COURSE 3

SAMPLE EXAM

Although a multiple choice format is not provided for some questions on this sample examination, the initial Course 3 examination will consist entirely of multiple choice type questions. The solutions to the questions on this sample examination are at the end.

Appendices A and B of Loss Models: From Data to Decisions, a normal table and an Illustrative Life table will be provided with the examination. The Illustrative Life Table used for this sample examination can be found in Appendix 2A of Actuarial Mathematics (Second Edition).

1.

A manufacturer offers a warranty paying 1000 at the time of failure for each machine that fails within 5 years of purchase. One customer purchases 500 machines. The manufacturer wants to establish a fund for warranty claims from this customer. The manufacturer wants to have at least a 95% probability that the fund is sufficient to pay the warranty claims.

You are given:

- The constant force of failure for each machine is $\mu = 0.02$.
- The force of interest is $\delta = 0.02$.
- The times until failure of the machines are independent.

Using the normal approximation, determine the minimum size of the fund.

(A) 27,900

(B) 55,600

(C) 78,200

(D) 86,400

(E) 90,600

2.

A reinsurer plans to assess the risk associated with a group of catastrophe insurance policies by examining the probability of ruin. Since it is believed that the time between claims has a gamma distribution with $\alpha = 2.5$ and $\theta = 1$, simulation is to be used. The inter-claim times will be generated using the rejection method with $g(x)=\lambda\exp(-\lambda x)$, $x>0$.

Using the choice of λ that minimizes the expected number of iterations required to generate each inter-claim time, calculate this expected number of iterations.

3.

A whole life insurance of 1 with benefits payable at the moment of death of (x) includes a double-indemnity provision. This provision pays an additional death benefit of 1 for death by accidental means. S is the single benefit premium for this insurance.

A second whole life insurance of 1 with benefits payable at the moment of death of (x) includes a triple-indemnity provision. This provision pays an additional death benefit of 2 for death by accidental means. T is the single benefit premium for this insurance.

You are given:

- μ is the force of decrement for death by accidental means.
- 5μ is the force of decrement for death by other means.
- There are no other decrements.

Determine $T - S$.

(A) $\dfrac{S}{12}$

(B) $\dfrac{S}{8}$

(C) $\dfrac{S}{7}$

(D) $\dfrac{S}{4}$

(E) $\dfrac{S}{2}$

4.

L is the loss-at-issue random variable for a fully continuous whole life insurance of 1 on (30). The premium is determined by the equivalence principle.

You are given:

- The actuarial present value of 1 payable at the moment of death of (50) is 0.70.
- The second moment of the present value of 1 payable at the moment of death of (30) is 0.30.
- The variance of L is 0.20.

Calculate the benefit reserve at $t = 20$ for a fully continuous whole life insurance on (30).

(A) 0.2

(B) 0.3

(C) 0.4

(D) 0.5

(E) 0.6

5.

You are given:

- The probability density function for the amount of a single loss is

$$f(x) = 0.01(1 - q + 0.01qx)e^{-0.01x}, \qquad x > 0.$$

- If an ordinary deductible of 100 is imposed, the expected payment (given that a payment is made) is 125.

Determine the expected payment (given that a payment is made) if the deductible is increased to 200.

6.

You are given:

- Hens lay an average of 30 eggs each month until death.

- The survival function for hens is $s(m) = 1 - \frac{m}{72}$, $0 \le m \le 72$, where m is in months.

- 100 hens have survived to age 12 months.

Calculate the expected total number of eggs to be laid by these 100 hens in their remaining lifetimes.

(A) 900

(B) 3000

(C) 9000

(D) 30,000

(E) 90,000

7.

An HMO plans to assess resource requirements for its elderly members by determining the distribution of numbers of members whose health is classified into one of three states:

- healthy
- moderately impaired
- severely impaired

Changes in health are to be modeled using a continuous-time Markov chain with three states. The distribution of the number in each state will be obtained by simulation. You are given:

- The state transition rates are

$$q_{12} = q_{13} = \frac{1}{2}$$
$$q_{21} = q_{23} = \frac{1}{3}$$
$$q_{31} = q_{32} = \frac{1}{4}$$

- For a given simulation, the process is in state 1 at time $t = 0$. The process enters state 2 at the time of the first transition.

- U_1 and U_2 are independent standard uniform random variables on (0,1).

Determine a function of U_1 and U_2 which will correctly generate the time, t, of the second transition.

8. - 9.

Use the following information for Questions 8. and 9.

You are given:

- Jim buys a new car for 20,000. He intends to keep the car for 3 years.
- A device called Car-Tracker can be attached to Jim's car. Car-Tracker will help police locate the car if stolen.
- The probability that that car will be stolen without Car-Tracker is 0.2 in each year. ($q_k = 0.2, \quad k = 0, \quad 1, \quad 2$)
- The probability that that car will be stolen with Car-Tracker is 0.1 in each year. ($q_k^{CT} = 0.1, \quad k = 0, \quad 1, \quad 2$)
- Theft is the only decrement.
- Insurance against theft is purchased with a single benefit premium.
- If the car is stolen in year j, the insurance company will pay 25,000 - 5000j at the end of the year, for j = 1, 2, 3. After the third year, no benefit is paid.
- $i = 0.06$

8.

Calculate the actuarial present value of the insurance benefit against theft assuming Jim does not purchase Car-Tracker.

(A) 3800

(B) 4800

(C) 7000

(D) 8000

(E) 9000

8. - 9.

(Repeated for convenience.) Use the following information for Questions 8. and 9.

You are given:

- Jim buys a new car for 20,000. He intends to keep the car for 3 years.
- A device called Car-Tracker can be attached to Jim's car. Car-Tracker will help police locate the car if stolen.
- The probability that that car will be stolen without Car-Tracker is 0.2 in each year. ($q_k = 0.2, \quad k = 0, \quad 1, \quad 2$)
- The probability that that car will be stolen with Car-Tracker is 0.1 in each year. ($q_k^{CT} = 0.1, \quad k = 0, \quad 1, \quad 2$)
- Theft is the only decrement.
- Insurance against theft is purchased with a single benefit premium.
- If the car is stolen in year j, the insurance company will pay 25,000 - 5000j at the end of the year, for j = 1, 2, 3. After the third year, no benefit is paid.
- $i = 0.06$

9.

Using the equivalence principle, calculate the greatest premium reduction that the insurance company can give if Jim buys Car-Tracker.

(A) 3000

(B) 3200

(C) 3400

(D) 4000

(E) 4100

10.

An insurance company is negotiating to settle a liability claim. If a settlement is not reached, the claim will be decided in the courts 3 years from now.

You are given:

- There is a 50% probability that the courts will require the insurance company to make a payment. The amount of the payment, if there is one, has a lognormal distribution with mean 10 and standard deviation 20.

- In either case, if the claim is not settled now, the insurance company will have to pay 5 in legal expenses, which will be paid when the claim is decided, 3 years from now.

- The most that the insurance company is willing to pay to settle the claim is the expected present value of the claim and legal expenses plus 0.02 times the variance of the present value.

- Present values are calculated using $i = 0.04$.

Calculate the insurance company's maximum settlement value for this claim.

(A) 8.89

(B) 9.93

(C) 12.45

(D) 12.89

(E) 13.53

11.

A special insurance program is designed to pay a benefit in the event a product fails. You are given:

- Benefits are payable at the moment of failure.

- $b_t = \begin{cases} 300, & 0 \le t < 25 \\ 100, & t \ge 25 \end{cases}$

- $\mu(t) = 0.04, \quad t \ge 0$

- $\delta_t = \begin{cases} 0.02, & 0 \le t < 25 \\ 0.03, & t \ge 25 \end{cases}$

Calculate the actuarial present value of this special insurance.

(A) 165

(B) 168

(C) 171

(D) 210

(E) 213

12.

The annual number of accidents for an individual driver has a Poisson distribution with mean λ. The Poisson means, λ, of a heterogeneous population of drivers have a gamma distribution with mean 0.1 and variance 0.01.

Calculate the probability that a driver selected at random from the population will have 2 or more accidents in one year.

(A) $\frac{1}{121}$

(B) $\frac{1}{110}$

(C) $\frac{1}{100}$

(D) $\frac{1}{90}$

(E) $\frac{1}{81}$

13.

For a special fully discrete modified premium whole life insurance of 1 on a life age 40, you are given:

- Benefit premiums P_1 and P_2 are determined according to the equivalence principle.
- The benefit premium is P_1 for the first 20 years and P_2 thereafter.
- The following values:

x	A_x	$\ddot{a}_x$
40	0.103	12.112
60	0.284	9.661

- $\ddot{a}_{40:\overline{20}|} = 10.290$
- ${}_{20}V$ is the benefit reserve at the end of year 20.
- If the benefit premium after year 20 is changed to P_1, and the benefit after year 20 is reduced to 0.75, then ${}_{20}V$ is unchanged.

Calculate ${}_{20}V$.

(A) 0.12

(B) 0.13

(C) 0.14

(D) 0.15

(E) 0.16

14. -15.

Use the following information for questions 14. and 15.

You are given:

- An aggregate loss distribution has a compound Poisson distribution with expected number of claims equal to 1.25.

- Individual claim amounts can take only the values 1, 2 or 3, with equal probability.

14.

Determine the probability that aggregate losses exceed 3.

14. - 15.

(Repeated for convenience.) Use the following information for questions 14. and 15.

You are given:

- An aggregate loss distribution has a compound Poisson distribution with expected number of claims equal to 1.25.
- Individual claim amounts can take only the values 1, 2 or 3, with equal probability.

15.

Calculate the expected aggregate losses if an aggregate deductible of 1.6 is applied.

16.

For a special 5-year term insurance of 1 on two lives, Kathy and Stan, you are given:

- The future lifetimes of Kathy, age 30, and Stan, age 50, are independent.
- Kathy is subject to a constant force of mortality of 0.02 for $0 < t \leq 5$.
  Stan is subject to a constant force of mortality of 0.04 for $0 < t \leq 5$.
- The force of interest is 0.03.
- The death benefit is payable at the moment of death of Kathy provided Kathy dies first.
- The policy pays nothing if Stan dies first.

Calculate the single benefit premium for this insurance.

(A) 0.08

(B) 0.10

(C) 0.12

(D) 0.14

(E) 0.16

17. - 18.

Use the following information for Questions 17. and 18.

You are given:

- Ground-up losses follow a lognormal distribution with parameters $\mu = 7$ and $\sigma = 2$.
- There is an ordinary deductible of 2000.
- 10 losses are expected each year.
- The number of losses and the individual loss amounts are independent.

17.

Determine the loss elimination ratio (LER) for the deductible.

17. - 18.

(Repeated for convenience.) Use the following information for Questions 17. and 18.

You are given:

- Ground-up losses follow a lognormal distribution with parameters $m = 7$ and $s = 2$.
- There is an ordinary deductible of 2000.
- 10 losses are expected each year.
- The number of losses and the individual loss amounts are independent.

18.

Determine the expected number of annual losses that exceed the deductible if all loss amounts increased uniformly by 20%, but the deductible remained the same.

19.

For two ice cream maker models, I and II, you are given:

- The failure rate for model I is $m^I(x) = \ln\frac{10}{8}$, $x \geq 0$.
- The failure rate for model II is $m^{II}(x) = \frac{1}{9-x}$, $0 \leq x < 9$.

Calculate the probability that if both models are of exact age 2, the first failure will occur between exact ages 3 and 6.

(A) 0.25

(B) 0.34

(C) 0.43

(D) 0.51

(E) 0.60

20.

You are given:

- An insured's claim severity distribution is described by an exponential distribution:
$$F(x) = 1 - e^{-x/1000}$$

- The insured's number of claims is described by a negative binomial distribution with $\beta = 2$ and $r = 2$.

- A 500 per claim deductible is in effect.

Calculate the standard deviation of the aggregate losses in excess of the deductible.

(A) Less than 2000

(B) At least 2000 but less than 3000

(C) At least 3000 but less than 4000

(D) At least 4000 but less than 5000

(E) At least 5000

21. - 22.

<u>Use the following information for questions 21. and 22.</u>

The force of mortality for a life selected at age x follows the model:

$$m_x(t) = j\,(x)m(t), \qquad\qquad t \geq 0$$

You are given:

- $j\,(x) = b + 0.006S + 0.003x$
- $m(t) = t$
- $S = \begin{cases} 1, & \quad\quad if\,(x) smokes \\ 0, & \quad\quad otherwise \end{cases}$
- ${}_{10}p^{n}_{[30]} = 0.96$
- A superscript of "s" indicates the case where S = 1 and "n" indicates the case where S = 0.

21.

Determine x such that $q^{s}_{[35]} = q^{n}_{[x]}$.

21. - 22.

<u>(Repeated for convenience.) Use the following information for questions 21. and 22.</u>

The force of mortality for a life selected at age *x* follows the model:

$$m_x(t) = j\ (x)m(t), \qquad t \geq 0$$

You are given:

- $j\ (x) = b + 0.006S + 0.003x$
- $m(t) = t$
- $S = \begin{cases} 1, & \textit{if } (x) \textit{smokes} \\ 0, & \textit{otherwise} \end{cases}$
- ${}_{10}p^{n}_{[30]} = 0.96$
- A superscript of "*s*" indicates the case where *S* = 1 and "*n*" indicates the case where *S* = 0.

22.

Calculate the probability that a life, drawn at random from a population of 30 year olds of which 40% are smokers, will survive at least 10 years.

23.

You are given:

- A loss occurrence in excess of 1 billion may be caused by a hurricane, an earthquake, or a fire.
- Hurricanes, earthquakes, and fires occur independently of one another.
- The number of hurricanes causing a loss occurrence in excess of 1 billion in a one-year period follows a Poisson distribution. The expected amount of time between such hurricanes is 2.0 years.
- The number of earthquakes causing a loss occurrence in excess of 1 billion in a one-year period follows a Poisson distribution. The expected amount of time between such earthquakes is 5.0 years.
- The number of fires causing a loss occurrence in excess of 1 billion in a one-year period follows a Poisson distribution. The expected amount of time between such fires is 10.0 years.

Determine the expected amount of time between loss occurrences in excess of 1 billion.

24.

The homeowners insurance policies of a large insurance company were all issued on January 1, 1992 and are removed from the in-force file for one of three reasons:

- The policyholder moves and cancels his or her policy. This decrement is uniformly distributed over each calendar year.
- The policyholder selects another insurance company and cancels his or her policy. This decrement occurs only on January 1 of each calendar year.
- The insurance company cancels the policy. This decrement occurs only on December 31 of each calendar year.

You are given the following partial table:

Calendar Year	Number of policies in-force at the end of the previous calendar year	Probability of policyholder moving and canceling his or her policy	Probability of policyholder selecting another insurance company and canceling his or her policy	Probability of insurance company canceling a policy
1996	100,000	0.0625	0.2000	0.2000
1997	--	--	0.1500	0.2000
1998	37,356	--	--	--

Calculate the probability of a policyholder on 1/1/1997 moving and canceling his or her policy during 1997.

25.

For aggregate losses $S = X_1 + X_2 + \ldots + X_N$, you are given:

- N has a Poisson distribution with mean 500.
- $X_1, X_2, \ldots$ have mean 100 and variance 100.
- $N, X_1, X_2 \ldots$ are mutually independent.

You are also given:

- For a portfolio of insurance policies, the loss ratio is the ratio of aggregate losses to aggregate premiums collected.
- The premium collected is 1.1 times the expected aggregate losses.

Using the normal approximation to the compound Poisson distribution, calculate the probability that the loss ratio exceeds 0.95.

26.

You are given:

- The Driveco Insurance company classifies all of its auto customers into two classes: preferred with annual expected losses of 400 and standard with annual expected losses of 900.

- There will be no change in the expected losses for either class over the next three years.

- The one year transition matrix between driver classes is given by:

		Driver's class in year k+1	
		Preferred	**Standard**
Driver's class in year k	**Preferred**	0.85	0.15
	Standard	0.60	0.40

- $i = 0.05$

- Losses are paid at the end of each year.

- There are no expenses.

- All drivers insured with Driveco at the start of the period will remain insured for the following three years.

Calculate the 3-year term insurance single benefit premium for a standard driver.

27.

You are given:

- The logarithm of the price of a stock can be modeled by Brownian motion with drift coefficient $m = 0$ and variance parameter $s^2 = 0.04$.

- The price of the stock at time $t = 0$ is 10.

Calculate the probability that the price of the stock will be 12 or greater at some time between $t = 0$ and $t = 1$.

(A) 0.023

(B) 0.181

(C) 0.273

(D) 0.362

(E) 0.543

28. - 31.

Use the following information for Questions 28. through 31.

Certain commercial loans are scheduled for repayment with continuous level payments over a 20-year period. You are given:

- The force of default (hazard rate) is 0.05.
- The force of prepayment is 0.03.

28.

Calculate the probability of default during the duration of a loan.

28. - 31.

(Repeated for convenience.) Use the following information for Questions 28. through 31.

Certain commercial loans are scheduled for repayment with continuous level payments over a 20-year period. You are given:

- The force of default (hazard rate) is 0.05.
- The force of prepayment is 0.03.

For Questions 29. - 31., you are also given:

- At the beginning of fiscal year Z, the government enters a program to guarantee 100 million of loans.
- At this time, the force of interest charged on these 20-year loans is 0.06.
- The government will collect a fee for a loan guarantee that is proportional to the initial size of the loan and is level over the actual loan repayment period. The fee level is determined so that the program is self-supporting, ignoring expenses, on the basis of the equivalence principle.
- $d = 0.05$ is used to calculate the government's liability and the fee level.

29.

Determine the government's liability, as estimated by the actuarial present value, for the guarantee of these loans.

28. - 31.

<u>(Repeated for convenience.) Use the following information for Questions 28. through 31.</u>

Certain commercial loans are scheduled for repayment with continuous level payments over a 20-year period. You are given:

- The force of default (hazard rate) is 0.05.
- The force of prepayment is 0.03.

<u>For Questions 29. - 31., you are also given:</u>

- At the beginning of fiscal year Z, the government enters a program to guarantee 100 million of loans.
- At this time, the force of interest charged on these 20-year loans is 0.06.
- The government will collect a fee for a loan guarantee that is proportional to the initial size of the loan and is level over the actual loan repayment period. The fee level is determined so that the program is self-supporting, ignoring expenses, on the basis of the equivalence principle.
- $d = 0.05$ is used to calculate the government's liability and the fee level.

30.

Calculate the annual fee level as a percentage of the initial loan amount.

28. - 31.

(Repeated for convenience.) Use the following information for Questions 28. through 31.

Certain commercial loans are scheduled for repayment with continuous level payments over a 20-year period. You are given:

- The force of default (hazard rate) is 0.05.
- The force of prepayment is 0.03.

For Questions 29. - 31., you are also given:

- At the beginning of fiscal year Z, the government enters a program to guarantee 100 million of loans.
- At this time, the force of interest charged on these 20-year loans is 0.06.
- The government will collect a fee for a loan guarantee that is proportional to the initial size of the loan and is level over the actual loan repayment period. The fee level is determined so that the program is self-supporting, ignoring expenses, on the basis of the equivalence principle.
- $d = 0.05$ is used to calculate the government's liability and the fee level.

31.

A lending institution's accounting study shows that the costs of administration of the guarantee on a loan is made up of the following three components:

- At initiation of the loan, 100 plus 0.1% of the loan amount.
- At default, 10% of the loan value outstanding.
- At prepayment, 1% of the loan value outstanding.

Using the equivalence principle, calculate the additional level fee required by the lending institution to cover the costs on a loan of 10,000.

32.

You are given:

- Annual losses for the portfolio of an insurer have an exponential distribution with mean 1 and are paid at the end of each year.
- The insurer has an initial surplus of 2 and collects a premium of 1.5 at the beginning of each year.
- No interest is earned or paid.

Using discretizing of losses by the method of rounding with a span of 2, determine the probability that the insurer becomes insolvent during the second year.

33.

Jim and Mary (both currently age 60) purchase an annuity payable continuously at the rate of:

- 1 per year with certainty for 15 years
- 1 per year after 15 years if both Jim and Mary are alive
- ¾ per year after 15 years if Jim is alive and Mary is dead
- ½ per year after 15 years if Mary is alive and Jim is dead

You are given:

- The future lifetimes for Jim and Mary are independent.

- Both are subject to a constant force of mortality, $\mu = 0.06$.

- The force of interest is $\delta = 0.04$.

- A 15-year temporary life annuity of 1 payable continuously on a life age 60 has a single benefit premium of 7.769.

- A 15-year temporary joint life annuity of 1 payable continuously on two lives aged 60 has an actuarial present value of 5.683.

Calculate the actuarial present value of this annuity.

(A) 6.25

(B) 11.28

(C) 13.93

(D) 17.53

(E) 20.17

34.

You are given:

- Two independent and identically distributed compound Poisson claim processes are insured by separate carriers at the same premium rate.
- The distribution of claim amounts is exponential with mean $\frac{1}{2}$.
- The relative security loading for each insurer is 100%.
- The surplus of the first insurer is $\log_e 2$ and the surplus of the second insurer is $\log_e 4$.
- The two companies merge.

Determine the probability of ruin for the merged company.

(A) $\frac{1}{16}$

(B) $\frac{1}{8}$

(C) $\frac{3}{16}$

(D) $\frac{1}{4}$

(E) $\frac{5}{16}$

35.

A worker has become disabled as a result of an occupational disease. The workers compensation laws of state *X* specify that, in addition to disability benefits, the worker's estate will receive a lump sum payment of 50,000 at the time of her death if the worker dies as a result of the occupational disease.

- The force of mortality for death attributable to the disease is $m^{(1)} = 0.020$.
- The force of mortality for death from all other causes $m^{(2)} = 0.015$.
- The force of interest is 0.05.

Determine the actuarial present value of this death benefit.

(A) 8,800

(B) 11,800

(C) 14,300

(D) 20,600

(E) 28,600

36.

For an insurer's surplus process, you are given:

- Aggregate claims follow a compound Poisson process.
- The claim amount distribution has mean 100 and standard deviation 100.
- The relative security loading is 0.20.
- The maximal aggregate loss is the maximal excess of aggregate claims over premiums received.

You are also given:

- Proportional reinsurance is available at a premium equal to 130% of the expected reinsured claims.
- α is the proportion of each claim reinsured that minimizes the expected maximal aggregate loss.

Calculate α.

(A) 0

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

(E) $\frac{1}{2}$

37. - 39.

Use the following information for questions 37. through 39.

A continuing care retirement community (CCRC) offers residents accommodation in an independent living unit (ILU) and nursing care in a skilled nursing facility (SNF). Residents are transferred either temporarily or permanently from their ILU to the SNF depending on an assessment of their recoverability. For the purpose of determining fees, the CCRC uses a continuous-time Markov chain with the following states:

1. ILU
2. SNF (temporary)
3. SNF (permanent)
4. Dead
5. Withdrawn

In a preliminary analysis, the CCRC uses the following state transition rates (per year):

$$\mu_{12} = 0.3 \quad \mu_{13} = 0.1 \quad \mu_{14} = 0.1$$
$$\mu_{15} = 0.2 \quad \mu_{21} = 0.5 \quad \mu_{23} = 0.3$$
$$\mu_{24} = 0.3 \quad \mu_{34} = 0.4$$

For other ij combinations, $\mu_{ij} = 0$.

37.

Calculate the probability that a resident will make more than one visit to the SNF.

(A) 0.11

(B) 0.16

(C) 0.21

(D) 0.26

(E) 0.31

37. - 39.

<u>(Repeated for convenience.) Use the following information for questions 37. through 39.</u>

A continuing care retirement community (CCRC) offers residents accommodation in an independent living unit (ILU) and nursing care in a skilled nursing facility (SNF). Residents are transferred either temporarily or permanently from their ILU to the SNF depending on an assessment of their recoverability. For the purpose of determining fees, the CCRC uses a continuous-time Markov chain with the following states:

1. ILU
2. SNF (temporary)
3. SNF (permanent)
4. Dead
5. Withdrawn

In a preliminary analysis, the CCRC uses the following state transition rates (per year):

$$\mu_{12} = 0.3 \quad \mu_{13} = 0.1 \quad \mu_{14} = 0.1$$
$$\mu_{15} = 0.2 \quad \mu_{21} = 0.5 \quad \mu_{23} = 0.3$$
$$\mu_{24} = 0.3 \quad \mu_{34} = 0.4$$

For other ij combinations, $\mu_{ij} = 0$.

38.

Calculate the expected duration of a visit to the SNF.

(A) 1.0 years

(B) 1.4 years

(C) 1.8 years

(D) 2.2 years

(E) 2.4 years

37. - 39.

(Repeated for convenience.) Use the following information for questions 37. through 39.

A continuing care retirement community (CCRC) offers residents accommodation in an independent living unit (ILU) and nursing care in a skilled nursing facility (SNF). Residents are transferred either temporarily or permanently from their ILU to the SNF depending on an assessment of their recoverability. For the purpose of determining fees, the CCRC uses a continuous-time Markov chain with the following states:

1. ILU
2. SNF (temporary)
3. SNF (permanent)
4. Dead
5. Withdrawn

In a preliminary analysis, the CCRC uses the following state transition rates (per year):

$$\mu_{12} = 0.3 \quad \mu_{13} = 0.1 \quad \mu_{14} = 0.1$$
$$\mu_{15} = 0.2 \quad \mu_{21} = 0.5 \quad \mu_{23} = 0.3$$
$$\mu_{24} = 0.3 \quad \mu_{34} = 0.4$$

For other ij combinations, $\mu_{ij} = 0$.

39. For Question 39., you are also given:

- The CCRC charges a fee of 100,000 upon admission of a resident.
- A refund of (100,000-20,000t) is provided to residents upon withdrawal at time t since admission, if withdrawal occurs within five years of admission and the resident has not made use of the SNF.
- $i = 0.05$

Calculate the actuarial present value at admission of the refund payable upon withdrawal to a resident who has not made use of the SNF.

(A)	16,411	(D)	19,744
(B)	17,522	(E)	20,855
(C)	18,633		

40.

A husband and wife purchase a 3-year term insurance with a benefit of 100,000 payable at the end of the year of the first death. You are given:

- $i = 0.08$
- The lives are independent.
- The following mortality information:

$\boldsymbol{k}$	**Men** q_{x+k}	**Women** q_{y+k}
0	0.08	0.06
1	0.10	0.10
2	0.12	0.13
3	0.14	0.17

Determine the actuarial present value of this insurance.

(A) 26,600

(B) 32,400

(C) 39,600

(D) 42,300

(E) 45,600

41. - 42.

Use the following information for questions 41. and 42.

The frequency distribution of the number of losses in a year is geometric-Poisson with geometric primary parameter $\beta = 3$ and Poisson secondary parameter $\lambda = 0.5$.

41.

Calculate the probability that the total number of losses in a year is at least 4.

41. - 42.

(Repeated for convenience.) Use the following information for questions 41. and 42.

The frequency distribution of the number of losses in a year is geometric-Poisson with geometric primary parameter $\beta = 3$ and Poisson secondary parameter $\lambda = 0.5$.

42.

If individual losses are all for exactly 100, determine the expected aggregate losses in excess of 400.

43.

A worker is injured on 1/1/99, resulting in a workers compensation claim. The insurer establishes a reserve, on that date, of 274,000.

You are given:

- Medical and indemnity payments will be made for 15 years.
- The medical payments will be 10,000 in 1999, increasing by 10% per year thereafter.
- The annual indemnity payment is P.
- The medical and indemnity payments are discounted at an interest rate of 5% per year to determine the reserve.
- Payments will be made at the end of each year.

Calculate P.

(A) 5300

(B) 5800

(C) 6300

(D) 6900

(E) 7200

44.

For a 10-year deferred whole life insurance of 1 payable at the moment of death on a life age 35, you are given:

- The force of interest is $\delta = 0.10$.
- The force of mortality is $\mu = 0.06$.
- Z is the present value random variable for this insurance.

Determine the 90th percentile of Z.

(A) 0.1335

(B) 0.1847

(C) 0.2631

(D) 0.4488

(E) 0.4512

45.

For a fully discrete 5-year term insurance of 1 on (40), you are given:

- Mortality follows the Illustrative Life Table.
- $i = 0.06$
- The insurer's utility of wealth function is $u(x) = -e^{-0.2x}$.
- The exponential premium for this insurance is 0.00328.
- ${}_4V$ is the exponential reserve at $t = 4$ for this insurance.

Calculate $1000\,{}_4V$.

(A) 0.567

(B) 1.437

(C) 10.329

(D) 21.777

(E) 45.508

Course 3 Sample Exam Answer Key

Number	***Answer***
1	B
2	$(2.5)^{2.5}\exp(-1.5)/\Gamma(2.5)$
3	C
4	C
5	120
6	E
7	$-\log U_1 - 3/2\log U_2$
8	C
9	B
10	C
11	B
12	A
13	C
14	0.277
15	1.43
16	A
17	0.139
18	4.17
19	D
20	B (2792)
21	37
22	0.86
23	1.25 years
24	0.0844
25	0.16
26	1854.9
27	D
28	0.4988
29	27,756,275
30	0.03898
31	56.76
32	0.02
33	C
34	A
35	B
36	D
37	A
38	C
39	D
40	C
41	0.1411
42	24.5
43	D
44	C
45	A

Course 3 Sample Exam Solutions

Solution – Question #1

$$\text{mean} = \frac{\mu}{\mu+\delta}\left(1-e^{-(\mu+\delta)s}\right) = .09063$$

for a \$1000 policy = \$90.63

$$\text{variance} = {}^2z - z^2 = .086393 - .0082138 = .0781792$$

for 1 policy of \$1000 = 78,179.20

standard dev for 1 policy of \$1000 = \$279.6054

$$\frac{F - n(\text{mean})}{\sqrt{n}\cdot \text{std. dev.}} = \frac{F - 500\ (90.63)}{\sqrt{500}\ (279.6054)} = 1.65 \Rightarrow 95\%\ \text{confidence}$$

$$F = 55631.08$$

Solution – Question #2

$$\text{Let } K_\lambda(X) = \frac{f(x)}{g(x)} = \frac{x^{1.5}e^{-x}\ /\ \Gamma(2.5)}{\lambda e^{-\lambda x}} = \frac{x^{1.5}e^{(\lambda-1)x}}{\lambda\Gamma(2.5)}$$

$$\text{Set } K_\lambda'(x) = \frac{1.5\ x^{0.5}e^{(\lambda-1)x} + (\lambda-1)\ x^{1.5}e^{(\lambda-1)x}}{\lambda\Gamma(2.5)} = \frac{x^{0.5}e^{(\lambda-1)x}\left[1.5 + x(\lambda-1)\right]}{\lambda\Gamma(2.5)} = 0$$

$$\Rightarrow x = \frac{1.5}{1-\lambda}\text{, which maximizes } K_\lambda(x) \text{ at } K_\lambda\left(\frac{1.5}{1-\lambda}\right) = \frac{\left(\frac{1.5}{1-\lambda}\right)^{1.5}e^{-1.5}}{\lambda\Gamma(2.5)}$$

$$\text{Next set } \frac{dk_\lambda\left(\frac{1.5}{1-\lambda}\right)}{d_\lambda} = \frac{\left(\frac{1.5}{e}\right)^{1.5}}{\Gamma(2.5)}\ \frac{d\left(\frac{1}{\lambda(1-\lambda)^{1.5}}\right)}{d_\lambda} = 0$$

$$\text{i.e. } \frac{d}{d\lambda}\left(\lambda(1-\lambda)^{1.5}\right) = -1.5(1-\lambda)^{0.5}\lambda + (1-\lambda)^{1.5} = (1-\lambda)^{0.5}(-1.5\lambda + 1 - \lambda) = 0$$

$$\Rightarrow \lambda = \frac{1}{2.5} = .4$$

$$\text{So } K_{.4}\left(\frac{1.5}{1-.4}\right) = \frac{\left(\frac{1.5}{.6}\right)^{1.5}e^{-1.5}}{.4\Gamma(2.5)} = \frac{(2.5)^{2.5}e^{-1.5}}{\Gamma(2.5)}$$

Solution – Question #3

$$\mu^{(\tau)} = 6\mu$$

$$S = \int_0^\infty 6\,\mu e^{-\delta t} e^{-6\mu t} dt + \int_0^\infty \mu e^{-\delta t} e^{-6\mu t} dt$$

$$= \frac{6\mu}{6\mu+\delta} + \frac{\mu}{6\mu+\delta}$$

$$= \frac{7\mu}{6\mu+\delta}$$

$$T = \int_0^\infty 6\mu e^{-\delta t} e^{-6\mu t} dt + 2\int_0^\infty \mu e^{-\delta t} e^{-6\mu t} dt$$

$$= \frac{6\mu}{6\mu+\delta} + \frac{2\mu}{6\mu+\delta}$$

$$= \frac{8\mu}{6\mu+\delta}$$

$$T - S = \frac{\mu}{6\mu+\delta}$$

$$= \frac{S}{7}$$

Solution – Question #4

$$Var(L) = 0.2 = \left({}^2\bar{A}_{30} - A_{30}^2\right)\left(\frac{1}{d\bar{a}_{30}}\right)^2 = \left(.3 - \bar{A}_{30}^2\right)\frac{1}{\left(1-\bar{A}_{30}\right)^2}$$

$$\bar{A}_{30} = \frac{.4 \pm \sqrt{0.16 + 4(0.1)(1.2)}}{2(1.2)} = 0.5$$

$${}_{20}\bar{V}\left(\bar{A}_{30}\right) = \frac{\bar{A}_{50} - \bar{A}_{30}}{1-\bar{A}_{30}} = \frac{0.7-0.5}{1-0.5} = 0.4$$

Solution – Question #5

$$S(x) = \int_x^\infty f(y)dy = (1+.01qx)e^{-.01x}$$

$$e(x) = E(X - x|X > x) = \frac{\int_x^\infty S(y)dy}{S(x)} = 100\left(1 + \frac{q}{1+.01qx}\right)$$

$$125 = e(100) = 100\left(1 + \frac{q}{1+q}\right) \Rightarrow q = \frac{1}{3}$$

$$e(200) = 100\left(1 + \frac{q}{1+2q}\right) = 120$$

Solution – Question #6

Hens live at most to 72 months. On average at age 12, there are 30 months left to live.
There are 100 hens, laying 30 eggs per month each.
30 months x 30 eggs x 100 hens = 90,000.

Solution – Question #7

Let T_i be the time until transition when currently in State i
T_i is exponential with parameter λ_i
where $\lambda_1 = q_{12} + q_{13} = \frac{1}{2} + \frac{1}{2} = 1$ and $\lambda_2 = q_{21} + q_{23} = \frac{1}{3} + \frac{1}{3} = \frac{2}{3}$

$U_i \sim$ Uniform (0,1). $1 - U_i$, i=1, 2 is also Uniform (0,1)

Let $1 - U_i = F(T_i) = 1 - e^{-\lambda 2 T_i}$, $T_i = \frac{\ln U_i}{\lambda_i}$, i = 1, 2

The time of the second transition = $T_1 + T_2 = -\frac{\ln U_1}{\lambda_1} - \frac{\ln U_2}{\lambda_2} = -\ln U_1 - \frac{3}{2}\ln U_2$

Solution – Question #8

$$\begin{aligned}\text{APV} &= \frac{20{,}000 \cdot 0.2}{1.06} + \frac{15{,}000 \cdot 0.8 \cdot 0.2}{1.06^2} + \frac{10{,}000 \cdot 0.8^2 \cdot 0.2}{1.06^3} \\ &= 3773.58 + 2135.99 + 1074.71 \\ &= 6984.29\end{aligned}$$

Solution – Question #9

APV of insurance with Car-Tracker:

$$\frac{20{,}000 \cdot 0.1}{1.06} + \frac{15{,}000 \cdot 0.9 \cdot 0.1}{1.06^2} + \frac{10{,}000 \cdot 0.9^2 \cdot 0.1}{1.06^3}$$

= 1886.79 + 1201.50 + 680.09

= 3768.38

Maximum discount = APV without Car-Tracker – APV with Car-Tracker

= 6894.29 – 3768.38

= 3215.91

Solution – Question #10

$1/(1.04)^3 = 0.889$, so the expected present value is 0.889(5 + 0.5(10)) = 8.88996.

The variance is calculated as follows:

We can ignore the legal expense payment, and the present value will simply add a factor of 0.889^2.

The second moment about the origin is $0.5(20^2 + 10^2) + 0.5(0) = 250$, so the variance of the loss payment is $250 - 5^2 = 225$, and the variance of the present value is $(0.889^2)(225) = 177.82$

Maximum settlement = 8.88996 + 0.02(177.82) = 12.446

Solution – Question #11

APV =

$$\int_0^{25} 300 \cdot 0.4 \cdot e^{-0.06t} dt + e^{-0.06 \cdot 25} \int_0^{\infty} 100 \cdot 0.4 \cdot e^{-0.07t} dt$$

$$= 12\int_0^{25} e^{-0.06t} dt + 4 \cdot e^{-1.5} \int_0^{\infty} e^{-0.07t} dt$$

$$= \left. \frac{12e^{-0.06t}}{0.06} \right|_0^{25} + \frac{4e^{-1.5}}{0.07}$$

$$= \frac{12}{0.06}(1 - e^{-1.5}) + \frac{4e^{-1.5}}{0.07}$$

= 155.37 + 12.75

= 168.12

Solution – Question #12

The accident count distribution is negative binomial with mean 0.1 and variance 0.11

The parameters of the negative binomial are r = 1 and β = 0.1

Using the formula for the negative binomial:

$$1 - \Pr\{N = 0\} - \Pr\{N = 1\} = 1 - 10/11 - 10/121 = 1/121$$

Solution – Question #13

$$_{20|}\ddot{a}_{40} = \ddot{a}_{40} - \ddot{a}_{40:\overline{20|}}$$
$$= 12.112 - 10.290$$
$$= 1.822$$
$$0.103 = 10.290P_1 + 1.822P_2$$
$$_{20}V = 0.75 \cdot 0.284 - 9.661P_1$$
$$_{20}V = 0.284 - 9.661P_2$$
$$\frac{0.071 + 9.661P_1}{9.661} = P_2$$
$$0.007349135 + P_1 = P_2$$
$$0.103 = 10.290P_1 + (0.007349135 + P_1)(1.822)$$
$$0.103 = 10.290P_1 + 0.013390123 + 1.822P_1$$
$$0.089609876 = 12.112P_1$$
$$P_1 = 0.0073984$$
$$_{20}V = 0.75 \cdot 0.284 - 0.0073984 \cdot 9.661$$
$$_{20}V = 0.1415$$

Solution – Question #14

Use convolutions to form a partial pdf table:

no. of claims	0	1	2	3
probability	0.287	0.358	0.224	0.093

claim amount		probabilities		
1		1/3		
2		1/3	1/9	
3		1/3	2/9	1/27

aggr. claims	0	1	2	3
probability	0.287	0.119	0.144	0.173

So probability that claims exceed 3 = 1 – (0.287 + 0.119 + 0.144 + 0.173) = 0.277

Solution – Question #15

$$E[S] = 1.25 * (1+2+3)/3 = 2.5$$
$$E[(S-1)] = E[S] - [1 - F_S(0)] = 2.5 - 1 + .287 = 1.787$$
$$E[(S-2)] = E[(S-1)] - [1 - F_S(1)] = 1.787 - 1 + .287 + .119 = 1.193$$
$$E[(S-1.6)] = .4 * 1.787 + .6 * 1.193 = 1.431$$

Solution – Question #16

$$\int_0^5 0.02\, e^{-0.02t}\; e^{-0.04t}\; e^{-0.03t}\; dt = \frac{0.02}{0.09}\left(1 - e^{-0.45}\right) = 0.0805$$

Solution – Question #17

$$\text{Since } E\left(X^k\right) = e^{\mu k + \frac{\sigma^2 k^2}{2}} \Rightarrow E(X) = e^{7 + \frac{4}{2}} = e^9$$

$$\text{Also } E\left[\left(X \Lambda d\right)^k\right] = e^{\mu k + \frac{\sigma^2 k^2}{2}} \Phi\left(\frac{\ln(d) - \mu - k\sigma^2}{\sigma}\right) + d^k\left[1 - \Phi\left(\frac{\ln(d) - \mu}{\sigma}\right)\right]$$

$$\text{so } E(X \Lambda 2000) = e^{7 + \frac{4}{2}} \Phi\left(\frac{\ln(2000) - 7 - 4}{2}\right) + 2000\left(1 - \Phi\left(\frac{\ln(2000) - 7}{2}\right)\right)$$

$$= e^9 \Phi(-1.70) + 2000\left(1 - \Phi(0.30)\right)$$

$$\text{LER} = \frac{E(X \Lambda 2000)}{E(X)} = \Phi(-1.07) + 2000\, e^{-9}\left(1 - \Phi(0.30)\right)$$

$$= 0.0446 + 2000\, e^{-9}\left(1 - 0.6179\right)$$

$$= 0.1389$$

Solution – Question #18

$$\Pr(1.2\ X > 2000) = \Pr\left(X > \frac{2000}{1.2}\right) = \Pr\left(\ln X > \ln\frac{2000}{1.2}\right)$$

$$= \Pr\left(\frac{\ln X - 7}{2} > \frac{\ln\frac{2000}{1.2} - 7}{2}\right)$$

$$= \Pr\left(N(0.1) > 0.2093\right) \approx 0.417$$

So the expected number of annual losses that exceed the deductible

$$= 10 \text{ x } 0.417 = 4.17$$

Solution – Question #19

$${}_tP_2^{I} = 0.8^t$$

$${}_tP_2^{I} = \frac{7-t}{7}$$

Want the probability that they both survive to age 3 minus the probability that they both survive to age 6.

$$= 0.8^1\,\frac{6}{7} - 0.8^4\,\frac{3}{7}$$

$$= 0.68571 - 0.17554$$

$$= 0.51017$$

Solution – Question #20

The distribution of the number of claims in excess of 500 is negative binomial with:

$$\beta = 2e^{-1/2} \text{ and } r = 2$$

which has mean = $4e^{-1/2}$ and variance = $4e^{-1/2} + 8e^{-1}$.

The distribution of the claim severity distribution in excess of 500 is also an exponential distribution:

$$F(x) = 1 - e^{-x/1000}$$

which has mean = 1000 and variance = 1000^2.

The aggregate variance = $E[N]Var[X] + Var[N]E[X]^2$

The standard deviation = $\sqrt{1000^2\left(8e^{-1/2} + 8e^{-1}\right)} = 2792.003$

Solution – Question #21

$${}_nP_{[x]} = e^{-\int_0^n \mu_x(t)dt} = e^{-\int_0^n \phi(x)tdt} = -e^{\frac{-n^2}{2}\phi(x)}$$

$$q^s_{[35]} = q^n_{[x]} \Rightarrow P^s_{[35]} = P^n_{[x]} \Rightarrow$$

$$e^{-\frac{1}{2}[\beta+.006+.003\,(35)]} = e^{-\frac{1}{2}[\beta+.003\,(x)]}$$

$$\frac{1}{2}[.006+.105] = \frac{1}{2}[.003x]$$

$$x = 37$$

Solution – Question #22

$$\Pr\{T(30) > 10\} = \Pr\{T(30) > 10 | S = 1\} \Pr\{S = 1\} + \Pr\{T(30) > 10 | S = 0\} \Pr\{S = 0\}$$
$$= {}_{10}P^{S}_{[30]}\,(.4) + {}_{10}P^{n}_{[30]}\,(.6)$$
$$= e^{-\frac{1}{2}(100)(\beta + .006 + .003(30))}(.4) + {}_{10}P^{n}_{[30]}(.6)$$
$$= (.4)\, e^{-.3} \cdot {}_{10}P^{n}_{[30]} + (.6)\ {}_{10}P^{n}_{[30]}$$
$$= (0.96)\left[.4e^{-.3} + .6\right]$$
$$= 0.86$$

Solution – Question #23

Let T_i be inter-arrival time between two successive loss occurrences of type i

Given $\Pr(T_H > t) = e^{-\lambda_H t}$, $\Pr(T_E > t) = e^{-\lambda_E t}$ and $\Pr(T_F > t) = e^{-\lambda_F t}$

Since $\Pr(\min\{T_H, T_E, T_F\} > t) = \Pr(T_H > t, T_E > t, T_F > t)$

$= \Pr(T_H > t)\Pr(T_E > t)\Pr(T_F > t)$ since independent events

$= e^{-(\lambda_H + \lambda_E + \lambda_F)t}$

So the expected amount of time between loss occurrences in excess of 1 billion is

$$\frac{1}{\lambda_H + \lambda_E + \lambda_F} = \frac{1}{\frac{1}{2} + \frac{1}{5} + \frac{1}{10}} = \frac{1}{\frac{8}{10}} = \frac{10}{8} = 1.25$$

Solution – Question #24

$$l_{97}^{(T)} = l_{96}^{(T)} \cdot p_{96}'^{(1)} \cdot p_{96}'^{(2)} \cdot p_{96}'^{(3)} = 100{,}000\left(1-0.0625\right)\left(1-0.2\right)\left(1-0.2\right) = 60{,}000$$

$$l_{98}^{(T)} = 37{,}356 = 60{,}000\; p_{97}'^{(1)}\left(1-0.15\right)\left(1-0.2\right) = 40{,}800\; p_{97}'^{(1)}$$

$$\Rightarrow p_{97}'^{(1)} = \frac{37356}{40800} = 0.9156 \Rightarrow q_{97}'^{(1)} = 0.0844$$

Since decrement (2) occurs on January 1 and decrement (3) occurs on December 31:

$$q_{97}^{(1)} = p_{97}'^{(2)} \cdot q_{97}'^{(1)} = (0.85)(0.0844) = 0.717$$

Solution – Question #25

$$E[S] = E[N]E[X] = (500)(100)$$

$$\mathrm{Var}(S) = E[N]\mathrm{Var}(X) + \mathrm{Var}(N)(E[X])^2 = 500\left(100 + (100)^2\right)$$

$$\Pr\left\{\frac{S}{(1.1)E[S]} > 0.95\right\} =$$

$$\Pr\left\{\frac{S - E[S]}{\sqrt{\mathrm{Var}(S)}} > \frac{0.95(1.1)E[S] - E[S]}{\sqrt{\mathrm{Var}(S)}}\right\} =$$

$$\Pr\left\{\frac{S - E[S]}{\sqrt{\mathrm{Var}(S)}} > \frac{0.045E[S]}{\sqrt{\mathrm{Var}(S)}}\right\}$$

$$\Pr\left\{\frac{S - E[S]}{\sqrt{\mathrm{Var}(S)}} > 1.00124\right\} \approx 1 - \Phi(1.00124) \approx 0.16$$

Solution – Question #26

Let T be the transition matrix. Then:

T	0.85	0.15
	0.6	0.4
T^2	0.8125	0.1875
	0.75	0.25

	Probabilities		Pure Prem			
	Pref	Std	Pref	Std	Expected	Disc Factor
Current	0	1	400	900	900	0.952380952
Year1	0.6	0.4	400	900	600	0.907029478
Year2	0.75	0.25	400	900	525	0.863837599

Answer: 1854.875

Solution – Question #27

The probability of a Brownian motion process exceeding a number a by time t is just twie the probability that the process will be a or greater at time t. In order to get this probability, convert to regular Brownian motion by taking log(10) = 2.3026 and log(12) = 2.4849. The standard deviation at time t =1 is just 0.2, so standardize by subtracting log(10) and dividing by σ =0.2 to get $\frac{\log(12)-\log(10)}{0.2}=0.9116$. The probability that the stock price is 12 or greater at time 1 is $1-\varphi(0.9116)=0.181$, so the probability that the price was 12 or greater at some time between 0 and 1 is 0.362.

Solution - Question #28

$$\begin{aligned}
\Pr\{J = d, T < 20\} &= \int_0^{20} \mu^{(d)}(t)\ {}_tP_0^{(T)} dt \\
&= \int_0^{20} 0.05\, e^{-(.05+.03)t} dt \\
&= \frac{0.05}{0.08} \int_0^{20} e^{-.08t} dt \\
&= \frac{5}{8}\left(1 - e^{-1.6}\right) \\
&= 0.4988
\end{aligned}$$

Solution – Question #29

On a loan of 1, the balance outstanding at *t* is $\frac{1}{\bar{a}_{\overline{20}|.06}}\ \bar{a}_{\overline{(20-t)}|.06} = \frac{1-e^{-0.6(20-t)}}{1-e^{-.06(20)}}$

At the discount rate of $\delta = 0.05$,
APV on default =

$$\begin{aligned}
\int_0^{20} \frac{1-e^{-.06(20-t)}}{1-e^{-.06(20)}}\, e^{-.05t}\, .05e^{-.08t} dt &= \frac{.05}{1-e^{-1.2}} \int_0^{20} \left(1-e^{-.06(20-t)}\right) e^{-.13t} dt \\
&= \left(\frac{.05}{1-e^{-1.2}}\right)\left[\int_0^{20} e^{-.13t} dt - \int_0^{20} e^{-(1.2+.07t)} dt\right] \\
&= \left(\frac{.05}{1-e^{-1.2}}\right)\left[\frac{1-e^{-2.6}}{.13} - \frac{1}{.07}\left(e^{-1.2} - e^{-2.6}\right)\right] \\
&= 0.27756275
\end{aligned}$$

For 100 million of loans APV = 27,756,275.

Solution – Question #30

APV of a fee of p on a loan of 1,

$$p\int_0^{20} v^t \; {}_tP_0^{(T)} dt = p\int_0^{20} e^{-.05t} e^{-(.05+.03)t} dt = p\int_0^{20} e^{-.13t} dt = \frac{p}{.13}\left(1-e^{-2.6}\right)$$

By equivalence principle and answer to #29

$$0.27756275 = \frac{p}{.13}\left(1-e^{-2.6}\right)$$
$$p = 0.03898$$

Solution – Question #31

APV of costs of administration –

10000 (.001) + 100 + (.10) APV of loan at default + (.01) APV of loan at prepayment

$$110 + (0.10)\,(10{,}000)\,(0.27756275) + (0.01)\,(10{,}000)\,\frac{.03}{.05}\,(0.27756275)$$

110 + (1000 + 60) (0.27756275) = 404.2165

By equivalence principle,

404.2165 = APV of level fee of 10,000 θ

$$404.2165 = 10{,}000\,\theta\,\frac{1-e^{-2.6}}{.13}$$

$$10{,}000\,\theta = 56.76$$

Solution – Question #32

Discretizing claims gives

$$f_0 = 1 - e^{-1} = .6321$$
$$f_1 = e^{-1} - e^{-3} = .3679 - .0498 = .3181$$
$$f_2 = e^{-3} - e^{-5} = .0498 - .0067 = .0431$$
$$f_3 = e^{-5} - e^{-7} = .0067 - .0009 = .0058$$

	Results	Probability
Year 1		
	2 + 1.5 – 0 = 3.5	.63
	2 + 1.5 – 2 = 1.5	.32
	2 + 1.5 – 4 = -.5	.04
	2 + 1.5 – 6 = -1.5	.01
Year 2		
	3.5 + 1.5 – 0 = 5	.63 * .63
	3.5 + 1.5 – 2 = 3	.63 * .32
	3.5 + 1.5 – 4 = 1	.63 * .04
	3.5 + 1.5 – 6 = -2	.63 * .01
	1.5 + 1.5 – 0 = 3	.32. * .63
	1.5 + 1.5 – 2 = 1	.32 * .32
	1.5 + 1.5 – 4 = -1	.32 * .04
	1.5 + 1.5 – 6 = -3	.32 * .01

Only 3 results consist of solvent after year 1 and insolvent after year 2.
Total probability is .63 * .01 + .32 * .04 + .32 * .01 = .0223

Solution – Question #33

$$\bar{a}_{60:\overline{15}|} = 7.769$$

$$\bar{a}_{60:60:\overline{15}|} = 5.683$$

$$\bar{a}_{60} = 10 = \frac{1}{\mu+\delta}; \ \bar{a}_{60:60} = 6.25$$

$$\bar{a}_{\overline{15}|} = \frac{1-e^{-15\delta}}{\delta} = 11.28$$

$${}_{15|}\bar{a}_{60} = \bar{a}_{60} - \bar{a}_{60:\overline{15}|} = 2.231; \quad {}_{15|}\bar{a}_{60:60} = \bar{a}_{60:60} - \ddot{a}_{60:60:\overline{15}|} = 0.567$$

$$\bar{a}_{\overline{15}|} + \left(\bar{a}_{60:60} - \bar{a}_{60:60:\overline{15}|}\right) + .75\left({}_{15|}\bar{a}_{60} - {}_{15|}\bar{a}_{60:60}\right) + .5\left({}_{15|}\bar{a}_{60} - {}_{15|}\bar{a}_{60:60}\right) = 13.927$$

Solution – Question #34

For the merged company:

The claim frequency is 2λ, where λ is from the initial companies.
Claim size distribution is exponential with mean $\frac{1}{2}$ relative security loading is $\theta = 1$.
Surplus is $\log_e 2 + \log_e 4 = \log_e 8$.

Then
$$\Psi(\log_e 8) = \frac{1}{1+1} e^{-\left(\frac{2\cdot 1\cdot 1/2\cdot \log_e 8}{(1+1)(1/4+1/4)}\right)}$$
$$= \frac{1}{2} e^{-\log_e 8}$$
$$= \frac{1}{2}\left(\frac{1}{8}\right) = \frac{1}{16}$$

Solution – Question #35

$$50{,}000\left(\frac{\mu^{(1)}}{\mu^{(T)} + \delta}\right) = 50{,}000\left(\frac{.02}{.085}\right) = 11764.71$$

Solution – Question #36

		Claim Process Parameters
	w/o reinsurance	w/proportional reinsurance@ α
claim frequency rate	λ	α
claim size mean	100	$100\ (1-\alpha)$
variance	$(100)^2$	$(100)^2(1-\alpha)^2$
2nd moment	$2(100)^2$	$2(100)^2(1-\alpha)^2$
premium rate	$\lambda 100(1.2)$	$\lambda 100(1.2)-\alpha\,\lambda(100)(1.3)$
R.S.L. (θ)	0.2	$\dfrac{(\lambda 120-\alpha\lambda 130)-100\lambda(1-\alpha)}{\lambda 100(1-\alpha)}$
$E[L]=\dfrac{P_2}{2\theta P_1}$	$\dfrac{2(100)^2}{2(.2)(100)}=500$	$\dfrac{2(100)^2(1-\alpha)^z}{2\left(\dfrac{20-30\alpha}{100(1-\alpha)}\right)100(1-\alpha)}$

$$E[L]=(100)^2\frac{(1-\alpha)^2}{20-30\alpha}$$

$$\text{minimize}\ \frac{(1-\alpha)^2}{20-30\alpha}=g(\alpha)$$

A local minimum will be at α where $g'(\alpha)=0$

$$2(\alpha-1)(20-30\alpha)-(-30)(\alpha-1)^2=0$$

$$40-60\alpha+30\alpha-30=0$$

$$10=30\alpha$$

$$\frac{1}{3}=\alpha$$

At $\alpha=0,\ \ g(0)=\dfrac{1}{20}$ and at $\alpha=\dfrac{1}{3},\ \ g\left(\dfrac{1}{3}\right)=\dfrac{4}{90}$

So absolute min. at $\alpha=\dfrac{1}{3}$.

Solution – Question #37

Pr (0 visits to SNF)

= Pr (resident leaves ILU by withdrawal or death)

$$= \frac{\mu_{14} + \mu_{15}}{\mu_{12} + \mu_{13} + \mu_{14} + \mu_{15}} = \frac{0.1 + 0.2}{0.3 + 0.1 + 0.1 + 0.2} = \frac{3}{7}$$

$= 0.42857$

Pr (1 visit to SNF)

= Pr(resident enters SNF by permanent transfer)

+ Pr(resident enters SNF by temporary transfer and leaves by death or permanent transfer)

+ Pr(resident enters SNF by temporary transfer then returns to ILU)

x Pr(0 visits to SNF)

$$= \frac{\mu_{13}}{\mu_{12} + \mu_{13} + \mu_{14} + \mu_{15}} + \frac{\mu_{12}}{\mu_{12} + \mu_{13} + \mu_{14} + \mu_{15}} \cdot \frac{\mu_{23} + \mu_{24}}{\mu_{21} + \mu_{23} + \mu_{24}} + \frac{\mu_{12}}{\mu_{12} + \mu_{13} + \mu_{14} + \mu_{15}} \cdot \frac{\mu_{21}}{\mu_{21} + \mu_{23} + \mu_{24}} \cdot \frac{3}{7}$$

$$= \frac{1}{7} + \frac{3}{7} \cdot \frac{6}{11} + \frac{3}{7} \cdot \frac{5}{11} \cdot \frac{3}{7}$$

$= 0.46011$

$\therefore$ Pr(>1 visit to SNF)

= 1 – Pr(0 visits) – Pr(1 visit)

= 0.11132

Solution – Question #38

Let D be the duration of a visit to the SNF.
Let S be the state the resident is in upon entering the SNF.
Want E[D]

Now $\Pr(S=2)=\dfrac{\mu_{12}}{\mu_{12}+\mu_{13}}=\dfrac{3}{4}$, and $\Pr(S=3)=\dfrac{1}{4}$

$$E[D]=E[D|S=2]\Pr(S=2)+E[D|S=3]\Pr(S=3)$$

$$E[D|S=3]=\frac{1}{\mu_{34}}=\frac{1}{0.4}=2.5$$

$$\begin{aligned}E[D|S=2]&=\text{expected duration in state }2+\text{expected duration in state }3\\&=\frac{1}{\mu_{21}+\mu_{23}+\mu_{24}}+\frac{\mu_{23}}{\mu_{21}+\mu_{23}+\mu_{24}}\cdot\frac{1}{\mu_{34}}\\&=\frac{1}{1.1}+\frac{3}{11}(2.5)=1.5909\end{aligned}$$

$$\therefore\ E[D]=(1.5909)\left(\frac{3}{4}\right)+(2.5)\left(\frac{1}{4}\right)=1.8182$$

Solution – Question #39

$$\begin{aligned}APV&=\int_0^5(100{,}000-20{,}000t)v^te^{-(\mu_{12}+\mu_{13}+\mu_{14}+\mu_{15})t}\mu_{15}\,dt\\&=\int_0^5(100{,}000-20{,}000t)v^te^{-0.7t}(0.2)dt\\&=20{,}000\int_0^5\left(v\,e^{-0.7}\right)^tdt-4000\int_0^5t\left(v\,e^{-0.7}\right)^tdt\\&=20{,}000\,\frac{\left(v\,e^{-0.7}\right)^5-1}{\log\left(v\,e^{-0.7}\right)}-4000\left[\left[t\,\frac{\left(v\,e^{-0.7}\right)^t}{\log\left(v\,e^{-0.7}\right)}\right]_0^5-\int_0^5\frac{\left(v\,e^{-0.7}\right)^t}{\log\left(v\,e^{-0.7}\right)}dt\right]\\&=19{,}744.40\end{aligned}$$

Solution – Question #40

Prob. failure year 1 = .08 (.94) + .06 (.92) + .06 (.08) = 0.1352
Prob. failure year 2 = .94(.92) [.1(.9) + .1(.9) + .1(.1)] = 0.164312
Prob. failure year 3 = .94(.92) (.9) (.9) [.12(.87) + .13(.88) + .12(.13)] = 0.164194

$$100{,}000\left[\frac{0.1352}{1.08}+\frac{0.164312}{(1.08)^2}+\frac{0.164194}{(1.08)^3}\right]=39{,}640$$

Solution – Question #41

The secondary distribution is Poisson (0.5) with

$$f_0 = e^{-0.5} = .60653$$

$$f_1 = (0.5)\ f_0 = .30327$$

$$f_2 = (.25)\ f_1 = .07582$$

$$f_3 = (1/6)\ f_2 = .01264$$

The recursive formula for the geometric $(\beta = 3)$ gives:

$$g_0 = P(f_0) = \{1+3(1-.60653)\}^{-1} = .45863$$

$$a = 3/4 = .75,\ b = 0$$

$$g_x = \frac{\sum_{y=1}^{x}\left(a+b\frac{y}{x}\right)f_y g_{x-y}}{1-af_0}$$

$$= 1.37589\sum_{y=1}^{x} f_y g_{x-y}$$

$$g_1 = (1.37589)\ (.30327)\ (.45863) = .19137$$

$$g_2 = (1.37589)\ \{(.30327)(.19137)+(.07582)(.45863)\} = .12770$$

$$g_3 = (1.37589)\ \{(.30327)(.12770)+(.07582)(.19137)+(.01264)(.45863)\} = .08122$$

$$1-g_0-g_1-g_2-g_3 = .14108$$

Solution – Question #42

E (# losses) = (3) (0.5) = 1.5

Stop loss premium:

$$100\sum_{x=4}^{\infty}(x-4)g_x$$

$$=100\left\{\sum_{x=0}^{\infty}xg_x-4+\sum_{x=0}^{3}(4-x)g_x\right\}$$

$$=100\left\{1.5-4+(4)\left(.45863\right)+(3)(.19137)+(2)(.12770)+(1)(.08122)\right\}$$

$$=24.525$$

Solution – Question #43

$$274{,}000=P\,a_{\overline{15}|}^{.05}+10000\left(\frac{1}{1.05}+\frac{1.1}{1.05^2}+\frac{1.1^2}{1.05^3}+\ldots+\frac{1.1^{14}}{1.05^{15}}\right)$$

$$274{,}000=P\,a_{\overline{15}|}^{.05}+10000\left(\frac{1}{1.05}\right)\left(\frac{1-(1.1/1.05)^{15}}{1-(1.1/1.05)}\right)$$

$$72134.44153=P\,a_{\overline{15}|}^{.05}$$

$$P=6949.60$$

Solution – Question #44

Let $\xi = 90\text{th percentile of } Z$

$\Pr(Z < \xi) = .9$

$$\Pr(Z = 0) = \Pr(T \le 10) = \int_0^{10} e^{-.06t}(.06)dt = 1 - e^{-.6} = .4512$$

$\Pr(Z = 0) + \Pr(0 < Z < \xi) = .9 \Rightarrow \Pr(0 < Z < \xi) = .4488$

$\Pr(v^T < \xi) = .4488 \Rightarrow \Pr(T > \ln \xi / \ln v) = .4488$

${}_T P_x = .4488 = e^{-.06T} \Rightarrow -.06T = -.8012$

$T = 13.3530 = \ln \xi / \ln v$

$\dfrac{1}{\ln v} = -\delta \qquad\qquad \xi = 0.26308$

Solution – Question #45

$$U(W - {}_4V) = E(U(W - {}_4L))$$

$$\Rightarrow -e^{-.2(W - {}_4V)} = E\left(-e^{-.2(W - {}_4L)}\right) = -e^{-.2W} E\left(e^{.2 \cdot {}_4L}\right)$$

$$\Rightarrow {}_4V = \frac{1}{.2} \ln E\left(e^{.2 \cdot {}_4L}\right) = 5 \cdot \ln\left[e^{.2 \cdot \left(\frac{1}{1.06} - .00328\right)} \cdot q_{44} + e^{.2(0 - .00328)} \cdot p_{44}\right]$$

$$= 5 \ln\left[e^{.18802} \cdot .00371 + e^{-.000656} \cdot (1 - .00371)\right]$$

$$= 5 \ln(1.000114)$$

$$= .00057$$

$1000\,{}_4V = .57$