Tables for Exam M

The reading material for Exam M includes a variety of textbooks. Each text has a set of probability distributions that are used in its readings. For those distributions used in more than one text, the choices of parameterization may not be the same in all of the books. This may be of educational value while you study, but could add a layer of uncertainty in the examination. For this latter reason, we have adopted one set of parameterizations to be used in examinations. This set will be based on Appendices A & B of *Loss Models: From Data to Decisions* by Klugman, Panjer and Willmot. A slightly revised version of these appendices is included in this note. A copy of this note will also be distributed to each candidate at the examination.

As an example of this adopted notation, consider the family of single-parameter exponential distributions. The distribution with mean 2 would be identified in three of the textbooks as follows:

Actuarial Mathematics	*the exponential distribution with* $\beta = ½$
Probability Models	*the exponential distribution with* $\lambda = ½$
Loss Models	*the exponential distribution with* $\theta = 2$

The last form is the one that will be used in examinations.

Another difference among the texts is the choice of generating functions for *discrete* distributions. *Loss Models* uses *probability* generating functions while *Actuarial Mathematics* and *Probability Models* use *moment* generating functions. The abridged tables from *Loss Models* will provide *only* the *probability* generating function for discrete distributions.

Each text also has its own system of dedicated notation and terminology. Sometimes these may conflict. If alternative meanings could apply in an examination question, the symbols will be defined.

In addition to the abridged table from *Loss Models*, an abridged version of the Illustrative Life Table from *Actuarial Mathematics* and a set of values from the standard normal distribution will be available for use in examinations. These are also included in this note.

NORMAL DISTRIBUTION TABLE

Entries represent the area under the standardized normal distribution from -∞ to z, Pr(Z<z)
The value of z to the first decimal is given in the left column. The second decimal place is given in the top row.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Values of z for selected values of Pr(Z<z)							
z	0.842	1.036	1.282	1.645	1.960	2.326	2.576
Pr(Z<z)	0.800	0.850	0.900	0.950	0.975	0.990	0.995

Excerpts from the Appendices to *Loss Models: From Data to Decisions, 2nd edition*

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Appendix A

An Inventory of Continuous Distributions

A.1 Introduction

The incomplete gamma function is given by

$$\Gamma(\alpha; x) = \frac{1}{\Gamma(\alpha)} \int_0^x t^{\alpha-1} e^{-t}\, dt, \quad \alpha > 0,\ x > 0$$

$$\text{with } \Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t}\, dt, \quad \alpha > 0.$$

Also, define

$$G(\alpha; x) = \int_x^\infty t^{\alpha-1} e^{-t}\, dt, \quad x > 0.$$

Integration by parts produces the relationship

$$G(\alpha; x) = -\frac{x^\alpha e^{-x}}{\alpha} + \frac{1}{\alpha} G(\alpha + 1; x)$$

For negative α, this can repeated until the first argument is postive, say at $\alpha + k$. Then the incomplete gamma function can be evaluated from

$$G(\alpha + k; x) = \Gamma(\alpha + k)[1 - \Gamma(\alpha + k; x)].$$

The incomplete beta function is given by

$$\beta(a, b; x) = \frac{\Gamma(a + b)}{\Gamma(a)\Gamma(b)} \int_0^x t^{a-1}(1 - t)^{b-1}\, dt, \quad a > 0,\ b > 0,\ 0 < x < 1.$$

A.2 Transformed beta family

A.2.3 Three-parameter distributions

A.2.3.1 Generalized Pareto (beta of the second kind)—α, θ, τ

$$
\begin{aligned}
f(x) &= \frac{\Gamma(\alpha+\tau)}{\Gamma(\alpha)\Gamma(\tau)} \frac{\theta^\alpha x^{\tau-1}}{(x+\theta)^{\alpha+\tau}} \qquad F(x) = \beta(\tau, \alpha; u), \quad u = \frac{x}{x+\theta} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(\tau+k)\Gamma(\alpha-k)}{\Gamma(\alpha)\Gamma(\tau)}, \quad -\tau < k < \alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k \tau(\tau+1)\cdots(\tau+k-1)}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\tau+k)\Gamma(\alpha-k)}{\Gamma(\alpha)\Gamma(\tau)} \beta(\tau+k, \alpha-k; u) + x^k[1-F(x)], \quad k > -\tau \\
\text{mode} &= \theta\frac{\tau-1}{\alpha+1}, \quad \tau > 1, \text{ else } 0
\end{aligned}
$$

A.2.3.2 Burr (Burr Type XII, Singh-Maddala)—α, θ, γ

$$
\begin{aligned}
f(x) &= \frac{\alpha\gamma(x/\theta)^\gamma}{x[1+(x/\theta)^\gamma]^{\alpha+1}} \qquad F(x) = 1-u^\alpha, \quad u = \frac{1}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(1+k/\gamma)\Gamma(\alpha-k/\gamma)}{\Gamma(\alpha)}, \quad -\gamma < k < \alpha\gamma \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(1+k/\gamma)\Gamma(\alpha-k/\gamma)}{\Gamma(\alpha)} \beta(1+k/\gamma, \alpha-k/\gamma; 1-u) + x^k u^\alpha, \quad k > -\gamma \\
\text{mode} &= \theta\left(\frac{\gamma-1}{\alpha\gamma+1}\right)^{1/\gamma}, \quad \gamma > 1, \text{ else } 0
\end{aligned}
$$

A.2.3.3 Inverse Burr (Dagum)—τ, θ, γ

$$
\begin{aligned}
f(x) &= \frac{\tau\gamma(x/\theta)^{\gamma\tau}}{x[1+(x/\theta)^\gamma]^{\tau+1}} \qquad F(x) = u^\tau, \quad u = \frac{(x/\theta)^\gamma}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \frac{\theta^k \Gamma(\tau+k/\gamma)\Gamma(1-k/\gamma)}{\Gamma(\tau)}, \quad -\tau\gamma < k < \gamma \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\tau+k/\gamma)\Gamma(1-k/\gamma)}{\Gamma(\tau)} \beta(\tau+k/\gamma, 1-k/\gamma; u) + x^k[1-u^\tau], \quad k > -\tau\gamma \\
\text{mode} &= \theta\left(\frac{\tau\gamma-1}{\gamma+1}\right)^{1/\gamma}, \quad \tau\gamma > 1, \text{ else } 0
\end{aligned}
$$

A.2.4 Two-parameter distributions

A.2.4.1 Pareto—α, θ

$$
\begin{aligned}
f(x) &= \frac{\alpha\theta^\alpha}{(x+\theta)^{\alpha+1}} \qquad F(x) = 1 - \left(\frac{\theta}{x+\theta}\right)^\alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}, \quad -1 < k < \alpha \\
\mathrm{E}[X^k] &= \frac{\theta^k k!}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer} \\
\mathrm{E}[X \wedge x] &= \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{x+\theta}\right)^{\alpha-1}\right], \quad \alpha \neq 1 \\
\mathrm{E}[X \wedge x] &= -\theta \ln\left(\frac{\theta}{x+\theta}\right), \quad \alpha = 1 \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k\Gamma(k+1)\Gamma(\alpha-k)}{\Gamma(\alpha)}\beta[k+1, \alpha-k; x/(x+\theta)] + x^k\left(\frac{\theta}{x+\theta}\right)^\alpha, \quad \text{all } k \\
\text{mode} &= 0
\end{aligned}
$$

A.2.4.2 Inverse Pareto—τ, θ

$$
\begin{aligned}
f(x) &= \frac{\tau\theta x^{\tau-1}}{(x+\theta)^{\tau+1}} \qquad F(x) = \left(\frac{x}{x+\theta}\right)^\tau \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(\tau+k)\Gamma(1-k)}{\Gamma(\tau)}, \quad -\tau < k < 1 \\
\mathrm{E}[X^k] &= \frac{\theta^k(-k)!}{(\tau-1)\cdots(\tau+k)}, \quad \text{if } k \text{ is a negative integer} \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\tau\int_0^{x/(x+\theta)} y^{\tau+k-1}(1-y)^{-k}dy + x^k\left[1 - \left(\frac{x}{x+\theta}\right)^\tau\right], \quad k > -\tau \\
\text{mode} &= \theta\frac{\tau-1}{2}, \quad \tau > 1, \text{ else } 0
\end{aligned}
$$

A.2.4.3 Loglogistic (Fisk)—γ, θ

$$
\begin{aligned}
f(x) &= \frac{\gamma(x/\theta)^\gamma}{x[1+(x/\theta)^\gamma]^2} \qquad F(x) = u, \quad u = \frac{(x/\theta)^\gamma}{1+(x/\theta)^\gamma} \\
\mathrm{E}[X^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma), \quad -\gamma < k < \gamma \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k\Gamma(1+k/\gamma)\Gamma(1-k/\gamma)\beta(1+k/\gamma, 1-k/\gamma; u) + x^k(1-u), \quad k > -\gamma \\
\text{mode} &= \theta\left(\frac{\gamma-1}{\gamma+1}\right)^{1/\gamma}, \quad \gamma > 1, \text{ else } 0
\end{aligned}
$$

A.2.4.4 Paralogistic—α, θ

This is a Burr distribution with $\gamma = \alpha$.

$$\begin{aligned}
f(x) &= \frac{\alpha^2(x/\theta)^\alpha}{x[1+(x/\theta)^\alpha]^{\alpha+1}} \qquad F(x) = 1 - u^\alpha, \quad u = \frac{1}{1+(x/\theta)^\alpha} \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(1+k/\alpha)\Gamma(\alpha-k/\alpha)}{\Gamma(\alpha)}, \quad -\alpha < k < \alpha^2 \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k\Gamma(1+k/\alpha)\Gamma(\alpha-k/\alpha)}{\Gamma(\alpha)}\beta(1+k/\alpha, \alpha-k/\alpha; 1-u) + x^k u^\alpha, \quad k > -\alpha \\
\text{mode} &= \theta\left(\frac{\alpha-1}{\alpha^2+1}\right)^{1/\alpha}, \quad \alpha > 1, \text{ else } 0
\end{aligned}$$

A.2.4.5 Inverse paralogistic—τ, θ

This is an inverse Burr distribution with $\gamma = \tau$.

$$\begin{aligned}
f(x) &= \frac{\tau^2(x/\theta)^{\tau^2}}{x[1+(x/\theta)^\tau]^{\tau+1}} \qquad F(x) = u^\tau, \quad u = \frac{(x/\theta)^\tau}{1+(x/\theta)^\tau} \\
\mathrm{E}[X^k] &= \frac{\theta^k\Gamma(\tau+k/\tau)\Gamma(1-k/\tau)}{\Gamma(\tau)}, \quad -\tau^2 < k < \tau \\
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k\Gamma(\tau+k/\tau)\Gamma(1-k/\tau)}{\Gamma(\tau)}\beta(\tau+k/\tau, 1-k/\tau; u) + x^k[1-u^\tau], \quad k > -\tau^2 \\
\text{mode} &= \theta\,(\tau-1)^{1/\tau}, \quad \tau > 1, \text{ else } 0
\end{aligned}$$

A.3 Transformed gamma family

A.3.2 Two-parameter distributions

A.3.2.1 Gamma—α, θ

$$\begin{aligned}
f(x) &= \frac{(x/\theta)^\alpha e^{-x/\theta}}{x\Gamma(\alpha)} \qquad F(x) = \Gamma(\alpha; x/\theta) \\
M(t) &= (1-\theta t)^{-\alpha}, \quad t < 1/\theta \qquad \mathrm{E}[X^k] = \frac{\theta^k\Gamma(\alpha+k)}{\Gamma(\alpha)}, \quad k > -\alpha \\
\mathrm{E}[X^k] &= \theta^k(\alpha+k-1)\cdots\alpha, \quad \text{if } k \text{ is an integer}
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k\Gamma(\alpha+k)}{\Gamma(\alpha)}\Gamma(\alpha+k; x/\theta) + x^k[1-\Gamma(\alpha; x/\theta)], \quad k > -\alpha \\
&= \alpha(\alpha+1)\cdots(\alpha+k-1)\theta^k\Gamma(\alpha+k; x/\theta) + x^k[1-\Gamma(\alpha; x/\theta)], \quad k \text{ an integer} \\
\text{mode} &= \theta(\alpha-1), \quad \alpha > 1, \text{ else } 0
\end{aligned}$$

A.3.2.2 Inverse gamma (Vinci)—α, θ

$$f(x) = \frac{(\theta/x)^\alpha e^{-\theta/x}}{x\Gamma(\alpha)} \qquad F(x) = 1 - \Gamma(\alpha; \theta/x)$$

$$\mathrm{E}[X^k] = \frac{\theta^k \Gamma(\alpha - k)}{\Gamma(\alpha)}, \quad k < \alpha \qquad \mathrm{E}[X^k] = \frac{\theta^k}{(\alpha-1)\cdots(\alpha-k)}, \quad \text{if } k \text{ is an integer}$$

$$\begin{aligned}
\mathrm{E}[(X \wedge x)^k] &= \frac{\theta^k \Gamma(\alpha - k)}{\Gamma(\alpha)}[1 - \Gamma(\alpha - k; \theta/x)] + x^k \Gamma(\alpha; \theta/x) \\
&= \frac{\theta^k \Gamma(\alpha - k)}{\Gamma(\alpha)} G(\alpha - k; \theta/x) + x^k \Gamma(\alpha; \theta/x), \text{ all } k \\
\text{mode} &= \theta/(\alpha + 1)
\end{aligned}$$

A.3.2.3 Weibull—θ, τ

$$f(x) = \frac{\tau (x/\theta)^\tau e^{-(x/\theta)^\tau}}{x} \qquad F(x) = 1 - e^{-(x/\theta)^\tau}$$

$$\begin{aligned}
\mathrm{E}[X^k] &= \theta^k \Gamma(1 + k/\tau), \quad k > -\tau \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k \Gamma(1 + k/\tau) \Gamma[1 + k/\tau; (x/\theta)^\tau] + x^k e^{-(x/\theta)^\tau}, \quad k > -\tau \\
\text{mode} &= \theta \left(\frac{\tau - 1}{\tau}\right)^{1/\tau}, \quad \tau > 1, \text{ else } 0
\end{aligned}$$

A.3.2.4 Inverse Weibull (log Gompertz)—θ, τ

$$f(x) = \frac{\tau (\theta/x)^\tau e^{-(\theta/x)^\tau}}{x} \qquad F(x) = e^{-(\theta/x)^\tau}$$

$$\begin{aligned}
\mathrm{E}[X^k] &= \theta^k \Gamma(1 - k/\tau), \quad k < \tau \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k \Gamma(1 - k/\tau)\{1 - \Gamma[1 - k/\tau; (\theta/x)^\tau]\} + x^k \left[1 - e^{-(\theta/x)^\tau}\right], \quad \text{all } k \\
&= \theta^k \Gamma(1 - k/\tau) G[1 - k/\tau; (\theta/x)^\tau] + x^k \left[1 - e^{-(\theta/x)^\tau}\right] \\
\text{mode} &= \theta \left(\frac{\tau}{\tau + 1}\right)^{1/\tau}
\end{aligned}$$

A.3.3 One-parameter distributions

A.3.3.1 Exponential—θ

$$f(x) = \frac{e^{-x/\theta}}{\theta} \qquad F(x) = 1 - e^{-x/\theta}$$

$$M(t) = (1 - \theta t)^{-1} \qquad \mathrm{E}[X^k] = \theta^k \Gamma(k + 1), \quad k > -1$$

$$\begin{aligned}
\mathrm{E}[X^k] &= \theta^k k!, \quad \text{if } k \text{ is an integer} \\
\mathrm{E}[X \wedge x] &= \theta(1 - e^{-x/\theta}) \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k \Gamma(k + 1) \Gamma(k + 1; x/\theta) + x^k e^{-x/\theta}, \quad k > -1 \\
&= \theta^k k! \Gamma(k + 1; x/\theta) + x^k e^{-x/\theta}, \quad k \text{ an integer} \\
\text{mode} &= 0
\end{aligned}$$

A.3.3.2 Inverse exponential—θ

$$
\begin{aligned}
f(x) &= \frac{\theta e^{-\theta/x}}{x^2} \qquad F(x) = e^{-\theta/x} \\
\mathrm{E}[X^k] &= \theta^k \Gamma(1-k), \quad k<1 \\
\mathrm{E}[(X \wedge x)^k] &= \theta^k \Gamma(1-k) G(1-k;\theta/x) + x^k(1-e^{-\theta/x}), \quad \text{all } k \\
\text{mode} &= \theta/2
\end{aligned}
$$

A.4 Other distributions

A.4.1.1 Lognormal—μ,σ (μ can be negative)

$$
\begin{aligned}
f(x) &= \frac{1}{x\sigma\sqrt{2\pi}} \exp(-z^2/2) = \phi(z)/(\sigma x), \quad z = \frac{\ln x - \mu}{\sigma} \qquad F(x) = \Phi(z) \\
\mathrm{E}[X^k] &= \exp(k\mu + k^2\sigma^2/2) \\
\mathrm{E}[(X \wedge x)^k] &= \exp(k\mu + k^2\sigma^2/2)\Phi\left(\frac{\ln x - \mu - k\sigma^2}{\sigma}\right) + x^k[1-F(x)] \\
\text{mode} &= \exp(\mu - \sigma^2)
\end{aligned}
$$

A.4.1.2 Inverse Gaussian—μ, θ

$$
\begin{aligned}
f(x) &= \left(\frac{\theta}{2\pi x^3}\right)^{1/2} \exp\left(-\frac{\theta z^2}{2x}\right), \quad z = \frac{x-\mu}{\mu} \\
F(x) &= \Phi\left[z\left(\frac{\theta}{x}\right)^{1/2}\right] + \exp\left(\frac{2\theta}{\mu}\right)\Phi\left[-y\left(\frac{\theta}{x}\right)^{1/2}\right], \quad y = \frac{x+\mu}{\mu} \\
M(t) &= \exp\left[\frac{\theta}{\mu}\left(1-\sqrt{1-\frac{2t\mu^2}{\theta}}\right)\right], \quad t < \frac{\theta}{2\mu^2}, \qquad \mathrm{E}[X] = \mu, \quad \mathrm{Var}[X] = \mu^3/\theta \\
\mathrm{E}[X \wedge x] &= x - \mu z \Phi\left[z\left(\frac{\theta}{x}\right)^{1/2}\right] - \mu y \exp\left(\frac{2\theta}{\mu}\right)\Phi\left[-y\left(\frac{\theta}{x}\right)^{1/2}\right]
\end{aligned}
$$

A.4.1.3 log-t—r, μ, σ (μ *can be negative*)

Let Y have a t distribution with r degrees of freedom. Then $X = \exp(\sigma Y + \mu)$ has the log-t distribution. Positive moments do not exist for this distribution. Just as the t distribution has a heavier tail than the normal distribution, this distribution has a heavier tail than the lognormal distribution.

$$
\begin{aligned}
f(x) &= \frac{\Gamma\left(\frac{r+1}{2}\right)}{x\sigma\sqrt{\pi r}\,\Gamma\left(\frac{r}{2}\right)\left[1+\frac{1}{r}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right]^{(r+1)/2}}, \\
F(x) &= F_r\left(\frac{\ln x - \mu}{\sigma}\right) \text{ with } F_r(t) \text{ the cdf of a } t \text{ distribution with } r \text{ d.f.,}
\end{aligned}
$$

$$F(x) = \begin{cases} \frac{1}{2}\beta\left[\frac{r}{2},\frac{1}{2};\frac{r}{r+\left(\frac{\ln x-\mu}{\sigma}\right)^2}\right], & 0 < x \le e^{\mu}, \\ 1-\frac{1}{2}\beta\left[\frac{r}{2},\frac{1}{2};\frac{r}{r+\left(\frac{\ln x-\mu}{\sigma}\right)^2}\right], & x \ge e^{\mu}. \end{cases}$$

A.4.1.4 Single-parameter Pareto—α, θ

$$\begin{aligned} f(x) &= \frac{\alpha\theta^\alpha}{x^{\alpha+1}}, \quad x > \theta & F(x) &= 1-(\theta/x)^\alpha, \quad x > \theta \\ \mathrm{E}[X^k] &= \frac{\alpha\theta^k}{\alpha-k}, \quad k < \alpha & \mathrm{E}[(X\wedge x)^k] &= \frac{\alpha\theta^k}{\alpha-k} - \frac{k\theta^\alpha}{(\alpha-k)x^{\alpha-k}} \\ \text{mode} &= \theta \end{aligned}$$

Note: Although there appears to be two parameters, only α is a true parameter. The value of θ must be set in advance.

A.5 Distributions with finite support

For these two distributions, the scale parameter θ is assumed known.

A.5.1.1 Generalized beta—a, b, θ, τ

$$\begin{aligned} f(x) &= \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}u^a(1-u)^{b-1}\frac{\tau}{x}, \quad 0 < x < \theta, \quad u = (x/\theta)^\tau \\ F(x) &= \beta(a,b;u) \\ \mathrm{E}[X^k] &= \frac{\theta^k\Gamma(a+b)\Gamma(a+k/\tau)}{\Gamma(a)\Gamma(a+b+k/\tau)}, \quad k > -a\tau \\ \mathrm{E}[(X\wedge x)^k] &= \frac{\theta^k\Gamma(a+b)\Gamma(a+k/\tau)}{\Gamma(a)\Gamma(a+b+k/\tau)}\beta(a+k/\tau,b;u) + x^k[1-\beta(a,b;u)] \end{aligned}$$

A.5.1.2 beta—a, b, θ

$$\begin{aligned} f(x) &= \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}u^a(1-u)^{b-1}\frac{1}{x}, \quad 0 < x < \theta, \quad u = x/\theta \\ F(x) &= \beta(a,b;u) \\ \mathrm{E}[X^k] &= \frac{\theta^k\Gamma(a+b)\Gamma(a+k)}{\Gamma(a)\Gamma(a+b+k)}, \quad k > -a \\ \mathrm{E}[X^k] &= \frac{\theta^k a(a+1)\cdots(a+k-1)}{(a+b)(a+b+1)\cdots(a+b+k-1)}, \quad \text{if } k \text{ is an integer} \\ \mathrm{E}[(X\wedge x)^k] &= \frac{\theta^k a(a+1)\cdots(a+k-1)}{(a+b)(a+b+1)\cdots(a+b+k-1)}\beta(a+k,b;u) \\ &\quad +x^k[1-\beta(a,b;u)] \end{aligned}$$

Appendix B

An Inventory of Discrete Distributions

B.2 The $(a, b, 0)$ class

B.2.1.1 Poisson—λ

$$\begin{aligned} p_0 &= e^{-\lambda}, \quad a = 0, \quad b = \lambda \qquad\qquad p_k = \frac{e^{-\lambda}\lambda^k}{k!} \\ \mathrm{E}[N] &= \lambda, \quad \mathrm{Var}[N] = \lambda \qquad\qquad P(z) = e^{\lambda(z-1)} \end{aligned}$$

B.2.1.2 Geometric—β

$$\begin{aligned} p_0 &= \frac{1}{1+\beta}, \quad a = \frac{\beta}{1+\beta}, \quad b = 0 \qquad\qquad p_k = \frac{\beta^k}{(1+\beta)^{k+1}} \\ \mathrm{E}[N] &= \beta, \quad \mathrm{Var}[N] = \beta(1+\beta) \qquad\qquad P(z) = [1-\beta(z-1)]^{-1}. \end{aligned}$$

This is a special case of the negative binomial with $r = 1$.

B.2.1.3 Binomial—q, m, $(0 < q < 1$, m an integer)

$$\begin{aligned} p_0 &= (1-q)^m, \quad a = -\frac{q}{1-q}, \quad b = \frac{(m+1)q}{1-q} \\ p_k &= \binom{m}{k} q^k (1-q)^{m-k}, \quad k = 0, 1, \ldots, m \\ \mathrm{E}[N] &= mq, \quad \mathrm{Var}[N] = mq(1-q) \qquad\qquad P(z) = [1+q(z-1)]^m. \end{aligned}$$

B.2.1.4 Negative binomial—β, r

$$\begin{aligned} p_0 &= (1+\beta)^{-r}, \quad a = \frac{\beta}{1+\beta}, \quad b = \frac{(r-1)\beta}{1+\beta} \\ p_k &= \frac{r(r+1)\cdots(r+k-1)\beta^k}{k!(1+\beta)^{r+k}} \\ \mathrm{E}[N] &= r\beta, \quad \mathrm{Var}[N] = r\beta(1+\beta) \qquad\qquad P(z) = [1-\beta(z-1)]^{-r}. \end{aligned}$$

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
0	10,000,000	20.42	16.8010	49.00	25.92	728.54	541.95	299.89	0
5	9,749,503	0.98	17.0379	35.59	8.45	743.89	553.48	305.90	5
10	9,705,588	0.85	16.9119	42.72	9.37	744.04	553.34	305.24	10
15	9,663,731	0.91	16.7384	52.55	11.33	743.71	552.69	303.96	15
20	9,617,802	1.03	16.5133	65.28	14.30	743.16	551.64	301.93	20
21	9,607,896	1.06	16.4611	68.24	15.06	743.01	551.36	301.40	21
22	9,597,695	1.10	16.4061	71.35	15.87	742.86	551.06	300.82	22
23	9,587,169	1.13	16.3484	74.62	16.76	742.68	550.73	300.19	23
24	9,576,288	1.18	16.2878	78.05	17.71	742.49	550.36	299.49	24
25	9,565,017	1.22	16.2242	81.65	18.75	742.29	549.97	298.73	25
26	9,553,319	1.27	16.1574	85.43	19.87	742.06	549.53	297.90	26
27	9,541,153	1.33	16.0873	89.40	21.07	741.81	549.05	297.00	27
28	9,528,475	1.39	16.0139	93.56	22.38	741.54	548.53	296.01	28
29	9,515,235	1.46	15.9368	97.92	23.79	741.24	547.96	294.92	29
30	9,501,381	1.53	15.8561	102.48	25.31	740.91	547.33	293.74	30
31	9,486,854	1.61	15.7716	107.27	26.95	740.55	546.65	292.45	31
32	9,471,591	1.70	15.6831	112.28	28.72	740.16	545.90	291.04	32
33	9,455,522	1.79	15.5906	117.51	30.63	739.72	545.07	289.50	33
34	9,438,571	1.90	15.4938	122.99	32.68	739.25	544.17	287.82	34
35	9,420,657	2.01	15.3926	128.72	34.88	738.73	543.18	286.00	35
36	9,401,688	2.14	15.2870	134.70	37.26	738.16	542.11	284.00	36
37	9,381,566	2.28	15.1767	140.94	39.81	737.54	540.92	281.84	37
38	9,360,184	2.43	15.0616	147.46	42.55	736.86	539.63	279.48	38
39	9,337,427	2.60	14.9416	154.25	45.48	736.11	538.22	276.92	39
40	9,313,166	2.78	14.8166	161.32	48.63	735.29	536.67	274.14	40
41	9,287,264	2.98	14.6864	168.69	52.01	734.40	534.99	271.12	41
42	9,259,571	3.20	14.5510	176.36	55.62	733.42	533.14	267.85	42
43	9,229,925	3.44	14.4102	184.33	59.48	732.34	531.12	264.31	43
44	9,198,149	3.71	14.2639	192.61	63.61	731.17	528.92	260.48	44
45	9,164,051	4.00	14.1121	201.20	68.02	729.88	526.52	256.34	45
46	9,127,426	4.31	13.9546	210.12	72.72	728.47	523.89	251.88	46
47	9,088,049	4.66	13.7914	219.36	77.73	726.93	521.03	247.08	47
48	9,045,679	5.04	13.6224	228.92	83.06	725.24	517.91	241.93	48
49	9,000,057	5.46	13.4475	238.82	88.73	723.39	514.51	236.39	49
50	8,950,901	5.92	13.2668	249.05	94.76	721.37	510.81	230.47	50
51	8,897,913	6.42	13.0803	259.61	101.15	719.17	506.78	224.15	51
52	8,840,770	6.97	12.8879	270.50	107.92	716.76	502.40	217.42	52
53	8,779,128	7.58	12.6896	281.72	115.09	714.12	497.64	210.27	53
54	8,712,621	8.24	12.4856	293.27	122.67	711.24	492.47	202.70	54
55	8,640,861	8.96	12.2758	305.14	130.67	708.10	486.86	194.72	55
56	8,563,435	9.75	12.0604	317.33	139.11	704.67	480.79	186.32	56
57	8,479,908	10.62	11.8395	329.84	147.99	700.93	474.22	177.53	57
58	8,389,826	11.58	11.6133	342.65	157.33	696.85	467.12	168.37	58
59	8,292,713	12.62	11.3818	355.75	167.13	692.41	459.46	158.87	59
60	8,188,074	13.76	11.1454	369.13	177.41	687.56	451.20	149.06	60
61	8,075,403	15.01	10.9041	382.79	188.17	682.29	442.31	139.00	61
62	7,954,179	16.38	10.6584	396.70	199.41	676.56	432.77	128.75	62
63	7,823,879	17.88	10.4084	410.85	211.13	670.33	422.54	118.38	63
64	7,683,979	19.52	10.1544	425.22	223.34	663.56	411.61	107.97	64
65	7,533,964	21.32	9.8969	439.80	236.03	656.23	399.94	97.60	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

x	l_x	$1000q_x$	$\ddot{a}_x$	$1000A_x$	$1000(^2A_x)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$	$1000\,{}_{20}E_x$	x
66	7,373,338	23.29	9.6362	454.56	249.20	648.27	387.53	87.37	66
67	7,201,635	25.44	9.3726	469.47	262.83	639.66	374.36	77.38	67
68	7,018,432	27.79	9.1066	484.53	276.92	630.35	360.44	67.74	68
69	6,823,367	30.37	8.8387	499.70	291.46	620.30	345.77	58.54	69
70	6,616,155	33.18	8.5693	514.95	306.42	609.46	330.37	49.88	70
71	6,396,609	36.26	8.2988	530.26	321.78	597.79	314.27	41.86	71
72	6,164,663	39.62	8.0278	545.60	337.54	585.25	297.51	34.53	72
73	5,920,394	43.30	7.7568	560.93	353.64	571.81	280.17	27.96	73
74	5,664,051	47.31	7.4864	576.24	370.08	557.43	262.31	22.19	74
75	5,396,081	51.69	7.2170	591.49	386.81	542.07	244.03	17.22	75
76	5,117,152	56.47	6.9493	606.65	403.80	525.71	225.46	13.04	76
77	4,828,182	61.68	6.6836	621.68	421.02	508.35	206.71	9.61	77
78	4,530,360	67.37	6.4207	636.56	438.42	489.97	187.94	6.88	78
79	4,225,163	73.56	6.1610	651.26	455.95	470.57	169.31	4.77	79
80	3,914,365	80.30	5.9050	665.75	473.59	450.19	151.00	3.19	80
81	3,600,038	87.64	5.6533	680.00	491.27	428.86	133.19	2.05	81
82	3,284,542	95.61	5.4063	693.98	508.96	406.62	116.06	1.27	82
83	2,970,496	104.28	5.1645	707.67	526.60	383.57	99.81	0.75	83
84	2,660,734	113.69	4.9282	721.04	544.15	359.79	84.59	0.42	84
85	2,358,246	123.89	4.6980	734.07	561.57	335.40	70.56	0.22	85
86	2,066,090	134.94	4.4742	746.74	578.80	310.56	57.83	0.11	86
87	1,787,299	146.89	4.2571	759.03	595.79	285.44	46.50	0.05	87
88	1,524,758	159.81	4.0470	770.92	612.51	260.21	36.61	0.02	88
89	1,281,083	173.75	3.8442	782.41	628.92	235.11	28.17	0.01	89
90	1,058,491	188.77	3.6488	793.46	644.96	210.36	21.13	0.00	90
91	858,676	204.93	3.4611	804.09	660.61	186.21	15.41	0.00	91
92	682,707	222.27	3.2812	814.27	675.83	162.90	10.91	0.00	92
93	530,959	240.86	3.1091	824.01	690.59	140.69	7.47	0.00	93
94	403,072	260.73	2.9450	833.30	704.86	119.79	4.93	0.00	94
95	297,981	281.91	2.7888	842.14	718.61	100.43	3.13	0.00	95
96	213,977	304.45	2.6406	850.53	731.83	82.78	1.90	0.00	96
97	148,832	328.34	2.5002	858.48	744.50	66.97	1.10	0.00	97
98	99,965	353.60	2.3676	865.99	756.60	53.09	0.60	0.00	98
99	64,617	380.20	2.2426	873.06	768.13	41.14	0.31	0.00	99
100	40,049	408.12	2.1252	879.70	779.08	31.12	0.15	0.00	100
101	23,705	437.28	2.0152	885.93	789.44	22.91	0.07	0.00	101
102	13,339	467.61	1.9123	891.76	799.21	16.37	0.03	0.00	102
103	7,101	498.99	1.8164	897.19	808.41	11.33	0.01	0.00	103
104	3,558	531.28	1.7273	902.23	817.02	7.56	0.00	0.00	104
105	1,668	564.29	1.6447	906.90	825.06	4.86	0.00	0.00	105
106	727	597.83	1.5685	911.22	832.53	2.99	0.00	0.00	106
107	292	631.64	1.4984	915.19	839.46	1.76	0.00	0.00	107
108	108	665.45	1.4341	918.82	845.84	0.98	0.00	0.00	108
109	36	698.97	1.3755	922.14	851.69	0.52	0.00	0.00	109
110	11	731.87	1.3223	925.15	857.04	0.26	0.00	0.00	110

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

Lives are independent.

x	$\ddot{a}_{xx}$	$1000A_{xx}$	$1000(^2A_{xx})$	$\ddot{a}_{x:x+10}$	$1000A_{x:x+10}$	$1000(^2A_{x:x+10})$	x
0	16.1345	86.73	50.89	16.2844	78.24	34.71	0
5	16.6432	57.93	16.51	16.4093	71.17	19.17	5
10	16.4660	67.96	18.13	16.1541	85.62	22.70	10
15	16.2187	81.96	21.67	15.8187	104.60	28.49	15
20	15.9005	99.97	27.00	15.3934	128.67	37.00	20
21	15.8272	104.12	28.33	15.2962	134.18	39.11	21
22	15.7502	108.48	29.77	15.1945	139.94	41.39	22
23	15.6696	113.04	31.33	15.0883	145.95	43.83	23
24	15.5851	117.82	33.01	14.9774	152.22	46.46	24
25	15.4967	122.83	34.82	14.8617	158.77	49.28	25
26	15.4041	128.07	36.77	14.7411	165.60	52.31	26
27	15.3073	133.55	38.87	14.6154	172.71	55.56	27
28	15.2062	139.27	41.12	14.4845	180.12	59.03	28
29	15.1005	145.26	43.55	14.3484	187.83	62.75	29
30	14.9901	151.50	46.16	14.2068	195.84	66.72	30
31	14.8750	158.02	48.96	14.0598	204.16	70.97	31
32	14.7549	164.82	51.96	13.9071	212.80	75.50	32
33	14.6298	171.90	55.18	13.7488	221.76	80.34	33
34	14.4995	179.27	58.63	13.5848	231.05	85.48	34
35	14.3640	186.94	62.32	13.4150	240.66	90.96	35
36	14.2230	194.92	66.26	13.2393	250.60	96.78	36
37	14.0766	203.21	70.48	13.0579	260.88	102.96	37
38	13.9246	211.81	74.98	12.8705	271.48	109.52	38
39	13.7670	220.74	79.77	12.6774	282.41	116.46	39
40	13.6036	229.99	84.89	12.4784	293.68	123.80	40
41	13.4344	239.56	90.32	12.2737	305.26	131.56	41
42	13.2594	249.47	96.11	12.0633	317.17	139.75	42
43	13.0786	259.70	102.25	11.8474	329.39	148.38	43
44	12.8919	270.27	108.76	11.6260	341.92	157.46	44
45	12.6994	281.16	115.66	11.3994	354.75	166.99	45
46	12.5011	292.39	122.95	11.1677	367.87	177.00	46
47	12.2971	303.94	130.67	10.9310	381.26	187.48	47
48	12.0873	315.81	138.80	10.6898	394.92	198.44	48
49	11.8720	328.00	147.38	10.4441	408.82	209.88	49
50	11.6513	340.49	156.41	10.1944	422.96	221.81	50
51	11.4252	353.29	165.89	9.9409	437.31	234.22	51
52	11.1941	366.37	175.85	9.6840	451.85	247.10	52
53	10.9580	379.74	186.28	9.4240	466.57	260.46	53
54	10.7172	393.37	197.18	9.1614	481.43	274.27	54
55	10.4720	407.24	208.57	8.8966	496.42	288.54	55
56	10.2227	421.35	220.44	8.6301	511.50	303.24	56
57	9.9696	435.68	232.79	8.3623	526.66	318.35	57
58	9.7131	450.20	245.62	8.0938	541.86	333.85	58
59	9.4535	464.90	258.93	7.8249	557.08	349.73	59
60	9.1911	479.75	272.69	7.5563	572.28	365.94	60
61	8.9266	494.72	286.91	7.2885	587.44	382.46	61
62	8.6602	509.80	301.56	7.0221	602.53	399.26	62
63	8.3926	524.95	316.62	6.7574	617.50	416.30	63
64	8.1241	540.15	332.09	6.4952	632.34	433.53	64
65	7.8552	555.36	347.92	6.2360	647.02	450.93	65

Illustrative Life Table: Basic Functions and Single Benefit Premiums at $i = 0.06$

Lives are independent.

x	$\ddot{a}_{xx}$	$1000A_{xx}$	$1000(^2A_{xx})$	$\ddot{a}_{x:x+10}$	$1000A_{x:x+10}$	$1000(^2A_{x:x+10})$	x
66	7.5866	570.57	364.09	5.9802	661.50	468.44	66
67	7.3187	585.74	380.58	5.7283	675.76	486.02	67
68	7.0520	600.83	397.35	5.4809	689.76	503.62	68
69	6.7872	615.82	414.36	5.2385	703.48	521.21	69
70	6.5247	630.68	431.58	5.0014	716.90	538.72	70
71	6.2650	645.37	448.96	4.7701	730.00	556.11	71
72	6.0088	659.88	466.46	4.5450	742.74	573.34	72
73	5.7565	674.16	484.03	4.3263	755.11	590.36	73
74	5.5086	688.19	501.64	4.1146	767.10	607.12	74
75	5.2655	701.95	519.23	3.9099	778.69	623.59	75
76	5.0278	715.41	536.75	3.7125	789.86	639.71	76
77	4.7959	728.54	554.16	3.5227	800.60	655.46	77
78	4.5700	741.32	571.41	3.3406	810.91	670.79	78
79	4.3507	753.74	588.45	3.1663	820.78	685.67	79
80	4.1381	765.77	605.25	2.9998	830.20	700.08	80
81	3.9326	777.40	621.75	2.8412	839.18	713.99	81
82	3.7344	788.62	637.91	2.6905	847.71	727.37	82
83	3.5438	799.41	653.70	2.5476	855.80	740.21	83
84	3.3607	809.77	669.08	2.4125	863.44	752.49	84
85	3.1855	819.69	684.02	2.2851	870.66	764.20	85
86	3.0181	829.16	698.48	2.1652	877.44	775.34	86
87	2.8587	838.19	712.45	2.0527	883.81	785.89	87
88	2.7071	846.77	725.89	1.9475	889.77	795.86	88
89	2.5633	854.91	738.79	1.8493	895.33	805.25	89
90	2.4274	862.60	751.14	1.7579	900.50	814.05	90
91	2.2991	869.86	762.91	1.6731	905.30	822.29	91
92	2.1784	876.70	774.11	1.5947	909.73	829.96	92
93	2.0651	883.11	784.73	1.5225	913.82	837.07	93
94	1.9590	889.11	794.77	1.4563	917.57	843.64	94
95	1.8600	894.72	804.22	1.3957	921.00	849.67	95
96	1.7678	899.93	813.09	1.3407	924.11	855.20	96
97	1.6823	904.77	821.39	1.2908	926.93	860.21	97
98	1.6032	909.25	829.12	1.2460	929.47	864.75	98
99	1.5304	913.38	836.29	1.2060	931.73	868.81	99
100	1.4634	917.16	842.92	1.1706	933.74	872.43	100
101	1.4023	920.63	849.02	1.1395	935.50	875.61	101
102	1.3466	923.78	854.60	1.1124	937.03	878.39	102
103	1.2962	926.63	859.67	1.0892	938.35	880.78	103
104	1.2509	929.20	864.26	1.0695	939.46	882.81	104
105	1.2103	931.49	868.38	1.0531	940.39	884.50	105
106	1.1744	933.53	872.04	1.0397	941.15	885.89	106
107	1.1428	935.32	875.27	1.0289	941.76	887.00	107
108	1.1153	936.87	878.10	1.0205	942.24	887.87	108
109	1.0916	938.21	880.53	1.0141	942.60	888.54	109
110	1.0715	939.35	882.60	1.0093	942.87	889.03	110

Interest Functions at $i = 0.06$

m	$i^{(m)}$	$d^{(m)}$	$i/i^{(m)}$	$d/d^{(m)}$	$\alpha(m)$	$\beta(m)$
1	0.06000	0.05660	1.00000	1.00000	1.00000	0.00000
2	0.05913	0.05743	1.01478	0.98564	1.00021	0.25739
4	0.05870	0.05785	1.02223	0.97852	1.00027	0.38424
12	0.05841	0.05813	1.02721	0.97378	1.00028	0.46812
∞	0.05827	0.05827	1.02971	0.97142	1.00028	0.50985

Special Note: Unless specified, the force of interest is constant in each question .