****BEGINNING OF EXAMINATION****

**1.** For a special whole life insurance on ($x$), payable at the moment of death:

(i) $\mu_x(t) = 0.05$, $t > 0$

(ii) $\delta = 0.08$

(iii) The death benefit at time $t$ is $b_t = e^{0.06t}$, $t > 0$.

(iv) $Z$ is the present value random variable for this insurance at issue.

Calculate $\mathrm{Var}(Z)$.

(A) 0.038

(B) 0.041

(C) 0.043

(D) 0.045

(E) 0.048

**2.** For a group of individuals all age $x$, you are given:

(i) 25% are smokers (s); 75% are nonsmokers (ns).

(ii)

| $k$ | $q^{s}_{x+k}$ | $q^{ns}_{x+k}$ |
|---|---|---|
| 0 | 0.10 | 0.05 |
| 1 | 0.20 | 0.10 |
| 2 | 0.30 | 0.15 |

(iii) $i = 0.02$

Calculate $10{,}000 A^{1}_{x:\overline{2}|}$ for an individual chosen at random from this group.

(A) 1690

(B) 1710

(C) 1730

(D) 1750

(E) 1770

**3.** For a fully continuous whole life insurance of 1 on ($x$), you are given:

(i) The forces of mortality and interest are constant.

(ii) ${}^2\bar{A}_x = 0.20$

(iii) $\bar{P}\left(\bar{A}_x\right) = 0.03$

(iv) ${}_0L$ is the loss-at-issue random variable based on the benefit premium.

Calculate $\mathrm{Var}\left({}_0L\right)$.

(A) 0.20

(B) 0.21

(C) 0.22

(D) 0.23

(E) 0.24

**4.** For a population which contains equal numbers of males and females at birth:

(i) For males, $\mu^m(x) = 0.10, \quad x \geq 0$

(ii) For females, $\mu^f(x) = 0.08, \quad x \geq 0$

Calculate $q_{60}$ for this population.

(A) 0.076

(B) 0.081

(C) 0.086

(D) 0.091

(E) 0.096

**5.** You are simulating the future lifetimes of newborns in a population.

(i) For any given newborn, mortality follows De Moivre's law with maximum lifetime $\Omega$.

(ii) $\Omega$ has distribution function $F(\omega) = (\omega/80)^2$, $0 \le \omega \le 80$.

(iii) You are using the inverse transform method, with small random numbers corresponding to small values of $\Omega$ or short future lifetimes.

(iv) Your first random numbers from $[0,1]$ for simulating $\Omega$ and the future lifetime are 0.4 and 0.7 respectively.

Calculate your first simulated value of the future lifetime.

(A) 22

(B) 35

(C) 46

(D) 52

(E) 56

**6.** You are simulating a compound claims distribution:

(i) The number of claims, $N$, is binomial with $m$ = 3 and mean 1.8.

(ii) Claim amounts are uniformly distributed on $\{1, 2, 3, 4, 5\}$.

(iii) Claim amounts are independent, and are independent of the number of claims.

(iv) You simulate the number of claims, $N$, then the amounts of each of those claims, $X_1, X_2, \ldots, X_N$. Then you repeat another $N$, its claim amounts, and so on until you have performed the desired number of simulations.

(v) When the simulated number of claims is 0, you do not simulate any claim amounts.

(vi) All simulations use the inverse transform method, with low random numbers corresponding to few claims or small claim amounts.

(vii) Your random numbers from (0, 1) are 0.7, 0.1, 0.3, 0.1, 0.9, 0.5, 0.5, 0.7, 0.3, and 0.1.

Calculate the aggregate claim amount associated with your third simulated value of $N$.

(A) 3

(B) 5

(C) 7

(D) 9

(E) 11

**7.** Annual prescription drug costs are modeled by a two-parameter Pareto distribution with $\theta = 2000$ and $\alpha = 2$.

A prescription drug plan pays annual drug costs for an insured member subject to the following provisions:

(i) The insured pays 100% of costs up to the ordinary annual deductible of 250.

(ii) The insured then pays 25% of the costs between 250 and 2250.

(iii) The insured pays 100% of the costs above 2250 until the insured has paid 3600 in total.

(iv) The insured then pays 5% of the remaining costs.

Determine the expected annual plan payment.

(A) 1120

(B) 1140

(C) 1160

(D) 1180

(E) 1200

**8.** For a tyrannosaur with a taste for scientists:

(i) The number of scientists eaten has a binomial distribution with $q = 0.6$ and $m = 8$.

(ii) The number of calories of a scientist is uniformly distributed on (7000, 9000).

(iii) The numbers of calories of scientists eaten are independent, and are independent of the number of scientists eaten.

Calculate the probability that two or more scientists are eaten and exactly two of those eaten have at least 8000 calories each.

(A) 0.23

(B) 0.25

(C) 0.27

(D) 0.30

(E) 0.35

**9.** For a special fully continuous last survivor insurance of 1 on ($x$) and ($y$), you are given:

(i) $T(x)$ and $T(y)$ are independent.

(ii) $\mu_x(t) = 0.08, \quad t > 0$

(iii) $\mu_y(t) = 0.04, \quad t > 0$

(iv) $\delta = 0.06$

(v) $\pi$ is the annual benefit premium payable until the first of ($x$) and ($y$) dies.

Calculate $\pi$.

(A) 0.055

(B) 0.080

(C) 0.105

(D) 0.120

(E) 0.150

**10.** For a special fully discrete whole life insurance of 1000 on (42):

(i) The contract premium for the first 4 years is equal to the level benefit premium for a fully discrete whole life insurance of 1000 on (40).

(ii) The contract premium after the fourth year is equal to the level benefit premium for a fully discrete whole life insurance of 1000 on (42).

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) ${}_3L$ is the prospective loss random variable at time 3, based on the contract premium.

(vi) $K(42)$ is the curtate future lifetime of $(42)$.

Calculate $\mathrm{E}\left[{}_3L \middle| K(42) \geq 3\right]$.

(A) 27

(B) 31

(C) 44

(D) 48

(E) 52

**11.** Your company is competing to sell a life annuity-due with an actuarial present value of 500,000 to a 50-year old individual.

Based on your company's experience, typical 50-year old annuitants have a complete life expectancy of 25 years. However, this individual is not as healthy as your company's typical annuitant, and your medical experts estimate that his complete life expectancy is only 15 years.

You decide to price the benefit using the issue age that produces a complete life expectancy of 15 years. You also assume:

(i) For typical annuitants of all ages, mortality follows De Moivre's Law with the same limiting age, $\omega$.

(ii) $i = 0.06$

Calculate the annual benefit that your company can offer to this individual.

(A) 38,000

(B) 41,000

(C) 46,000

(D) 49,000

(E) 52,000

**12.** For a double decrement table, you are given:

(i) $q'^{(1)}_x = 0.2$

(ii) $q'^{(2)}_x = 0.3$

(iii) Each decrement is uniformly distributed over each year of age in the double decrement table.

Calculate ${}_{0.3}q^{(1)}_{x+0.1}$.

(A) 0.020

(B) 0.031

(C) 0.042

(D) 0.053

(E) 0.064

**13-14.** *Use the following information for questions* 13 *and* 14.

For a Markov model for an insured population:

(i) Annual transition probabilities between health states of individuals are as follows:

| | Healthy | Sick | Terminated |
|---|---|---|---|
| Healthy | 0.7 | 0.1 | 0.2 |
| Sick | 0.3 | 0.6 | 0.1 |
| Terminated | 0.0 | 0.0 | 1.0 |

(ii) The mean annual healthcare cost each year for each health state is:

| | Mean |
|---|---|
| Healthy | 500 |
| Sick | 3000 |
| Terminated | 0 |

(iii) Transitions occur at the end of the year.

(iv) $i = 0$

**13.** Calculate the expected future healthcare costs (including the current year) for an insured individual whose current state is healthy.

Recall:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} = \begin{pmatrix} a_{22}/d & -a_{12}/d \\ -a_{21}/d & a_{11}/d \end{pmatrix} \quad \text{where } d = a_{11}a_{22} - a_{12}a_{21}$$

(A) 5100

(B) 5600

(C) 6100

(D) 6600

(E) 7100

**13-14.** *(Repeated for convenience) Use the following information for questions* 13 *and* 14.

For a Markov model for an insured population:

(i) Annual transition probabilities between health states of individuals are as follows:

| | Healthy | Sick | Terminated |
|---|---|---|---|
| Healthy | 0.7 | 0.1 | 0.2 |
| Sick | 0.3 | 0.6 | 0.1 |
| Terminated | 0.0 | 0.0 | 1.0 |

(ii) The mean annual healthcare cost each year for each health state is:

| | Mean |
|---|---|
| Healthy | 500 |
| Sick | 3000 |
| Terminated | 0 |

(iii) Transitions occur at the end of the year.

(iv) $i = 0$

**14.** A contract premium of 800 is paid each year by an insured not in the terminated state.

Calculate the expected value of contract premiums less healthcare costs over the first 3 years for a new healthy insured.

(A) –390

(B) –200

(C) –20

(D) 160

(E) 340

**15.** Two types of insurance claims are made to an insurance company. For each type, the number of claims follows a Poisson distribution and the amount of each claim is uniformly distributed as follows:

| Type of Claim | Poisson Parameter $\lambda$ for Number of Claims | Range of Each Claim Amount |
|---|---|---|
| I | 12 | (0, 1) |
| II | 4 | (0, 5) |

The numbers of claims of the two types are independent and the claim amounts and claim numbers are independent.

Calculate the normal approximation to the probability that the total of claim amounts exceeds 18.

(A) 0.37

(B) 0.39

(C) 0.41

(D) 0.43

(E) 0.45

**16.** For a water reservoir:

(i) The present level is 4999 units.

(ii) 1000 units are used uniformly daily.

(iii) The only source of replenishment is rainfall.

(iv) The number of rainfalls follows a Poisson process with $\lambda = 0.2$ per day.

(v) The distribution of the amount of a rainfall is as follows:

| Amount | Probability |
|---|---|
| 8000 | 0.2 |
| 5000 | 0.8 |

(vi) The numbers and amounts of rainfalls are independent.

Calculate the probability that the reservoir will be empty sometime within the next 10 days.

(A) 0.27

(B) 0.37

(C) 0.39

(D) 0.48

(E) 0.50

**17.** The number of annual losses has a Poisson distribution with a mean of 5. The size of each loss has a two-parameter Pareto distribution with $\theta = 10$ and $\alpha = 2.5$. An insurance for the losses has an ordinary deductible of 5 per loss.

Calculate the expected value of the aggregate annual payments for this insurance.

(A) 8

(B) 13

(C) 18

(D) 23

(E) 28

**18.** Losses in 2003 follow a two-parameter Pareto distribution with $\alpha = 2$ and $\theta = 5$. Losses in 2004 are uniformly 20% higher than in 2003. An insurance covers each loss subject to an ordinary deductible of 10.

Calculate the Loss Elimination Ratio in 2004.

(A) 5/9

(B) 5/8

(C) 2/3

(D) 3/4

(E) 4/5

**19.** A company insures a town for liability during its annual parade. The company will charge an annual premium equal to expected claims plus a relative security load of 1/3.

You are given:

(i) Annual claims are 3, 5, or 7 and are independent.

(ii) $p(3) = 0.75$

(iii) $p(5) = 0.15$

(iv) Premiums are payable at the beginning of the year.

(v) $i = 0$

(vi) Initial surplus = 3

Calculate the probability of ruin within the first two years.

(A) 0.01

(B) 0.02

(C) 0.05

(D) 0.08

(E) 0.10

**20.** The mortality of $(x)$ and $(y)$ follows a common shock model with components $T^*(x)$, $T^*(y)$ and $Z$.

(i) $T^*(x)$, $T^*(y)$ and $Z$ are independent and have exponential distributions with respective forces $\mu_1$, $\mu_2$ and $\lambda$.

(ii) The probability that $(x)$ survives 1 year is 0.96.

(iii) The probability that $(y)$ survives 1 year is 0.97.

(iv) $\lambda = 0.01$

Calculate the probability that both $(x)$ and $(y)$ survive 5 years.

(A) 0.65

(B) 0.67

(C) 0.70

(D) 0.72

(E) 0.74

**21.** For a fully discrete whole life insurance of 100,000 on each of 10,000 lives age 60, you are given:

(i) The future lifetimes are independent.

(ii) Mortality follows the Illustrative Life Table.

(iii) $i = 0.06$.

(iv) $\pi$ is the premium for each insurance of 100,000.

Using the normal approximation, calculate $\pi$, such that the probability of a positive total loss is 1%.

(A) 3340

(B) 3360

(C) 3380

(D) 3390

(E) 3400

**22.** For a special fully discrete 3-year endowment insurance on (75), you are given:

(i) The maturity value is 1000.

(ii) The death benefit is 1000 plus the benefit reserve at the end of the year of death.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.05$

Calculate the level benefit premium for this insurance.

(A) 321

(B) 339

(C) 356

(D) 364

(E) 373

**23.** For a special fully discrete 3-year term insurance on (55), whose mortality follows a double decrement model:

(i) Decrement 1 is accidental death; decrement 2 is all other causes of death.

(ii)

| $x$ | $q_x^{(1)}$ | $q_x^{(2)}$ |
|---|---|---|
| 55 | 0.002 | 0.020 |
| 56 | 0.005 | 0.040 |
| 57 | 0.008 | 0.060 |

(iii) $i = 0.06$

(iv) The death benefit is 2000 for accidental deaths and 1000 for deaths from all other causes.

(v) The level annual contract premium is 50.

(vi) ${}_1L$ is the prospective loss random variable at time 1, based on the contract premium.

(vii) $K(55)$ is the curtate future lifetime of (55).

Calculate $\mathrm{E}\left[{}_1L \middle| K(55) \geq 1\right]$.

(A) 5

(B) 9

(C) 13

(D) 17

(E) 20

**24.** The future lifetime of (0) follows a two-parameter Pareto distribution with $\theta = 50$ and $\alpha = 3$.

Calculate $\mathring{e}_{20}$.

(A) 5

(B) 15

(C) 25

(D) 35

(E) 45

**25.** For a modeled insurance company:

(i) P is the continuous time, infinite horizon survival probability.

(ii) Q is the discrete time, infinite horizon survival probability.

(iii) R is the continuous time, finite horizon survival probability.

(iv) S is the discrete time, finite horizon survival probability.

Which of the following is always true?

(A) $S \le Q$

(B) $S \le P$

(C) $Q \le P$

(D) $R \le P$

(E) $2P \le Q + R$

**26.** Customers arrive at a store at a Poisson rate that increases linearly from 6 per hour at 1:00 p.m. to 9 per hour at 2:00 p.m.

Calculate the probability that exactly 2 customers arrive between 1:00 p.m. and 2:00 p.m.

(A) 0.016

(B) 0.018

(C) 0.020

(D) 0.022

(E) 0.024

**27.** For independent stochastic processes $X(t)$ and $Y(t)$:

(i) $X(t)$ is a Brownian motion process with $X(0) = 0$, drift coefficient $\mu = 0$, and variance parameter $\sigma^2 = 0.5$.

(ii) $Y(t)$ is a Brownian motion process with $Y(0) = 2$, drift coefficient $\mu = 0$, and variance parameter $\sigma^2 = 1$.

Calculate the probability that $X(t) \geq Y(t)$ for some $t$ in $[0, 5]$.

(A) 0.23

(B) 0.29

(C) 0.35

(D) 0.41

(E) 0.47

**28.** For (80) and (84), whose future lifetimes are independent:

| $x$ | $p_x$ |
|---|---|
| 80 | 0.50 |
| 81 | 0.40 |
| 82 | 0.60 |
| 83 | 0.25 |
| 84 | 0.20 |
| 85 | 0.15 |
| 86 | 0.10 |

Calculate the change in the value ${}_{2|}q_{\overline{80:84}}$ if $p_{82}$ is decreased from 0.60 to 0.30.

(A) 0.03

(B) 0.06

(C) 0.10

(D) 0.16

(E) 0.19

**29.** At interest rate $i$:

(i) $\ddot{a}_x = 5.6$

(ii) The actuarial present value of a 2-year certain and life annuity-due of 1 on ($x$) is $\ddot{a}_{\overline{\overline{x:\overline{2|}}}} = 5.6459$.

(iii) $e_x = 8.83$

(iv) $e_{x+1} = 8.29$

Calculate $i$.

(A) 0.077

(B) 0.079

(C) 0.081

(D) 0.083

(E) 0.084

**30.** For a deferred whole life annuity-due on (25) with annual payment of 1 commencing at age 60, you are given:

(i) Level benefit premiums are payable at the beginning of each year during the deferral period.

(ii) During the deferral period, a death benefit equal to the benefit reserve is payable at the end of the year of death.

Which of the following is a correct expression for the benefit reserve at the end of the 20th year?

(A) $\left(\ddot{a}_{60} / \ddot{s}_{\overline{35}|}\right)\ddot{s}_{\overline{20}|}$

(B) $\left(\ddot{a}_{60} / \ddot{s}_{\overline{20}|}\right)\ddot{s}_{\overline{35}|}$

(C) $\left(\ddot{s}_{\overline{20}|} / \ddot{a}_{60}\right)\ddot{s}_{\overline{35}|}$

(D) $\left(\ddot{s}_{\overline{35}|} / \ddot{a}_{60}\right)\ddot{s}_{\overline{20}|}$

(E) $\left(\ddot{a}_{60} / \ddot{s}_{\overline{35}|}\right)$

**31.** You are given:

(i) The future lifetimes of (50) and (50) are independent.

(ii) Mortality follows the Illustrative Life Table.

(iii) Deaths are uniformly distributed over each year of age.

Calculate the force of failure at duration 10.5 for the last survivor status of (50) and (50).

(A) 0.001

(B) 0.002

(C) 0.003

(D) 0.004

(E) 0.005

**32.** Bob is a carnival operator of a game in which a player receives a prize worth $W = 2^N$ if the player has $N$ successes, $N = 0, 1, 2, 3, \ldots$ Bob models the probability of success for a player as follows:

(i) $N$ has a Poisson distribution with mean $\Lambda$.

(ii) $\Lambda$ has a uniform distribution on the interval (0, 4).

Calculate $\mathrm{E}[W]$.

(A) 5

(B) 7

(C) 9

(D) 11

(E) 13

**33.** You are simulating the gain/loss from insurance where:

(i) Claim occurrences follow a Poisson process with $\lambda = 2/3$ per year.

(ii) Each claim amount is 1, 2 or 3 with $p(1) = 0.25$, $p(2) = 0.25$, and $p(3) = 0.50$.

(iii) Claim occurrences and amounts are independent.

(iv) The annual premium equals expected annual claims plus 1.8 times the standard deviation of annual claims.

(v) $i = 0$

You use 0.75, 0.60, 0.40, and 0.20 from the unit interval to simulate time between claims, where small numbers correspond to longer times.

You use 0.30, 0.60, 0.20, and 0.70 from the unit interval to simulate claim size, where small numbers correspond to smaller claims.

Calculate the gain or loss during the first 2 years from this simulation.

(A) loss of 5

(B) loss of 4

(C) 0

(D) gain of 4

(E) gain of 5

**34.** Annual dental claims are modeled as a compound Poisson process where the number of claims has mean 2 and the loss amounts have a two-parameter Pareto distribution with $\theta = 500$ and $\alpha = 2$.

An insurance pays 80% of the first 750 of annual losses and 100% of annual losses in excess of 750.

You simulate the number of claims and loss amounts using the inverse transform method with small random numbers corresponding to small numbers of claims or small loss amounts.

The random number to simulate the number of claims is 0.8. The random numbers to simulate loss amounts are 0.60, 0.25, 0.70, 0.10 and 0.80.

Calculate the total simulated insurance claims for one year.

(A) 294

(B) 625

(C) 631

(D) 646

(E) 658

**35.** For a special whole life insurance:

(i) The benefit for accidental death is 50,000 in all years.

(ii) The benefit for non-accidental death during the first 2 years is return of the single benefit premium without interest.

(iii) The benefit for non-accidental death after the first 2 years is 50,000.

(iv) Benefits are payable at the moment of death.

(v) Force of mortality for accidental death: $\mu_x^{(1)}(t) = 0.01, \quad t \geq 0$

(vi) Force of mortality for non-accidental death: $\mu_x^{(2)}(t) = 2.29, \quad t \geq 0$

(vii) $\delta = 0.10$

Calculate the single benefit premium for this insurance.

(A) 1,000

(B) 4,000

(C) 7,000

(D) 11,000

(E) 15,000

**36.** A special whole life insurance on ($x$) pays 10 times salary if the cause of death is an accident and 500,000 for all other causes of death.

You are given:

(i) $\mu_x^{(\tau)}(t) = 0.01, \quad t \geq 0$

(ii) $\mu_x^{(\text{accident})}(t) = 0.001, \quad t \geq 0$

(iii) Benefits are payable at the moment of death.

(iv) $\delta = 0.05$

(v) Salary of ($x$) at time $t$ is $50{,}000e^{0.04t}, \quad t \geq 0$.

Calculate the actuarial present value of the benefits at issue.

(A) 78,000

(B) 83,000

(C) 92,000

(D) 100,000

(E) 108,000

**37.** $Z$ is the present value random variable for a 15-year pure endowment of 1 on ($x$):

(i) The force of mortality is constant over the 15-year period.

(ii) $v = 0.9$

(iii) $\mathrm{Var}(Z) = 0.065\,\mathrm{E}[Z]$

Calculate $q_x$.

(A) 0.020

(B) 0.025

(C) 0.030

(D) 0.035

(E) 0.040

**38.** You are given:

(i) ${}_kV^A$ is the benefit reserve at the end of year $k$ for type A insurance, which is a fully discrete 10-payment whole life insurance of 1000 on $(x)$.

(ii) ${}_kV^B$ is the benefit reserve at the end of year $k$ for type B insurance, which is a fully discrete whole life insurance of 1000 on $(x)$.

(iii) $q_{x+10} = 0.004$

(iv) The annual benefit premium for type B is 8.36.

(v) ${}_{10}V^A - {}_{10}V^B = 101.35$

(vi) $i = 0.06$

Calculate ${}_{11}V^A - {}_{11}V^B$.

(A) 91

(B) 93

(C) 95

(D) 97

(E) 99

**39.** For a special fully discrete 3-year term insurance on $(x)$:

(i) $b_{k+1} = \begin{cases} 0 & \text{for } k = 0 \\ 1,000(11-k) & \text{for } k = 1, 2 \end{cases}$

(ii)

| $k$ | $q_{x+k}$ |
|---|---|
| 0 | 0.200 |
| 1 | 0.100 |
| 2 | 0.097 |

(iii) $i = 0.06$

Calculate the level annual benefit premium for this insurance.

(A) 518

(B) 549

(C) 638

(D) 732

(E) 799

**40.** For a special 3-year temporary life annuity-due on ($x$), you are given:

(i)

| $t$ | Annuity Payment | $p_{x+t}$ |
|---|---|---|
| 0 | 15 | 0.95 |
| 1 | 20 | 0.90 |
| 2 | 25 | 0.85 |

(ii) $i = 0.06$

Calculate the variance of the present value random variable for this annuity.

(A) 91

(B) 102

(C) 114

(D) 127

(E) 139

****END OF EXAMINATION****

# COURSE 3, Fall 2004
# Final Answer Key

| *Question #* | *Answer* | | *Question #* | *Answer* |
|---|---|---|---|---|
| **1** | **D** | | **21** | **C** |
| **2** | **C** | | **22** | **C** |
| **3** | **A** | | **23** | **D** |
| **4** | **B** | | **24** | **D** |
| **5** | **B** | | **25** | **E** |
| **6** | **C** | | **26** | **A** |
| **7** | **C** | | **27** | **E** |
| **8** | **D** | | **28** | **B** |
| **9** | **A** | | **29** | **B** |
| **10** | **B** | | **30** | **A** |
| **11** | **E** | | **31** | **B** |
| **12** | **D** | | **32** | **E** |
| **13** | **B** | | **33** | **A-or-E** |
| **14** | **D** | | **34** | **C** |
| **15** | **A** | | **35** | **D** |
| **16** | **D** | | **36** | **D** |
| **17** | **C** | | **37** | **B** |
| **18** | **B** | | **38** | **E** |
| **19** | **A** | | **39** | **A** |
| **20** | **E** | | **40** | **C** |