

CASUALTY ACTUARIAL SOCIETY

AND THE

CANADIAN INSTITUTE OF ACTUARIES



Thomas G. Myers Vice President-Admissions

Richard P. Yocius Chairperson Examination, Committee

Exam 3

Actuarial Models

Examination Committee
General Officers
Russell Frank
Arthur Placek
Virginia Prevosto
Julia C. Stenberg
Rhonda Port Walker
Floyd M. Yager

4 HOURS

November 1, 2005

INSTRUCTIONS TO CANDIDATES

- 1. This 80 point examination consists of 40 multiple choice questions worth 2 points each.
- 2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only. Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Please make your marks dark and fill in the spaces completely. Fill in that it is Fall, 2005, and the exam number 3.

Darken the spaces corresponding to your Candidate ID number. Four rows are available. If your Candidate ID number is fewer than 4 digits, include leading zeros. (For example, if your Candidate ID number is 987, consider that your Candidate ID number is 0987, enter a zero on the first row, 9 on the second row, 8 on the third row, and 7 on the fourth [last] row.) Please write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.

For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

- 3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.
- 4. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. <u>Do not remove this label</u>. Keep a record of your Candidate ID number for future inquiries regarding this exam.
- 5. At the beginning of the examination, check through the exam booklet for any missing or defective pages. The supervisor has additional exams for those candidates who have defective exam booklets. Verify that you have a copy of "Tables for CAS Exam 3" included in your exam packet.
- 6. Candidates must remain in the examination center until two hours after the start of the examination. You may leave the examination room to use the restroom with permission from the supervisor. To avoid excessive noise during the end of the examination, candidates may not leave the exam room during the last fifteen minutes of the examination.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

©2005 Casualty Actuarial Society

- 7. At the end of the examination, place the short-answer card in the Examination Envelope.

 BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE
 SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

 Anything written in the examination booklet will not be graded. Only the short-answer card and the answer sheets will be graded.
- 8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. (Do not put the self-addressed stamped envelope inside the Examination Envelope.)
 - If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may <u>not</u> take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.
 - Candidates may obtain a copy of the examination from the CAS website.
 - All extra answer sheets, scrap paper, etc., must be returned to the supervisor for disposal.
- 9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.
- 10. The exam survey is available on the CAS website in the "Admissions" section. <u>Please submit your survey to the CAS Office by November 21, 2005.</u>

END OF INSTRUCTIONS

1) The following sample was taken from a distribution with probability density function $f(x) = \theta x^{\theta-1}$, where 0 < x < 1 and $\theta > 0$.

0.21 0.43 0.56 0.67 0.72

Let R and S be the estimators of θ using the maximum likelihood and method of moments, respectively.

Calculate the value of R - S.

- A. Less than 0.3
- B. At least 0.3, but less than 0.4
- C. At least 0.4, but less than 0.5
- D. At least 0.5, but less than 0.6
- E. At least 0.6

2) Let Y_1 , Y_2 , Y_3 , Y_4 , Y_5 be the order statistics of a random sample of size 5 from a distribution having p.d.f. $f(x) = e^{-x}$ for $x \ge 0$, and f(x) = 0 elsewhere.

Calculate the probability that $Y_5>1$.

- A. Less than 0.55
- B. At least 0.55, but less than 0.65
- C. At least 0.65, but less than 0.75
- D. At least 0.75, but less than 0.85
- E. At least 0.85

3) For a Poisson distribution you are to test the null hypothesis H_0 : $\lambda = 1$ against the alternative hypothesis H_1 : $\lambda = 2$ at a significance level of 10% using a single observation.

Calculate the power of this test at $\lambda = 2$.

- A. Less than 20%
- B. At least 20%, but less than 40%
- C. At least 40%, but less than 60%
- D. At least 60%, but less than 80%
- E. At least 80%

4) When Mr. Jones visits his local race track, he places three independent bets. In his last 20 visits, he lost all of his bets 10 times, won one bet 7 times, and won two bets 3 times. He has never won all three of his bets.

Calculate the maximum likelihood estimate of the probability that Mr. Jones wins an individual bet.

- A. 13/60
- B. 4/15
- C. 19/60
- D. 11/30
- E. 5/12

5) The following information is based on a sample of 10 observations from a normal distribution:

$$\sum_{i=1}^{10} X_i = 110$$

$$\sum_{i=1}^{10} X_i^2 = 1,282$$

Which of the following are 95% confidence intervals for σ^2 ?

- 1. (0.0, 21.6)
- 2. (3.8, 26.7)
- 3. $(4.3, \infty)$
- A. 2. only
- B. 1. and 2. only
- C. 1. and 3. only
- D. 2. and 3. only
- E. 1., 2. and 3.

6) Claim sizes are uniformly distributed over the interval $[0, \theta]$. A sample of 10 claims, denoted $X_1, X_2, X_3, \dots X_{10}$, was observed and an estimate of θ was obtained as follows:

$$\hat{\theta} = Y = Max(X_1, X_2, ..., X_{10})$$

Recall that the probability density function for Y is:

$$f_r(y) = \frac{10y^9}{\theta^{10}} \qquad \text{for } 0 \le y \le \theta$$

Calculate the mean square error of $\hat{\theta}$ for $\theta = 100$.

- A. Less than 75
- B. At least 75, but less than 100
- C. At least 100, but less than 125
- D. At least 125, but less than 150
- E. At least 150

- 7) You are given the following information about an experiment:
 - The population is from a normal distribution.
 - Normal distribution values:

$\Phi(\mathbf{x})$	x
0.93	1.476
0.94	1.555
0.95	1.645
0.97	1.751

- H_0 : $\mu = 10$
- H_1 : $\mu = 11$
- $\bullet \quad \sigma^2 = 1$
- The probability of Type I error is 0.05.
- The probability of Type II error is no more than 0.06.

Calculate the minimum sample size for the experiment.

- A. 10
- B. 11
- C. 12
- D. 13
- E. 14

8) Claim sizes for a certain line of business are known to follow a normal distribution. A sample of claim sizes from this line of business are shown below:

Claim Number	Claim Size
1	3.3
2	5.4
3	7.1
4	8.9
5	23.5
6	29.8

For the hypothesis σ^2 <50, in which of the following ranges does the *p*-value fall?

- A. $p \le 0.005$
- B. 0.005
- C. 0.010
- D. 0.025
- E. p > 0.050

9) The following information is known about average claim sizes:

<u>Year</u>	Average Claim Size
1	\$1,020
2	1,120
3	1,130
4	1,210
5	1.280

Average claim sizes, Y, in year X are modeled by:

$$Y = \alpha e^{\beta X}$$

Using linear regression to estimate α and β , calculate the predicted average claim size in year 6.

- A. Less than \$1,335
- B. At least \$1,335, but less than \$1,340
- C. At least \$1,340, but less than \$1,345
- D. At least \$1,345, but less than \$1,350
- E. At least \$1,350

10) You are given the survival function s(x) as described below:

•
$$s(x) = 1 - \frac{x}{40}$$
 for $0 \le x \le 40$

• s(x) is zero elsewhere.

Calculate $\stackrel{\circ}{e}_{25}$, the complete expectation of life at age 25.

- A. Less than 7.7
- B. At least 7.7, but less than 8.2
- C. At least 8.2, but less than 8.7
- D. At least 8.7, but less than 9.2
- E. At least 9.2

11) Individuals with Flapping Gum Disease are known to have a constant force of mortality μ . Historically, 10% will die within 20 years.

A new, more serious strain of the disease has surfaced with a constant force of mortality equal to 2μ .

Calculate the probability of death in the next 20 years for an individual with this new strain.

- A. 17%
- B. 18%
- C. 19%
- D. 20%
- E. 21%

12) For a life age x, the force of mortality is $\mu_{x+t} = \frac{x+t-58}{200}$, for x>58 and t\ge 0.

Calculate q_{60} , the probability that (60) will die between ages 70 and 71.

- A. Less than 0.015
- B. At least 0.015, but less than 0.030
- C. At least 0.030, but less than 0.045
- D. At least 0.045, but less than 0.060
- E. At least 0.060

13) You are given:

- $s(x) = 1 \frac{x}{100}$ for $0 \le x \le 100$
- . s(x) is zero elsewhere.
- Uniform distribution of deaths within each year of age.

Calculate $_{0.25}$ $q_{45.75}$.

- A. Less than 0.002
- B. At least 0.002, but less than 0.004
- C. At least 0.004, but less than 0.006
- D. At least 0.006, but less than 0.008
- E. At least 0.008

- 14) You are given:
 - $\mu_x = \frac{1}{100 x}$ for x < 100
 - The future lifetimes of (45) and (50) are independent.

Calculate $_{5}$ $p_{\overline{45.50}}$, the probability that the last-survivor status of (45) and (50) survives 5 years.

- A. Less than 0.80
- B. At least 0.80, but less than 0.85
- C. At least 0.85, but less than 0.90
- D. At least 0.90, but less than 0.95
- E. At least 0.95

15) A new communications device uses 2 independent, non-rechargeable, non-replaceable batteries. Failure of either of the batteries results in device failure. The device has a 15-year warranty against failure.

Each battery has the following probability of survival to age t, where t is measured in years:

- $_t p_0 = 1 0.02t$ for t < 50
- $_{t}p_{0} = 0$ elsewhere

Calculate the probability that the device fails before the warranty runs out.

- A. Less than 0.10
- B. At least 0.10, but less than 0.30
- C. At least 0.30, but less than 0.50
- D. At least 0.50, but less than 0.70
- E. At least 0.70

16) For two independent lives, both age x, you are given $p_x = \left(1 - \frac{t}{20}\right)^2$ for t < 20.

Calculate the difference in the expected time between the first and second death.

- A. Less than 5
- B. At least 5, but less than 7
- C. At least 7, but less than 9
- D. At least 9, but less than 11
- E. At least 11

- 17) For 100 people entering an assisted-living facility, you are given the following:
 - Residents leave the facility each year either by (1) death or (2) all other reasons.
 - # of Residents Leaving Due to Cause

 Year (1) (2)

 1 14 12

 2 17 9

 3 21 3

 4 23 1
 - A room deposit of \$1,000 is returned to the resident if that person leaves the facility before death. The return is made at the end of the year of departure.
 - The interest rate is 5%.

Calculate the actuarial present value of the deposit return for a person who has lived at the facility for 2 years.

- A. 75.59
- B. 78.42
- C. 83.33
- D. 86.12
- E. 94.95

- 18) For a 20-year term life insurance on (x), you are given:
 - \bullet i = 0
 - A benefit of 2 is paid if death occurs by accident.
 - A benefit of 1 is paid if death occurs from all other causes.
 - Benefits are paid at the moment of death.
 - The force of mortality for death by accident is $\mu_{x+t}^{(1)} = \frac{t}{20}$.
 - The force of mortality for all other causes of death is $\mu_{x+t}^{(2)} = \frac{t}{10}$.

Calculate the net single premium for this insurance.

- A. $2/3(1 e^{-20})$ B. $1 e^{-20}$

- C. $1 e^{-30}$ D. $1 e^{-40}$ E. $4/3(1 e^{-30})$

19) Claim size, X, follows a Pareto distribution with parameters α and θ . A transformed distribution, Y, is created such that $Y = X^{1/\tau}$.

Which of the following is the probability density function of Y?

A.
$$\frac{\tau \theta y^{\tau-1}}{(y+\theta)^{\tau+1}}$$

B.
$$\frac{\alpha \theta^{\alpha} t y^{\tau-1}}{(y^{\tau} + \theta)^{\alpha+1}}$$

C.
$$\frac{\theta \alpha^{\theta}}{(y+\alpha)^{\theta+1}}$$

D.
$$\frac{\alpha \tau (y/\theta)^{\tau}}{y[1+(y/\theta)^{\tau}]^{\alpha+1}}$$

E.
$$\frac{\alpha\theta^{\alpha}}{(y^{\tau}+\theta)^{\alpha+1}}$$

20) Losses follow an exponential distribution with parameter θ . For a deductible of 100, the expected payment per loss is 2,000.

Which of the following represents the expected payment per loss for a deductible of 500?

- Α. θ
- B. $\theta (1 e^{-500/\theta})$
- C. 2,000 $e^{-400/\theta}$
- D. 2,000 e^{-5/ θ} E. 2,000 $(1 e^{-500/\theta}) / (1 e^{-100/\theta})$

21) Losses during the current year follow a Pareto distribution with $\alpha = 2$ and $\theta = 400,000$. Annual inflation is 10%.

Calculate the ratio of the expected proportion of claims that will exceed \$750,000 next year to the proportion of claims that exceed \$750,000 this year.

- A. Less than 1.105
- B. At least 1.105, but less than 1.115
- C. At least 1.115, but less than 1.125
- D. At least 1.125, but less than 1.135
- E. At least 1.135

22) An insurance agent gets a bonus based on the underlying losses, L, from his book of business. L follows a Pareto distribution with parameters $\alpha = 3$ and $\theta = 600,000$. His bonus, B, is calculated as (650,000 - L)/3 if this quantity is positive and 0 otherwise.

Calculate his expected bonus.

- A. Less than 100,000
- B. At least 100,000, but less than 120,000
- C. At least 120,000, but less than 140,000
- D. At least 140,000, but less than 160,000
- E. At least 160,000

23) New drivers enter at time 0 and will be classified as Preferred with probability 0.10, Standard with probability 0.30, and Substandard with probability 0.60. At time n, each driver is reclassified based on the probabilities indicated by the following transition matrices, Q_n , where state 1 is Preferred, state 2 is Standard, and state 3 is Substandard.

$$Q_1 = \begin{bmatrix} 0.40 & 0.40 & 0.20 \\ 0.10 & 0.60 & 0.30 \\ 0.00 & 0.30 & 0.70 \end{bmatrix}$$

$$Q_2 = \begin{bmatrix} 0.50 & 0.40 & 0.10 \\ 0.20 & 0.60 & 0.20 \\ 0.00 & 0.20 & 0.80 \end{bmatrix}$$

Calculate the probability that a new driver will transition from Standard to Substandard at time 2.

- A. 0.080
- B. 0.108
- C. 0.120
- D. 0.200
- E. 0.260

- 24) For a compound loss model, you are given:
 - The claim count follows a Poisson distribution with $\lambda = 0.01$.
 - Individual losses are distributed as follows:

<u>x</u>	$\underline{\mathbf{F}}(\mathbf{x})$
100	0.10
300	0.20
500	0.25
600	0.40
700	0.50
800	0.70
900	0.80
1,000	0.90
1,200	1.00

Calculate the probability of paying at least one claim after implementing a \$500 deductible.

- A. Less than 0.005
- B. At least 0.005, but less than 0.010
- C. At least 0.010, but less than 0.015
- D. At least 0.015, but less than 0.020
- E. At least 0.020

25) ABC Insurance Company plans to sell insurance policies that provide coverage for damage due to hail storms. The annual premium of \$125 per policy is collected at the moment the policy is sold. ABC expects policies to be sold according to a Poisson process, with a rate of 200 per year. Hail storm events also follow a Poisson process, with a rate of 10 per year.

Calculate the probability that ABC collects at least \$10,000 in premiums by the time the first storm occurs.

- A. 0.01
- B. 0.02
- C. 0.03
- D. 0.04
- E. 0.05

26) The number of reindeer injuries on December 24 follows a Poisson process with intensity function:

$$\lambda(t) = (t/12)^{1/2}$$
 0\leq t\leq 24, where t is measured in hours

Calculate the probability that no reindeer will be injured during the last hour of the day.

- A. Less than 30%
- B. At least 30%, but less than 40%
- C. At least 40%, but less than 50%
- D. At least 50%, but less than 60%
- E. At least 60%

27) The number of accidents reported to a local insurance adjusting office is a Poisson process with parameter $\lambda = 3$ claims per hour.

The number of claimants associated with each reported accident follows a negative binomial distribution with parameters r = 3 and $\beta = 0.75$.

If the adjusting office opens at 8:00 a.m., calculate the variance in the distribution of the number of claimants before noon.

- A. 9
- B. 16
- C. 47
- D. 108
- E. 189

28) Big National Bank has 3 teller windows open for customer service. Each teller services customers at a Poisson rate of 6 customers per hour.

There is a single line to wait for the next available teller and all tellers are currently serving customers.

If there are 2 people in line when the next customer arrives, calculate the probability that he must wait more than 10 minutes for the next available teller.

- A. Less than 30%
- B. At least 30%, but less than 40%
- C. At least 40%, but less than 50%
- D. At least 50%, but less than 60%
- E. At least 60%

29) ABC Insurance Company estimates that the time between reported claims is exponentially distributed with mean 0.50 years. Times between claims are independent. Each time a claim is reported, a payment is made with probability 0.70.

Calculate the probability that no payment will be made on claims reported during the next two years.

- A. 0.06
- B. 0.14
- C. 0.25
- D. 0.30
- E. 0.50

- 30) On January 1, 2005, Dreamland Insurance sold 10,000 insurance policies that pay \$100 for each day in 2005 that a policyholder is in the hospital. The following assumptions were used in pricing the policies:
 - The probability that a given policyholder will be hospitalized during the year is 0.05. No policyholder will be hospitalized more than one time during the year.
 - If a policyholder is hospitalized, the number of days spent in the hospital follows a lognormal distribution with $\mu = 1.039$ and $\sigma = 0.833$.

Using the normal approximation, calculate the premium per policy such that there is a 90% probability that total premiums will exceed total losses.

- A. Less than 21.20
- B. At least 21.20, but less than 21.50
- C. At least 21.50, but less than 21.80
- D. At least 21.80, but less than 22.10
- E. At least 22.10

31) The Toronto Bay Leaves attempt shots in a hockey game according to a Poisson process with mean 30. Each shot is independent. For each attempted shot, the probability of scoring a goal is 0.10.

Calculate the standard deviation of the number of goals scored by the Bay Leaves in a game.

- A. Less than 1.4
- B. At least 1.4, but less than 1.6
- C. At least 1.6, but less than 1.8
- D. At least 1.8, but less than 2.0
- E. At least 2.0

32) For a certain insurance company, 60% of claims have a normal distribution with mean 5,000 and variance 1,000,000. The remaining 40% have a normal distribution with mean 4,000 and variance 1,000,000.

Calculate the probability that a randomly selected claim exceeds 6,000.

- A. Less than 0.10
- B. At least 0.10, but less than 0.15
- C. At least 0.15, but less than 0.20
- D. At least 0.20, but less than 0.25
- E. At least 0.25

33) In year 2005, claim amounts have the following Pareto distribution:

$$F(x) = 1 - \left(\frac{800}{x + 800}\right)^3$$

The annual inflation rate is 8%. A franchise deductible of 300 will be implemented in 2006.

Calculate the loss elimination ratio of the franchise deductible.

- A. Less than 0.15
- B. At least 0.15, but less than 0.20
- C. At least 0.20, but less than 0.25
- D. At least 0.25, but less than 0.30
- E. At least 0.30

34) Claim frequency follows a Poisson process with rate of 10 per year. Claim severity is exponentially distributed with mean 2,000.

The method of moments is used to estimate the parameters of a lognormal distribution for the aggregate losses.

Using the lognormal approximation, calculate the probability that annual aggregate losses exceed 105% of expected annual losses.

- A. Less than 34.5%
- B. At least 34.5%, but less than 35.5%
- C. At least 35.5%, but less than 36.5%
- D. At least 36.5%, but less than 37.5%
- E. At least 37.5%

- 35) For a fully continuous whole life policy on (45), you are given:
 - The force of mortality is $\mu = 0.10$.
 - The force of interest is $\delta = 0.05$.

Calculate the variance of the loss random variable.

- A. Less than 0.25
- B. At least 0.25, but less than 0.35
- C. At least 0.35, but less than 0.45
- D. At least 0.45, but less than 0.55
- E. At least 0.55

36) A 60-year-old lottery winner has the choice of receiving (i) a single lump sum payment of P_1 in 10 years if she is still alive or (ii) a 20-year life annuity-due of P_2 per year beginning today.

Given:

- The payouts in (i) and (ii) are actuarially equivalent.
- d = 6%
- $A_{60: \frac{1}{10|}} = 0.4188$
- $^2A_{60: \frac{1}{10|}} = 0.2339$
- $A_{60:\overline{20|}} = 0.4427$
- ${}^{2}A_{60:\overline{20}|} = 0.2312$
- Calculate the difference in the variances of the payouts in (i) and (ii).
- A. $0.04P_1^2$
- B. $0.12P_1^2$
- C. $0.20P_1^2$
- D. $0.33 P_1^2$
- E. $0.41P_1^2$

- 37) For a fully discrete whole life policy of 1 on (95), you are given:
 - $l_{95} = 1,000$
 - $l_{96} = 900$
 - $l_{97} = 250$
 - $l_{98} = 0$
 - v = 0.90

Calculate the annual benefit premium.

- A. Less than 0.41
- B. At least 0.41, but less than 0.43
- C. At least 0.43, but less than 0.45
- D. At least 0.45, but less than 0.47
- E. At least 0.47

38) For a fully discrete 20-year term life insurance on (50) that pays \$1,000 at the end of the year of death, the annual premium is \$29.52.

You are given:

- v = 0.95
- $q_{52} = 0.0263$

Calculate ${}_{3}V^{1}_{50\overline{20}|}$, the benefit reserve for this insurance at time 3.

- A. 6.90
- B. 18.26
- C. 24.15
- D. 34.92
- E. 45.25

- 39) An investor wishes to invest in an oil field. Oil fields can be in one of three states: "Gusher," "Normal," or "Dry."
 - A "Gusher" pays a dividend of \$1,000 at the end of the year.
 - A "Normal" pays a dividend of \$200 at the end of the year.
 - A "Dry" pays no dividend.

At the end of each year after paying the dividend, the oil field is reclassified as follows:

- A "Gusher" has a 50% chance of remaining a "Gusher," a 30% chance of becoming "Normal," and a 20% chance of becoming "Dry."
- A "Normal" has a 50% chance of remaining a "Normal" and a 50% chance of becoming "Dry."
- A "Dry" has a 100% chance of remaining a "Dry."

The interest rate is 10%.

Calculate the actuarial present value of a "Gusher" at the beginning of the year.

- A. Less than \$1,800
- B. At least \$1,800, but less than \$1,850
- C. At least \$1,850, but less than \$1,900
- D. At least \$1,900, but less than \$1,950
- E. At least \$1,950

40) QRS Insurance has a beginning surplus of 20. Premium collected at the beginning of each year is 20 for the first year and 25 for the next year. Losses, paid at the end of each year, are distributed as follows:

Year 1Year 2		Year 2
Losses	Probability	Losses Probability
0	10%	0 10%
10	60%	15 70%
50	30%	60 20%

There is no investment income.

Calculate the probability that QRS will survive at least 2 years.

- A. 0.42
- B. 0.48
- C. 0.52
- D. 0.56
- E. 0.58

Exam 3 Fall 2005

Multiple Choice Answers

Question	Answer
1	A
2	E
3	В
4	A
5	E
6	E
7	В
8	D
9	D
10	A
11	C C C
12	C
13	C
14	E
15	D
16	В
17	В
18	E
19	В
20	C
21	D
22	C
23	A
24	В
25	В
26	A D
27	D
28	C
29	A C
30	C
31	C
32	В
33	В
34	D
35	D
36	A
37	A
38	В
39	В
40	E