May 2001
Course 3

****BEGINNING OF EXAMINATION****

1. For a given life age 30, it is estimated that an impact of a medical breakthrough will be an increase of 4 years in $\overset{\circ}{e}_{30}$, the complete expectation of life.

Prior to the medical breakthrough, s(x) followed de Moivre's law with $\omega = 100$ as the limiting age.

Assuming de Moivre's law still applies after the medical breakthrough, calculate the new limiting age.

(A) 104

(B) 105

(C) 106

(D) 107

(E) 108

2. On January 1, 2002, Pat, age 40, purchases a 5-payment, 10-year term insurance of 100,000:

(i) Death benefits are payable at the moment of death.

(ii) Contract premiums of 4000 are payable annually at the beginning of each year for 5 years.

(iii) $i = 0.05$

(iv) L is the loss random variable at time of issue.

Calculate the value of L if Pat dies on June 30, 2004.

(A) 77,100

(B) 80,700

(C) 82,700

(D) 85,900

(E) 88,000

3. Glen is practicing his simulation skills.

He generates 1000 values of the random variable X as follows:

(i) He generates the observed value λ from the gamma distribution with $\alpha = 2$ and $\theta = 1$ (hence with mean 2 and variance 2).

(ii) He then generates x from the Poisson distribution with mean λ.

(iii) He repeats the process 999 more times: first generating a value λ, then generating x from the Poisson distribution with mean λ.

(iv) The repetitions are mutually independent.

Calculate the expected number of times that his simulated value of X is 3.

(A) 75

(B) 100

(C) 125

(D) 150

(E) 175

4. Lucky Tom finds coins on his way to work at a Poisson rate of 0.5 coins per minute. The denominations are randomly distributed:

(i) 60% of the coins are worth 1;

(ii) 20% of the coins are worth 5;

(iii) 20% of the coins are worth 10.

Calculate the variance of the value of the coins Tom finds during his one-hour walk to work.

(A) 379

(B) 487

(C) 566

(D) 670

(E) 768

5. For a fully discrete 20-payment whole life insurance of 1000 on (x), you are given:

(i) $i = 0.06$

(ii) $q_{x+19} = 0.01254$

(iii) The level annual benefit premium is 13.72.

(iv) The benefit reserve at the end of year 19 is 342.03.

Calculate $1000\ P_{x+20}$, the level annual benefit premium for a fully discrete whole life insurance of 1000 on $(x+20)$.

(A) 27

(B) 29

(C) 31

(D) 33

(E) 35

6. For a multiple decrement model on (60):

(i) $\mu_{60}^{(1)}(t), \quad t \geq 0,$ follows the Illustrative Life Table.

(ii) $\mu_{60}^{(\tau)}(t) = 2\mu_{60}^{(1)}(t), \quad t \geq 0$

Calculate ${}_{10|}q_{60}^{(\tau)}$, the probability that decrement occurs during the 11th year.

(A) 0.03

(B) 0.04

(C) 0.05

(D) 0.06

(E) 0.07

7. A coach can give two types of training, “ light” or “heavy,” to his sports team before a game. If the team wins the prior game, the next training is equally likely to be light or heavy. But, if the team loses the prior game, the next training is always heavy.

The probability that the team will win the game is 0.4 after light training and 0.8 after heavy training.

Calculate the long run proportion of time that the coach will give heavy training to the team.

(A) 0.61

(B) 0.64

(C) 0.67

(D) 0.70

(E) 0.73

8. For a simulation of the movement of a stock's price:

(i) The price follows geometric Brownian motion, with drift coefficient $\mu = 0.01$ and variance parameter $\sigma^2 = 0.0004$.

(ii) The simulation projects the stock price in steps of time 1.

(iii) Simulated price movements are determined using the inverse transform method.

(iv) The price at $t = 0$ is 100.

(v) The random numbers, from the uniform distribution on $[0,1]$, for the first 2 steps are 0.1587 and 0.9332, respectively.

(vi) F is the price at $t = 1$; G is the price at $t = 2$.

Calculate $G - F$.

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5

9. (x) and (y) are two lives with identical expected mortality.

You are given:

$P_x = P_y = 0.1$

$P_{\overline{xy}} = 0.06$, where $P_{\overline{xy}}$ is the annual benefit premium for a fully discrete insurance of 1 on $(\overline{xy})$.

$d = 0.06$

Calculate the premium P_{xy}, the annual benefit premium for a fully discrete insurance of 1 on (xy).

(A) 0.14

(B) 0.16

(C) 0.18

(D) 0.20

(E) 0.22

10. For students entering a college, you are given the following from a multiple decrement model:

(i) 1000 students enter the college at $t = 0$.

(ii) Students leave the college for failure (1) or all other reasons (2).

(iii) $\mathbf{m}^{(1)}(t) = \mathbf{m} \qquad 0 \le t \le 4$

$\mathbf{m}^{(2)}(t) = 0.04 \qquad 0 \le t < 4$

(iv) 48 students are expected to leave the college during their first year due to all causes.

Calculate the expected number of students who will leave because of failure during their fourth year.

(A) 8

(B) 10

(C) 24

(D) 34

(E) 41

11. You are using the inverse transform method to simulate Z, the present value random variable for a special two-year term insurance on (70). You are given:

(i) (70) is subject to only two causes of death, with

k	${}_{k|}q_{70}^{(1)}$	${}_{k|}q_{70}^{(2)}$
0	0.10	0.10
1	0.10	0.50

(ii) Death benefits, payable at the end of the year of death, are:

During year	Benefit for Cause 1	Benefit for Cause 2
1	1000	1100
2	1100	1200

(iii) $i = 0.06$

(iv) For this trial your random number, from the uniform distribution on $[0,1]$, is 0.35.

(v) High random numbers correspond to high values of Z.

Calculate the simulated value of Z for this trial.

(A) 943

(B) 979

(C) 1000

(D) 1038

(E) 1068

12. You are simulating one year of death and surrender benefits for 3 policies. Mortality follows the Illustrative Life Table. The surrender rate, occurring at the end of the year, is 15% for all ages. The simulation procedure is the inverse transform algorithm, with low random numbers corresponding to the decrement occurring. You perform the following steps for each policy:

(1) Simulate if the policy is terminated by death. If not, go to Step 2; if yes, continue with the next policy.

(2) Simulate if the policy is terminated by surrender.

The following values are successively generated from the uniform distribution on $[0,1]$:

0.3, 0.5, 0.1, 0.4, 0.8, 0.2, 0.3, 0.4, 0.6, 0.7,….

You are given:

Policy #	Age	Death Benefit	Surrender Benefit
1	100	10	10
2	91	25	20
3	96	20	15

Calculate the total benefits generated by the simulation.

(A) 30

(B) 35

(C) 40

(D) 45

(E) 50

13. Mr. Ucci has only 3 hairs left on his head and he won't be growing any more.

(i) The future mortality of each hair follows

$${}_{k|}q_x = 0.1(k+1), \quad k = 0, 1, 2, 3 \text{ and } x \text{ is Mr. Ucci's age}$$

(ii) Hair loss follows the hyperbolic assumption at fractional ages.

(iii) The future lifetimes of the 3 hairs are independent.

Calculate the probability that Mr. Ucci is bald (has no hair left) at age $x + 2.5$.

(A) 0.090

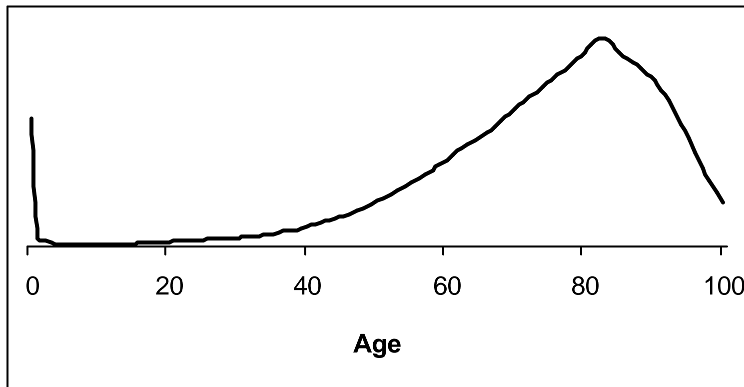
(B) 0.097

(C) 0.104

(D) 0.111

(E) 0.118

14. The following graph is related to current human mortality:



Which of the following functions of age does the graph most likely show?

(A) $\boldsymbol{m}(x)$

(B) $l_x \boldsymbol{m}(x)$

(C) $l_x p_x$

(D) l_x

(E) $l_x^{\,2}$

15. An actuary for an automobile insurance company determines that the distribution of the annual number of claims for an insured chosen at random is modeled by the negative binomial distribution with mean 0.2 and variance 0.4.

The number of claims for each individual insured has a Poisson distribution and the means of these Poisson distributions are gamma distributed over the population of insureds.

Calculate the variance of this gamma distribution.

(A) 0.20

(B) 0.25

(C) 0.30

(D) 0.35

(E) 0.40

16. A dam is proposed for a river which is currently used for salmon breeding. You have modeled:

(i) For each hour the dam is opened the number of salmon that will pass through and reach the breeding grounds has a distribution with mean 100 and variance 900.

(ii) The number of eggs released by each salmon has a distribution with mean of 5 and variance of 5.

(iii) The number of salmon going through the dam each hour it is open and the numbers of eggs released by the salmon are independent.

Using the normal approximation for the aggregate number of eggs released, determine the least number of whole hours the dam should be left open so the probability that 10,000 eggs will be released is greater than 95%.

(A) 20

(B) 23

(C) 26

(D) 29

(E) 32

17. For a special 3-year term insurance on (x), you are given:

(i) Z is the present-value random variable for the death benefits.

(ii) $q_{x+k} = 0.02(k+1) \qquad k = 0, 1, 2$

(iii) The following death benefits, payable at the end of the year of death:

k	b_{k+1}
0	300,000
1	350,000
2	400,000

(iv) $i = 0.06$

Calculate $\mathrm{E}(Z)$.

(A) 36,800

(B) 39,100

(C) 41,400

(D) 43,700

(E) 46,000

18. For a special fully discrete 20-year endowment insurance on (55):

(i) Death benefits in year k are given by $b_k = (21 - k)$, $k = 1, 2, \ldots, 20$.

(ii) The maturity benefit is 1.

(iii) Annual benefit premiums are level.

(iv) ${}_kV$ denotes the benefit reserve at the end of year k, $k = 1, 2, \ldots, 20$.

(v) ${}_{10}V = 5.0$

(vi) ${}_{19}V = 0.6$

(vii) $q_{65} = 0.10$

(viii) $i = 0.08$

Calculate ${}_{11}V$.

(A) 4.5

(B) 4.6

(C) 4.8

(D) 5.1

(E) 5.3

19. For a stop-loss insurance on a three person group:

(i) Loss amounts are independent.

(ii) The distribution of loss amount for each person is:

Loss Amount	Probability
0	0.4
1	0.3
2	0.2
3	0.1

(iii) The stop-loss insurance has a deductible of 1 for the group.

Calculate the net stop-loss premium.

(A) 2.00

(B) 2.03

(C) 2.06

(D) 2.09

(E) 2.12

20. An insurer's claims follow a compound Poisson claims process with two claims expected per period. Claim amounts can be only 1, 2, or 3 and these are equal in probability.

Calculate the continuous premium rate that should be charged each period so that the adjustment coefficient will be 0.5.

(A) 4.8

(B) 5.9

(C) 7.8

(D) 8.9

(E) 11.8

21-22. *Use the following information for questions* 21 *and* 22.

The Simple Insurance Company starts at time $t = 0$ with a surplus of $S = 3$. At the beginning of every year, it collects a premium of $P = 2$. Every year, it pays a random claim amount:

Claim Amount	Probability of Claim Amount
0	0.15
1	0.25
2	0.50
4	0.10

Claim amounts are mutually independent.

If, at the end of the year, Simple's surplus is more than 3, it pays a dividend equal to the amount of surplus in excess of 3. If Simple is unable to pay its claims, or if its surplus drops to 0, it goes out of business. Simple has no administrative expenses and its interest income is 0.

21. Determine the probability that Simple will ultimately go out of business.

(A) 0.00

(B) 0.01

(C) 0.44

(D) 0.56

(E) 1.00

21-22. *(Repeated for convenience) Use the following information for questions* 21 *and* 22.

The Simple Insurance Company starts at time $t = 0$ with a surplus of $S = 3$. At the beginning of every year, it collects a premium of $P = 2$. Every year, it pays a random claim amount:

Claim Amount	Probability of Claim Amount
0	0.15
1	0.25
2	0.50
4	0.10

Claim amounts are mutually independent.

If, at the end of the year, Simple's surplus is more than 3, it pays a dividend equal to the amount of surplus in excess of 3. If Simple is unable to pay its claims, or if its surplus drops to 0, it goes out of business. Simple has no administrative expenses and its interest income is 0.

22. Calculate the expected dividend at the end of the third year.

(A) 0.115

(B) 0.350

(C) 0.414

(D) 0.458

(E) 0.550

23. A continuous two-life annuity pays:

100 while both (30) and (40) are alive;
70 while (30) is alive but (40) is dead; and
50 while (40) is alive but (30) is dead.

The actuarial present value of this annuity is 1180. Continuous single life annuities paying 100 per year are available for (30) and (40) with actuarial present values of 1200 and 1000, respectively.

Calculate the actuarial present value of a two-life continuous annuity that pays 100 while at least one of them is alive.

(A) 1400

(B) 1500

(C) 1600

(D) 1700

(E) 1800

24. For a disability insurance claim:

(i) The claimant will receive payments at the rate of 20,000 per year, payable continuously as long as she remains disabled.

(ii) The length of the payment period in years is a random variable with the gamma distribution with parameters $\boldsymbol{a} = 2$ and $\boldsymbol{q} = 1$.

(iii) Payments begin immediately.

(iv) $\boldsymbol{d} = 0.05$

Calculate the actuarial present value of the disability payments at the time of disability.

(A) 36,400

(B) 37,200

(C) 38,100

(D) 39,200

(E) 40,000

25. For a discrete probability distribution, you are given the recursion relation

$$p(k) = \frac{2}{k} * p(k-1), \quad k = 1, 2, \ldots .$$

Determine $p(4)$.

(A) 0.07

(B) 0.08

(C) 0.09

(D) 0.10

(E) 0.11

26. A company insures a fleet of vehicles. Aggregate losses have a compound Poisson distribution. The expected number of losses is 20. Loss amounts, regardless of vehicle type, have exponential distribution with $\theta = 200$.

In order to reduce the cost of the insurance, two modifications are to be made:

(i) a certain type of vehicle will not be insured. It is estimated that this will reduce loss frequency by 20%.

(ii) a deductible of 100 per loss will be imposed.

Calculate the expected aggregate amount paid by the insurer after the modifications.

(A) 1600

(B) 1940

(C) 2520

(D) 3200

(E) 3880

27. An actuary is modeling the mortality of a group of 1000 people, each age 95, for the next three years.

The actuary starts by calculating the expected number of survivors at each integral age by

$$l_{95+k} = 1000\,{}_k p_{95}, \quad k = 1, 2, 3$$

The actuary subsequently calculates the expected number of survivors at the middle of each year using the assumption that deaths are uniformly distributed over each year of age.

This is the result of the actuary's model:

Age	Survivors
95	1000
95.5	800
96	600
96.5	480
97	--
97.5	288
98	--

The actuary decides to change his assumption for mortality at fractional ages to the constant force assumption. He retains his original assumption for each ${}_k p_{95}$.

Calculate the revised expected number of survivors at age 97.5.

(A) 270

(B) 273

(C) 276

(D) 279

(E) 282

28. For a population of individuals, you are given:

(i) Each individual has a constant force of mortality.

(ii) The forces of mortality are uniformly distributed over the interval (0,2).

Calculate the probability that an individual drawn at random from this population dies within one year.

(A) 0.37

(B) 0.43

(C) 0.50

(D) 0.57

(E) 0.63

29-30. *Use the following information for questions* 29 *and* 30.

You are the producer of a television quiz show that gives cash prizes. The number of prizes, N, and prize amounts, X, have the following distributions:

n	$\Pr(N = n)$
1	0.8
2	0.2

x	$\Pr(X = x)$
0	0.2
100	0.7
1000	0.1

29. Your budget for prizes equals the expected prizes plus the standard deviation of prizes.

Calculate your budget.

(A) 306

(B) 316

(C) 416

(D) 510

(E) 518

29-30. *(Repeated for convenience) Use the following information for questions* 29 *and* 30.

You are the producer of a television quiz show that gives cash prizes. The number of prizes, N, and prize amounts, X, have the following distributions:

n	$\Pr(N=n)$
1	0.8
2	0.2

x	$\Pr(X=x)$
0	0.2
100	0.7
1000	0.1

30. You buy stop-loss insurance for prizes with a deductible of 200. The cost of insurance includes a 175% relative security load.

Calculate the cost of the insurance.

(A) 204

(B) 227

(C) 245

(D) 273

(E) 357

31. For a special fully discrete 3-year term insurance on (x):

(i) Level benefit premiums are paid at the beginning of each year.

(ii)

k	b_{k+1}	q_{x+k}
0	200,000	0.03
1	150,000	0.06
2	100,000	0.09

(iii) $i = 0.06$

Calculate the initial benefit reserve for year 2.

(A) 6,500

(B) 7,500

(C) 8,100

(D) 9,400

(E) 10,300

32. For a special fully continuous whole life insurance on (x):

(i) The level premium is determined using the equivalence principle.

(ii) Death benefits are given by $b_t = (1+i)^t$ where i is the interest rate.

(iii) L is the loss random variable at $t = 0$ for the insurance.

(iv) T is the future lifetime random variable of (x).

Which of the following expressions is equal to L?

(A) $\dfrac{\left(\boldsymbol{n}^T - \bar{A}_x\right)}{\left(1 - \bar{A}_x\right)}$

(B) $\left(\boldsymbol{n}^T - \bar{A}_x\right)\left(1 + \bar{A}_x\right)$

(C) $\dfrac{\left(\boldsymbol{n}^T - \bar{A}_x\right)}{\left(1 + \bar{A}_x\right)}$

(D) $\left(\boldsymbol{n}^T - \bar{A}_x\right)\left(1 - \bar{A}_x\right)$

(E) $\dfrac{\left(v^T + \bar{A}_x\right)}{\left(1 + \bar{A}_x\right)}$

33. For a 4-year college, you are given the following probabilities for dropout from all causes:

$$q_0 = 0.15$$
$$q_1 = 0.10$$
$$q_2 = 0.05$$
$$q_3 = 0.01$$

Dropouts are uniformly distributed over each year.

Compute the temporary 1.5-year complete expected college lifetime of a student entering the second year, $\mathring{e}_{1:\overline{1.5}|}$.

(A) 1.25

(B) 1.30

(C) 1.35

(D) 1.40

(E) 1.45

34. Lee, age 63, considers the purchase of a single premium whole life insurance of 10,000 with death benefit payable at the end of the year of death.

The company calculates benefit premiums using:

(i) mortality based on the Illustrative Life Table,

(ii) $i = 0.05$

The company calculates contract premiums as 112% of benefit premiums.

The single contract premium at age 63 is 5233.

Lee decides to delay the purchase for two years and invests the 5233.

Calculate the minimum annual rate of return that the investment must earn to accumulate to an amount equal to the single contract premium at age 65.

(A) 0.030

(B) 0.035

(C) 0.040

(D) 0.045

(E) 0.050

35. You have calculated the actuarial present value of a last-survivor whole life insurance of 1 on (x) and (y). You assumed:

(i) The death benefit is payable at the moment of death.

(ii) The future lifetimes of (x) and (y) are independent, and each life has a constant force of mortality with $\boldsymbol{m} = 0.06$.

(iii) $\boldsymbol{d} = 0.05$

Your supervisor points out that these are not independent future lifetimes. Each mortality assumption is correct, but each includes a common shock component with constant force 0.02.

Calculate the increase in the actuarial present value over what you originally calculated.

(A) 0.020

(B) 0.039

(C) 0.093

(D) 0.109

(E) 0.163

36. The number of accidents follows a Poisson distribution with mean 12. Each accident generates 1, 2, or 3 claimants with probabilities $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, respectively.

Calculate the variance in the total number of claimants.

(A) 20

(B) 25

(C) 30

(D) 35

(E) 40

37. For a claims process, you are given:

(i) The number of claims $\{N(t),\ t \geq 0\}$ is a nonhomogeneous Poisson process with intensity function:

$$\lambda(t) = \begin{cases} 1, & 0 \leq t < 1 \\ 2, & 1 \leq t < 2 \\ 3, & 2 \leq t \end{cases}$$

(ii) Claims amounts Y_i are independently and identically distributed random variables that are also independent of $N(t)$.

(iii) Each Y_i is uniformly distributed on [200,800].

(iv) The random variable P is the number of claims with claim amount less than 500 by time $t = 3$.

(v) The random variable Q is the number of claims with claim amount greater than 500 by time $t = 3$.

(vi) R is the conditional expected value of P, given $Q = 4$.

Calculate R.

(A) 2.0

(B) 2.5

(C) 3.0

(D) 3.5

(E) 4.0

38. Lottery Life issues a special fully discrete whole life insurance on (25):

(i) At the end of the year of death there is a random drawing. With probability 0.2, the death benefit is 1000. With probability 0.8, the death benefit is 0.

(ii) At the start of each year, including the first, while (25) is alive, there is a random drawing. With probability 0.8, the level premium ***p*** is paid. With probability 0.2, no premium is paid.

(iii) The random drawings are independent.

(iv) Mortality follows the Illustrative Life Table.

(v) $i = 0.06$

(vi) ***p*** is determined using the equivalence principle.

Calculate the benefit reserve at the end of year 10.

(A) 10.25

(B) 20.50

(C) 30.75

(D) 41.00

(E) 51.25

39. A government creates a fund to pay this year's lottery winners.

You are given:

(i) There are 100 winners each age 40.

(ii) Each winner receives payments of 10 per year for life, payable annually, beginning immediately.

(iii) Mortality follows the Illustrative Life Table.

(iv) The lifetimes are independent.

(v) $i = 0.06$

(vi) The amount of the fund is determined, using the normal approximation, such that the probability that the fund is sufficient to make all payments is 95%.

Calculate the initial amount of the fund.

(A) 14,800

(B) 14,900

(C) 15,050

(D) 15,150

(E) 15,250

40. For a special fully discrete 35-payment whole life insurance on (30):

(i) The death benefit is 1 for the first 20 years and is 5 thereafter.

(ii) The initial benefit premium paid during the each of the first 20 years is one fifth of the benefit premium paid during each of the 15 subsequent years.

(iii) Mortality follows the Illustrative Life Table.

(iv) $i = 0.06$

(v) $A_{30:\overline{20|}} = 0.32307$

(vi) $\ddot{a}_{30:\overline{35|}} = 14.835$

Calculate the initial annual benefit premium.

(A) 0.010

(B) 0.015

(C) 0.020

(D) 0.025

(E) 0.030

****END OF EXAMINATION****

COURSE 3
MAY 2001

MULTIPLE-CHOICE ANSWER KEY

1	E
2	A
3	C
4	E
5	D
6	B
7	C
8	D
9	C
10	A
11	B
12	A
13	E
14	B
15	A
16	B
17	A
18	E
19	C
20	C
21	E
22	D
23	A
24	B
25	C

26	B
27	D
28	D
29	E
30	D
31	D
32	A
33	D
34	A
35	B
36	E
37	C
38	A
39	E
40	B