) = 2 \times .7 = 1.4$

$\mathrm{Var}(S) = \mathrm{E}(N)\ \mathrm{Var}(X) + \mathrm{E}^2(X)\ \mathrm{Var}(N) =$

$= .7 \times 16 + 4 \times 1.21 = 16.04$

$\text{Standard Dev}(S) = 4$

$\mathrm{E}(S) + 2 \times \text{Standard Dev}(S) = 1.4 + 2 \times 4 = 9.4$

Since there are no possible values of S between 0 and 10,

$\Pr(S > 9.4) = 1 - \Pr(S = 0)$

$= 1 - .7 - .2 \times .8^2 - .1 \times .8^3 = .12$

Test Question: 135 Key: D

APV of regular death benefit $= \int_0^{\infty} (100000)\left(e^{-\delta t}\right)(0.008)\left(e^{-\mu t}\right) dt$

$= \int_0^{\infty} (100000)\left(e^{-0.06t}\right)(0.008)\left(e^{-0.008t}\right) dt$

$= 100000\left[0.008 / (0.06 + 0.008)\right] = 11{,}764.71$

APV of accidental death benefit $= \int_0^{30} (100000)\left(e^{-\delta t}\right)(0.001)\left(e^{-\mu t}\right) dt$

$= \int_0^{30} (100000)\left(e^{-0.06t}\right)(0.001)\left(e^{-0.008t}\right) dt$

$= 100\left[1 - e^{-2.04}\right] / 0.068 = 1{,}279.37$

Total APV $= 11765 + 1279 = 13044$

Test Question: 136 Key: B

$$l_{[60]+.6} = (.6)(79{,}954) + (.4)(80{,}625)$$
$$= 80{,}222.4$$
$$l_{[60]+1.5} = (.5)(79{,}954) + (.5)(78{,}839)$$
$$= 79{,}396.5$$

$${}_{0.9}q_{[60]+.6} = \frac{80222.4 - 79{,}396.5}{80{,}222.4}$$
$$= 0.0103$$

$$P_0 = \tfrac{1}{11} = 9.09\overline{09}\%$$

Test Question: 137 Key: A

$$P(0) = \tfrac{1}{5}\int_0^5 e^{-\lambda} d\lambda = \tfrac{1}{5}\left(-e^{-\lambda}\right)\Big|_0^5 = \tfrac{1}{5}\left(1 - e^{-5}\right) = 0.1987$$

$$P(1) = \tfrac{1}{5}\int_0^5 \lambda e^{-\lambda} d\lambda = \tfrac{1}{5}\left(-\lambda e^{-\lambda} - e^{-\lambda}\right)\Big|_0^5 = \tfrac{1}{5}\left(1 - 6e^{-5}\right) = 0.1919$$

$$P(N \ge 2) = 1 - .1987 - .1919 = .6094$$

Test Question: 138 Key: A

$$q_{40}^{(\tau)} = q_{40}^{(1)} + q_{40}^{(2)} = 0.34$$
$$= 1 - p_{40}'^{(1)} p_{40}'^{(2)}$$
$$0.34 = 1 - 0.75 p_{40}'^{(2)}$$

$$p_{40}'^{(2)} = 0.88$$
$$q_{40}'^{(2)} = 0.12 = y$$

$$q_{41}'^{(2)} = 2y = 0.24$$

$q_{41}^{(\tau)} = 1 - (0.8)(1 - 0.24) = 0.392$

$l_{42}^{(\tau)} = 2000(1 - 0.34)(1 - 0.392) = 803$

Test Question: 139 Key: C

$\Pr[L(\pi') > 0] < 0.5$

$\Pr\left[10{,}000v^{K+1} - \pi' \ddot{a}_{\overline{K+1}|} > 0\right] < 0.5$

From Illustrative Life Table, ${}_{47}p_{30} = 0.50816$ and ${}_{48}p_{30} = .47681$

Since *L* is a decreasing function of *K*, to have
$\Pr[L(\pi') > 0] < 0.5$ means we must have $L(\pi') \le 0$ for $K \ge 47$.
Highest value of $L(\pi')$ for $K \ge 47$ is at *K* = 47.

$L(\pi')[\text{at } K = 47] = 10{,}000\, v^{47+1} - \pi'\, \ddot{a}_{\overline{47+1}|}$

$= 609.98 - 16.589\pi'$

$L(\pi') \le 0 \Rightarrow (609.98 - 16.589\pi') \le 0$

$\Rightarrow \pi' > \dfrac{609.98}{16.589} = 36.77$

Test Question: 140 Key: B

$\Pr(K = 0) = 1 - p_x = 0.1$

$\Pr(K = 1) = {}_1p_x - {}_2p_x = 0.9 - 0.81 = 0.09$

$\Pr(K > 1) = {}_2p_x = 0.81$

$E(Y) = .1 \times 1 + .09 \times 1.87 + .81 \times 2.72 = 2.4715$

$E(Y^2) = .1 \times 1^2 + .09 \times 1.87^2 + .81 \times 2.72^2 = 6.407$

$VAR(Y) = 6.407 - 2.4715^2 = 0.299$

Test Question: 141 Key: D

Let X be the occurrence amount, Y = max(X-100, 0) be the amount paid.
E[X] = 1,000
Var[X] = $(1{,}000)^2$
P(X>100) = exp(-100/1,000) = .904837

The distribution of Y given that X>100, is also exponential with mean 1,000 (memoryless property).

So Y is $\begin{cases} \text{0 with prob .095163} \\ \text{exponential mean 1000 with prob .904837} \end{cases}$

$$\mathrm{E}\left[Y\right] = .095163 \times 0 + .904837 \times 1{,}000 = 904.837$$
$$\mathrm{E}\left[Y^2\right] = .095163 \times 0 + .904837 \times 2 \times \left(1{,}000\right)^2 = 1{,}809{,}675$$

$$\mathrm{Var}\left[Y\right] = 1{,}809{,}675 - \left(904.837\right)^2 = 990{,}944$$

Alternatively, think of this as a compound distribution whose frequency is Bernoulli with p = .904837, and severity is exponential with mean 1,000.

$$\mathrm{Var} = \mathrm{Var}\left[N\right] \times \mathrm{E}\left[X\right]^2 + \mathrm{Var}\left[X\right] \times \mathrm{E}\left[N\right] = p\left(1-p\right)\left(1{,}000{,}000\right) + p\left(1{,}000{,}000\right)$$

Test Question: 142 Key: B

In general $\mathrm{Var}(L) = \left(1+\frac{P}{\delta}\right)^2 \left({}^2\overline{A}_x - \overline{A}_x^2\right)$

Here $\overline{P}(\overline{A}_x) = \dfrac{1}{\overline{a}_x} - \delta = \dfrac{1}{5} - .08 = .12$

So $\mathrm{Var}(L) = \left(1+\dfrac{.12}{.08}\right)^2 \left({}^2\overline{A}_x - \overline{A}_x^2\right) = .5625$

and $\mathrm{Var}(L^*) = \left(1+\dfrac{\frac{5}{4}(.12)}{.08}\right)^2 \left({}^2\overline{A}_x - \overline{A}_x^2\right)$

So $\mathrm{Var}(L^*) = \dfrac{\left(1+\frac{15}{8}\right)^2}{\left(1+\frac{12}{8}\right)^2}(0.5625) = .744$

$\mathrm{E}[L^*] = \overline{A}_x - .15\overline{a}_x = 1 - \overline{a}_x(\delta + .15) = 1 - 5(.23) = -.15$

$\mathrm{E}[L^*] + \sqrt{\mathrm{Var}(L^*)} = .7125$

Test Question: 143 Key: C

Serious claims are reported according to a Poisson process at an average rate of 2 per month. The chance of seeing at least 3 claims is (1 – the chance of seeing 0, 1, or 2 claims).

$P(3+) \geq 0.9$ is the same as $P(0,1,2) \leq 0.1$ is the same as $\left[P(0) + P(1) + P(2)\right] \leq 0.1$

$0.1 \geq e^{-\lambda} + \lambda e^{-\lambda} + \left(\lambda^2 / 2\right)e^{-\lambda}$

The expected value is 2 per month, so we would expect it to be at least 2 months $(\lambda = 4)$.

Plug in and try

$e^{-4} + 4e^{-4} + \left(4^2 / 2\right)e^{-4} = .238,$ too high, so try 3 months $(\lambda = 6)$

$e^{-6} + 6e^{-6} + \left(6^2 / 2\right)e^{-6} = .062,$ okay. The answer is 3 months.

[While 2 is a reasonable first guess, it was not critical to the solution. Wherever you start, you should conclude 2 is too few, and 3 is enough].

Test Question: 144 Key: B

Let $l_0^{(\tau)} =$ number of students entering year 1
superscript (f) denote academic failure
superscript (w) denote withdrawal
subscript is "age" at start of year; equals year - 1

$$p_0^{(\tau)} = 1 - 0.40 - 0.20 = 0.40$$

$$l_2^{(\tau)} = 10\, l_2^{(\tau)}\, q_2^{(f)} \Rightarrow q_2^{(f)} = 0.1$$

$$q_2^{(w)} = q_2^{(\tau)} - q_2^{(f)} = (1.0 - 0.6) - 0.1 = 0.3$$

$$l_1^{(\tau)} q_1^{(f)} = 0.4\left[l_1^{(\tau)}\left(1 - q_1^{(f)} - q_1^{(w)}\right)\right]$$

$$q_1^{(f)} = 0.4\left(1 - q_1^{(f)} - 0.3\right)$$

$$q_1^{(f)} = \frac{0.28}{1.4} = 0.2$$

$$p_1^{(\tau)} = 1 - q_1^{(f)} - q_1^{(w)} = 1 - 0.2 - 0.3 = 0.5$$

$${}_3q_0^{(w)} = q_0^{(w)} + p_0^{(\tau)} q_1^{(w)} + p_0^{(\tau)} p_1^{(\tau)} q_2^{(w)}$$

$$= 0.2 + (0.4)(0.3) + (0.4)(0.5)(0.3)$$

$$= 0.38$$

Test Question: 145 Key: D

$$e_{25} = p_{25}(1+e_{26})$$

$$e_{26}^N = e_{26}^M \quad \text{since same } \mu$$

$$p_{25}^N = e^{-\int_0^1 \left[\mu_{25}^M(t)+0.1(1-t)\right]dt}$$
$$= e^{-\int_0^1 \mu_{25}^M(t)dt - \int_0^1 0.1(1-t)dt}$$

$$= e^{-\int_0^1 \mu_{25}^M(t)dt} e^{-\int_0^1 0.1(1-t)dt}$$

$$= p_{25}^M e^{-\left[0.1\left(t-\frac{t^2}{2}\right)\right]_0^1}$$

$$= e^{-0.05} p_{25}^M$$

$$e_{25}^N = p_{25}^N(1+e_{26})$$

$$= e^{-0.05} p_{25}^M (1+e_{26})$$

$$= 0.951\, e_{25}^M = (0.951)(10.0) = 9.5$$

Test Question: 146 Key: D

$$\mathrm{E}\left[Y_{AGG}\right] = 100\mathrm{E}\left[Y\right] = 100(10{,}000)\bar{a}_x$$

$$= 100(10{,}000)\left(\frac{\left(1-\bar{A}_x\right)}{\delta}\right) = 10{,}000{,}000$$

$$\sigma_Y = \sqrt{\mathrm{Var}\left[Y\right]} = \sqrt{(10{,}000)^2 \frac{1}{\delta^2}\left({}^2\bar{A}_x - \bar{A}_x^2\right)}$$

$$= \frac{(10{,}000)}{\delta}\sqrt{(0.25)-(0.16)} = 50{,}000$$

$$\sigma_{AGG} = \sqrt{100}\sigma_Y = 10(50{,}000) = 500{,}000$$

$$0.90 = \Pr\left[\frac{F-\mathrm{E}\left[Y_{AGG}\right]}{\sigma_{AGG}} > 0\right]$$

$$\Rightarrow 1.282 = \frac{F-\mathrm{E}\left[Y_{AGG}\right]}{\sigma_{AGG}}$$

$$F = 1.282\sigma_{AGG} + \mathrm{E}\left[Y_{AGG}\right]$$

$$F = 1.282(500{,}000) + 10{,}000{,}000 = 10{,}641{,}000$$

Test Question: 147 Key: C

Expected claims under current distribution = 500

$\theta =$ parameter of new distribution

$X =$ claims

$\mathrm{E}(X) = \theta$

bonus $= .5 \times \left[500 - X \wedge 500\right]$

E (claims + bonus) $= \theta + .5\left(500 - \theta\left(1 - \frac{\theta}{500+\theta}\right)\right) = 500$

$$\theta - \frac{\theta}{2}\left(\frac{500}{500+\theta}\right) = 250$$

$$2(500+\theta)\theta - 500\theta = 250(500+\theta)\cdot 2$$

$$1000\theta + \theta^2 \cdot 2 - 500\theta = 2 \times 250 \times 500 + 500\theta$$

$$\theta = \sqrt{250 \times 500} = 354$$

Test Question: 148 Key: E

$$(DA)^{1}_{80:\overline{20|}} = 20vq_{80} + vp_{80}\left((DA)^{1}_{81:\overline{19|}}\right)$$

$q_{80} = .2$ $\quad 13 = \dfrac{20(.2)}{1.06} + \dfrac{.8}{1.06}(DA)^{1}_{81:\overline{19|}}$

$$\therefore (DA)^{1}_{81:\overline{19|}} = \frac{13(1.06) - 4}{.8} = 12.225$$

$q_{80} = .1$ $\quad DA^{1}_{80:\overline{20|}} = 20v(.1) + v(.9)(12.225)$

$$= \frac{2 + .9(12.225)}{1.06} = 12.267$$

Test Question: 149 Key: B

Let T denote the random variable of time until the college graduate finds a job
Let $\{N(t), t \geq 0\}$ denote the job offer process

Each offer can be classified as either

$$\begin{cases} \text{Type I - - accept with probability } p \Rightarrow \{N_1(t)\} \\ \text{Type II - - reject with probability } (1-p) \Rightarrow \{N_2(t)\} \end{cases}$$

By proposition 5.2, $\{N_1(t)\}$ is Poisson process with $\lambda_1 = \lambda \cdot p$

$$p = \Pr(w > 28{,}000) = \Pr(\ln w > \ln 28{,}000)$$

$$= \Pr(\ln w > 10.24) = \Pr\left(\frac{\ln w - 10.12}{0.12} > \frac{10.24 - 10.12}{0.12}\right) = 1 - \Phi(1)$$

$$= 0.1587$$

$$\lambda_1 = 0.1587 \times 2 = 0.3174$$

T has an exponential distribution with $\theta = \dfrac{1}{.3174} = 3.15$

$$\Pr(T > 3) = 1 - F(3)$$

$$= e^{\frac{-3}{3.15}} = 0.386$$

Test Question: 150 Key: A

$$ {}_t p_x = \exp\left[-\int_0^t \frac{ds}{100-x-s}\right] = \exp\left[\ln(100-x-s)\Big|_0^t\right] = \frac{100-x-t}{100-x} $$

$$ \mathring{e}_{\overline{50:60}} = \mathring{e}_{50} + \mathring{e}_{60} - \mathring{e}_{50:60} $$

$$ \mathring{e}_{50} = \int_0^{50} \frac{50-t}{50}\,dt = \frac{1}{50}\left[50t - \frac{t^2}{2}\right]_0^{50} = 25 $$

$$ \mathring{e}_{60} = \int_0^{40} \frac{40-t}{40}\,dt = \frac{1}{40}\left[40t - \frac{t^2}{2}\right]_0^{40} = 20 $$

$$ \mathring{e}_{50:60} = \int_0^{40} \left(\frac{50-t}{50}\right)\left(\frac{40-t}{40}\right)dt = \int_0^{40} \frac{1}{2000}\left(2000 - 90t + t^2\right)dt $$

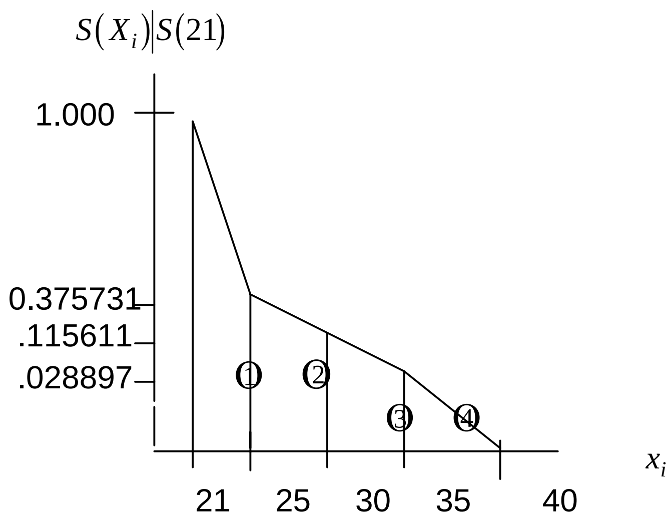
$$ = \frac{1}{2000}\left(2000t - 45t^2 + \frac{t^3}{3}\Big|_0^{40}\right) = 14.67 $$

$$ \mathring{e}_{\overline{50:60}} = 25 + 20 - 14.67 = 30.33 $$

Test Question: 151 Key: A

$$UDD \Rightarrow l_{21} = (0.8)(53{,}488) + (0.2)(17{,}384) = 46{,}267.2$$

$$\begin{aligned} Mrl(21) = \overset{\circ}{e}_{21} &= \int_0^\infty {}_t p_{21} \int_0^\infty \frac{S(21+t)}{S(21)} dt \\ &= \sum \text{areas} \\ &= 2.751 + 1.228 + 0.361 + 0.072 \\ &= 4.412 \end{aligned}$$



Test: 152 Key: Question D

$$\mu'_{n1} = \mathrm{E}[N] = 25$$
$$\mu_{x2} = \mathrm{Var}[X] = 675$$
$$\mu_{N2} = \mathrm{Var}[N] = 25$$
$$E[X] = 50$$

$$\mathrm{E}[S] = \mathrm{E}[X]\mathrm{E}[N] = 25 \times 50 = 1250$$

$$\begin{aligned} \mathrm{Var}[S] &= E[N]\mathrm{Var}[X] + \mathrm{Var}[N]E[X]^2 \\ &= 25 \times 675 + 25 \times 2500 = 79{,}375 \end{aligned}$$

Standard Deviation$\left[S\right]=\sqrt{79{,}375}=281.74$

$\Pr\left(S>2000\right)=\Pr\left[\left(S-1250\right)/281.74>\left(2000-1250\right)/281.74\right]=1-\Phi\left(2.66\right)$

Test Question: 153 Key: E

$$\begin{aligned}
\mathrm{Var}\left({}_0L\right) &= \mathrm{Var}\left(\Lambda_0\right)+v^2\,\mathrm{Var}\left(\Lambda_1\right) \quad \text{since } Var\left(\Lambda_2\right)=0\\
\mathrm{Var}\left(\Lambda_0\right) &= \left[v\left(b_1-{}_1V\right)\right]^2 p_{50}q_{50}\\
&= \frac{\left(10{,}000-3{,}209\right)^2\left(0.00832\right)\left(0.99168\right)}{1.03^2}\\
&= 358664.09
\end{aligned}$$

$$\begin{aligned}
\mathrm{Var}\left(\Lambda_1\right) &= \left[v\left(b_2-{}_2V\right)\right]^2 p_{50}q_{51}p_{51}\\
&= \frac{\left(10{,}000-6{,}539\right)^2\left(0.99168\right)\left(0.00911\right)\left(0.99089\right)}{1.03^2}\\
&= 101075.09
\end{aligned}$$

$$\mathrm{Var}\left({}_0L\right) = 358664.09+\frac{101075.09}{1.03^2}=453937.06$$

Alternative solution:

$$\pi=10{,}000\,v-{}_2V_{50:\overline{3|}}=9708.74-6539=3169.74$$

$$_0L=\begin{cases}10{,}000\,v-\pi\,\ddot{a}_{\overline{1|}}=6539 & \text{for} \quad K=0\\ 10{,}000\,v^2-\pi\,\ddot{a}_{\overline{2|}}=3178.80 & \text{for} \quad K=1\\ 10{,}000\,v^3-\pi\,\ddot{a}_{\overline{3|}}=-83.52 & \text{for} \quad K>1\end{cases}$$

$\Pr(K=0) = q_{50} = 0.00832$

$\Pr(K=1) = p_{50}\, q_{51} = (0.99168)(0.00911) = 0.0090342$

$\Pr(K>1) = 1 - \Pr(K=0) - \Pr(K=1) = 0.98265$

$\text{Var}({}_0L) = \text{E}\left[{}_0L^2\right] - \text{E}\left[{}_0L\right]^2 = \text{E}\left[{}_0L^2\right]$ since π is benefit premium

$= 0.00832 \times 6539^2 + 0.00903 \times 3178.80^2 + 0.98265 \times (-83.52)^2$

$= 453{,}895$ [difference from the other solution is due to rounding]

Test Question: 154 Key: C

Let π denote the single benefit premium.

$\pi = {}_{30|}\ddot{a}_{35} + \pi A^{\;1}_{35:\overline{30|}}$

$$\pi = \frac{{}_{30|}\ddot{a}_{35}}{1 - A^{\;1}_{35:\overline{30|}}} = \frac{\left(A_{35:\overline{30|}} - A^{\;1}_{35:\overline{30|}}\right)\ddot{a}_{65}}{1 - A^{\;1}_{35:\overline{30|}}} =$$

$$= \frac{(.21-.07)9.9}{(1-.07)}$$

$$= \frac{1.386}{.93}$$

$$= 1.49$$

Test Question: 155 Key: E

$${}_{0.4}p_0 = .5 = e^{-\int_0^{0.4}\left(F+e^{2x}\right)dx}$$

$$= e^{-.4F-\left[\frac{e^{2x}}{2}\right]_0^{.4}}$$

$$= e^{-.4F-\left(\frac{e^{0.8}-1}{2}\right)}$$

$$.5 = e^{-.4F-.6128}$$

$\Rightarrow \ln(.5) = -.4F - .6128$

$\Rightarrow -.6931 = -.4F - .6128$

$\Rightarrow F = 0.20$

Test Question: 156 Key: C

$$\mathrm{E}[X] = 2000(1!)/(1!) = 2000$$

$$\mathrm{E}[X \wedge 3000] = \left(\frac{2000}{1}\right) \times \left[1 - \frac{2000}{(3000+2000)}\right] = 2000 \times \left(1 - \frac{2}{5}\right) = 2000 \times \frac{3}{5} = 1200$$

So the fraction of the losses expected to be covered by the reinsurance is $\frac{2000-1200}{2000} = 0.4.$

The expected ceded losses are 4,000,000 $\Rightarrow$ the ceded premium is 4,400,000.

Test Question: 157 Key: E

$$X_{2002} = 1.05 \times X_{2001}$$

so: $$F\left(\frac{x_{2002}}{1.05}\right) = 1 - \left[\frac{2000}{(x_{2002}/1.05 + 2000)}\right]^2$$

$$= 1 - \left[\frac{2100}{x_{2002} + 2100}\right]^2$$

This is just another Pareto distribution with $\alpha = 2, \theta = 2100.$

$\mathrm{E}[X_{2002}] = 2100.$

and

$$\mathrm{E}[X_{2002} \wedge 3000] = \left(\frac{2100}{1}\right) \times \left[1 - \left(\frac{2100}{(3000+2100)}\right)\right]$$

$$= 2100 \times \left[\frac{3000}{5100}\right] = 1235$$

So the fraction of the losses expected to be covered by the reinsurance is

$$\frac{2100-1235}{2100} = 0.412.$$

The total expected losses have increased to 10,500,000, so

$$C_{2002} = 1.1 \times 0.412 \times 10{,}500{,}000 = 4{,}758{,}600$$

And $\dfrac{C_{2002}}{C_{2001}} = \dfrac{4{,}758{,}600}{4{,}400{,}000} = 1.08$

SOCIETY OF ACTUARIES

EXAM M ACTUARIAL MODELS

ADDITIONAL EXAM M SAMPLE QUESTIONS AND SOLUTIONS

1. The CAS Insurance Company classifies its auto drivers as Preferred (State #1) or Standard (State #2) starting at time 0 at the start of the first year when they are first insured, with reclassifications occurring at the start of each new policy year. The transition-probability matrices $\boldsymbol{Q}_n$ from the state at time n at the start of year $n+1$ to the state at time $n+1$ are

$$\boldsymbol{Q}_n = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} + \frac{1}{n+1} \begin{bmatrix} 0.1 & -0.1 \\ -0.2 & 0.2 \end{bmatrix}.$$

Given that an insured is classified as Preferred at the start of the second year, find the probability that that insured is Preferred at the start of the fourth year.

Question 1 solution

This is the probability ${}_2Q_1^{(1,1)}$, which is just the $(1,1)$-entry of

$$\boldsymbol{Q}_1\boldsymbol{Q}_2 = \begin{bmatrix} 0.75 & 0.25 \\ 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.73333 & 0.26667 \\ 0.33333 & 0.66667 \end{bmatrix},$$

namely $0.75(0.73333) + 0.25(0.33333) = 0.63333$.

2. The CAS Insurance Company classifies its auto drivers as Preferred (State #1) or Standard (State #2) starting at time 0 at the start of the first year when they are first insured, with reclassifications occurring at the start of each new policy year. The transition-probability matrices $\boldsymbol{Q}_n$ from the state at time n at the start of year $n+1$ to the state at time $n+1$ are

$$\boldsymbol{Q}_n = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} + \frac{1}{n+1} \begin{bmatrix} 0.1 & -0.1 \\ -0.2 & 0.2 \end{bmatrix}.$$

Given that an insured is classified as Preferred at the start of the second year, find the probability that that insured transitions from being Preferred at the start of the fourth year to being Standard at the start of the fifth year.

Question 2 solution

This can be computed as ${}_2Q_1^{(1,1)}Q_3^{(1,2)}$. ${}_2Q_1^{(1,1)}$ is just the $(1,1)$-entry of

$$\boldsymbol{Q}_1\boldsymbol{Q}_2 = \begin{bmatrix} 0.75 & 0.25 \\ 0.3 & 0.7 \end{bmatrix} \begin{bmatrix} 0.73333 & 0.26667 \\ 0.33333 & 0.66667 \end{bmatrix},$$

namely $0.75(0.73333) + 0.25(0.33333) = 0.63333$. So the answer is $(0.63333)(0.275) = 0.17417$.

3. The CAS Insurance Company classifies its auto drivers as Preferred (State #1) or Standard (State #2) starting at time 0 at the start of the first year when they are first insured, with reclassifications occurring at the start of each new policy year. The transition-probability matrices $\boldsymbol{Q}_n$ from the state at time n at the start of year $n+1$ to the state at time $n+1$ are

$$\boldsymbol{Q}_n = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} + \frac{1}{n+1} \begin{bmatrix} 0.1 & -0.1 \\ -0.2 & 0.2 \end{bmatrix}.$$

Given that an insured is classified as Standard at the start of the second year, find the probability that that insured remains Standard at the start of each of the next three years.

Question 3 solution

This probability is just ${}_3P_2^{(2)} = Q_1^{(2,2)} Q_2^{(2,2)} Q_3^{(2,2)} = (0.7)(0.66667)(0.65) = 0.30333.$

4. The CAS Insurance Company classifies its auto drivers as Preferred (State #1) or Standard (State #2) starting at time 0 at the start of the first year when they are first insured, with reclassifications occurring at the start of each new policy year. The transition-probability matrices $\boldsymbol{Q}_n$ from the state at time n at the start of year $n+1$ to the state at time $n+1$ are

$$\boldsymbol{Q}_n = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} + \frac{1}{n+1} \begin{bmatrix} 0.1 & -0.1 \\ -0.2 & 0.2 \end{bmatrix}.$$

Driver A is Standard now, at the start of the fourth year. For $k = 0, 1$, there is a cost of $10(1.1)^k$ at the end of year $4+k$ for a transition from Standard at the start of that year to Preferred at the start of the next year. Find the actuarial present value now of these cash flows for Driver A using 15% interest.

Question 4 solution

The triple-product summation for this actuarial present value is $Q_3^{(2,1)}(10)(\frac{1}{1.15}) + Q_3^{(2,2)}Q_4^{(2,1)}[10(1.1)](\frac{1}{1.15^2}) = \frac{(0.35)(10)}{1.15} + \frac{(0.65)(0.36)[10(1.1)]}{1.15^2} = 4.9898.$

5. The CAS Insurance Company classifies its auto drivers as Preferred (State #1) or Standard (State #2) starting at time 0 at the start of the first year when they are first insured, with reclassifications occurring at the start of each new policy year. The transition-probability matrices $\boldsymbol{Q}_n$ from the state at time n at the start of year $n+1$ to the state at time $n+1$ are

$$\boldsymbol{Q}_n = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} + \frac{1}{n+1} \begin{bmatrix} 0.1 & -0.1 \\ -0.2 & 0.2 \end{bmatrix}.$$

Driver B is Preferred now, at the start of the fourth year. For $k = 0, 1$, there is a cost of $10(1.1)^k$ at the end of year $4+k$ for a transition from Standard at the start of that year to Preferred at the start of the next year. Find the actuarial present value now of these cash flows for Driver B using 15% interest.

Question 5 solution

The triple-product summation for this actuarial present value is

$$[Q_3^{(1,2)} Q_4^{(2,1)}][10(1.1)](\frac{1}{1.15^2}) = \frac{(0.275)(0.36)(11)}{1.15^2} = 0.82344$$

6. A non-homogeneous Markov Chain has transition-probability matrices $\boldsymbol{Q}_n$ and cash-flow matrices ${}_{\ell+1}\boldsymbol{C}$ defining cash flows at time $\ell+1$ for transitions from states at time ℓ to states at time $\ell+1$. You are given that

$$\boldsymbol{Q}_3 = \begin{bmatrix} 0.8 & 0.2 \\ 0.4 & 0.6 \end{bmatrix}, \qquad {}_4\boldsymbol{C} = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}, \qquad i = 25\%.$$

You also are given that the actuarial present value at time 4 of future cash flows at transition for a subject in State #1 at time 4 equals 5, while it equals 7 for a subject in State #2 at time 4. Find the actuarial present value at time 3 of future cash flows for a subject in State #2 at time 3.

Question 6 solution

You can compute $APV_{2@3}$, the actuarial present value of these cash flows as seen from State #2 at time 3, by splitting off the first time period from the remaining periods:

$$APV_{2@3} = Q_3^{(2,1)}\,{}_4C^{(2,1)}v + Q_3^{(2,1)}vAPV_{1@4} + Q_3^{(2,2)}\,{}_4C^{(2,2)}v + Q_3^{(2,2)}vAPV_{2@4},$$

which equals $(0.4)(4)(0.8) + (0.4)(0.8)(5) + (0.6)(5)(0.8) + (0.6)(0.8)(7) = 8.64$.

7. The CAS Insurance Company classifies its auto drivers as Preferred (State #1) or Standard (State #2) starting at time 0 at the start of the first year when they are first insured, with reclassifications occurring at the start of each new policy year. The transition-probability matrices $\boldsymbol{Q}_n$ from the state at time n at the start of year $n+1$ to the state at time $n+1$ are

$$\boldsymbol{Q}_n = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} + \frac{1}{n+1} \begin{bmatrix} 0.1 & -0.1 \\ -0.2 & 0.2 \end{bmatrix}.$$

Driver C is now Standard at the start of the fourth year. For $k = 0, 1$ there is a cost of 5 at time $3+k$ if Driver C is Standard at the start of year $3+k+1$. Find the actuarial present value now of these costs for Driver C using 15% interest.

Question 7 solution

The triple-product summation for this actuarial present value is

$$(1)(5)(1) + Q_3^{(2,2)}(5)v = 5 + \frac{(0.65)(5)}{1.15} = 7.8261.$$

8. The CAS Insurance Company classifies its auto drivers as Preferred (State #1) or Standard (State #2) starting at time 0 at the start of the first year when they are first insured, with reclassifications occurring at the start of each new policy year. The transition-probability matrices $\boldsymbol{Q}_n$ from the state at time n at the start of year $n+1$ to the state at time $n+1$ are

$$\boldsymbol{Q}_n = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} + \frac{1}{n+1}\begin{bmatrix} 0.1 & -0.1 \\ -0.2 & 0.2 \end{bmatrix}.$$

Driver F is Standard now, at the start of the fourth year. For $k = 0, 1$, there is a cost of $10(1.1)^k$ at the end of year $4+k$ for a transition from Standard at the start of that year to Preferred at the start of the next year. These costs will be funded by allocations ("premiums") P paid at time 3 if Driver F is Standard at time 3 and paid at time 4 if Driver F is Standard at time 4. The allocation is determined to be $P = 3.1879$ by the Equivalence Principle, using 15% interest. Suppose that Driver F is Standard at the start of the fifth year; find the benefit reserve.

Question 8 solution

The benefit reserve is the actuarial present value of future benefits minus that of future benefit premiums, as computed for a driver in State #2 at time 4. Since there is only one year of possible benefits and one certain premium, the benefit reserve is

$$Q_4^{(2,1)}[10(1.1)](\frac{1}{1.15}) - 3.1879 = \frac{(0.36)(11)}{1.15} - 3.1879 = 0.25558.$$

9. The status of residents in a Continuing Care Retirement Community (CCRC) is modeled by a non-homogeneous Markov Chain with three states: Independent Living (#1), Health Center (#2), and Gone (#3). The transition-probability matrices for a new entrant (time 0) are

$$\boldsymbol{Q}_0 = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.3 \\ 0 & 0 & 1 \end{bmatrix}, \boldsymbol{Q}_1 = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0 & 0.4 & 0.6 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{Q}_2 = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{bmatrix}, \boldsymbol{Q}_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

A new entrant, Resident G, enters Independent Living at time 0. The CCRC undergoes a cost of 100 at the end of year k for transition from Independent Living at the start of that year to Health Center at the start of the next year, for all k. The CCRC wishes to charge a fee P at the start of each year when Resident G is in Independent Living, with P determined by the Equivalence Principle to be $P = 17.97$ using 25% interest. Suppose that Resident G is in Independent Living at the start of the third year. Find the benefit reserve.

Question 9 solution

The benefit reserve is the actuarial present value of future benefits minus that of future benefit premiums, as computed for a resident in State #1 at time 2. Since there is only one year of possible benefits and one certain premium, the benefit reserve is

$$Q_2^{(1,2)}(100)v - 17.97 = (0.2)(100)(0.8) - 17.97 = -1,97.$$

10. The CAS Insurance Company classifies its auto drivers as Preferred (State #1) or Standard (State #2) starting at time 0 at the start of the first year when they are first insured, with reclassifications occurring at the start of each new policy year. The transition-probability matrices $\boldsymbol{Q}_n$ from the state at time n at the start of year $n+1$ to the state at time $n+1$ are

$$\boldsymbol{Q}_n = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix} + \frac{1}{n+1} \begin{bmatrix} 0.1 & -0.1 \\ -0.2 & 0.2 \end{bmatrix}.$$

Driver D is Standard now, at the start of the fourth year. For $k = 0, 1$, there is a cost of $10(1.1)^k$ at the end of year $4+k$ for a transition from Standard at the start of that year to Preferred at the start of the next year. These costs will be funded by allocations ("premiums") P paid at time 3 if Driver D is Standard at time 3 and paid at time 4 if Driver D is Standard at time 4. The allocation is determined by the Equivalence Principle, using 15% interest. Find P.

Question 10 solution

The benefit-premium can be computed as the actuarial present value of the benefits divided by the actuarial present value of premiums of 1. The triple-product summation for the actuarial present value of the benefits is $Q_3^{(2,1)}(10)(\frac{1}{1.15})+Q_3^{(2,2)}Q_4^{(2,1)}[10(1.1)](\frac{1}{1.15^2}) = \frac{(0.35)(10)}{1.15} + \frac{(0.65)(0.36)[10(1.1)]}{1.15^2} = 4.9898$. That for the actuarial present value of premiums of 1 is

$$(1)(1)(1) + Q_3^{(2,2)}(1)v = 1 + \frac{(0.65)(1)}{1.15} = 1.5652.$$

Thus the benefit premium is $\frac{4.9898}{1.5652} = 3.1879$.

11. The status of residents in a Continuing Care Retirement Community (CCRC) is modeled by a non-homogeneous Markov Chain with three states: Independent Living (#1), Health Center (#2), and Gone (#3). The transition-probability matrices for a new entrant (time 0) are

$$\boldsymbol{Q}_0 = \begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.1 & 0.6 & 0.3 \\ 0 & 0 & 1 \end{bmatrix}, \boldsymbol{Q}_1 = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0 & 0.4 & 0.6 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\boldsymbol{Q}_2 = \begin{bmatrix} 0.3 & 0.2 & 0.5 \\ 0 & 0.2 & 0.8 \\ 0 & 0 & 1 \end{bmatrix}, \boldsymbol{Q}_3 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

A new entrant, Resident E, enters Independent Living at time 0. The CCRC undergoes a cost of 100 at the end of year k for transition from Independent Living at the start of that year to Health Center at the start of the next year, for all k. The CCRC wishes to charge a fee P at the start of each year when Resident E is in Independent Living, with P determined by the Equivalence Principle using 25% interest. Find P.

Question 11 solution

The benefit-premium can be computed as the actuarial present value of the benefits divided by the actuarial present value of premiums of 1. The triple-product summation for the actuarial present value of the benefits is

$$Q_0^{(2,2)}(100)v + Q_0^{(1,1)}Q_1^{(1,2)}(100)v^2 + {}_2Q_0^{(1,1)}Q_2^{(1,2)}(100)v^3,$$

which equals $(0.2)(100)(0.8)+(0.7)(0.3)(100)(0.64)+(0.35)(0.2)(100)(0.512) = 33.024$; here ${}_2Q_0^{(1,1)}$ was computed as the $(1,1)$-entry of $\boldsymbol{Q}_0\boldsymbol{Q}_1$, namely 0.35. The actuarial present value of premiums of 1 is

$$1 + Q_0^{(1,1)}v + {}_2Q_0^{(1,1)}v^2 + {}_3Q_0^{(1,1)}v^3,$$

which is $1+(0.7)(0.8)+(0.35)(0.64)+(0.105)(0.512) = 1.83776$; here ${}_3Q_0^{(1,1)}$ was computed as the $(1,1)$-entry of $\boldsymbol{Q}_0\boldsymbol{Q}_1\boldsymbol{Q}_2$, namely 0.105. Finally, the benefit premium is $\frac{33.024}{1.83776} = 17.970$.