1 Exam FM questions

1. (# 12, May 2001). Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns an annual effective discount rate of d. The amount of interest earned in Bruce's account during the 11th year is equal to X. The amount of interest earned in Robbie's account during the 17th year is also equal to X. Calculate X.
   (A) 28.0 (B) 31.3 (C) 34.6 (D) 36.7 (E) 38.9

2. (# 12, May 2003). Eric deposits X into a savings account at time 0, which pays interest at a nominal rate of i, compounded semiannually. Mike deposits $2X$ into a different savings account at time 0, which pays simple interest at an annual rate of i. Eric and Mike earn the same amount of interest during the last 6 months of the 8-th year. Calculate i.
   (A) 9.06% (B) 9.26% (C) 9.46% (D) 9.66% (E) 9.86%

3. (# 50, May 2003). Jeff deposits 10 into a fund today and 20 fifteen years later. Interest is credited at a nominal discount rate of d compounded quarterly for the first 10 years, and at a nominal interest rate of 6% compounded semiannually thereafter. The accumulated balance in the fund at the end of 30 years is 100. Calculate d.
   (A) 4.33% (B) 4.43% (C) 4.53% (D) 4.63% (E) 4.73

4. (#25 Sample Test). Brian and Jennifer each take out a loan of X. Jennifer will repay her loan by making one payment of 800 at the end of year 10. Brian will repay his loan by making one payment of 1120 at the end of year 10. The nominal semi-annual rate being charged to Jennifer is exactly one–half the nominal semi–annual rate being charged to Brian. Calculate X.
   A. 562 B. 565 C. 568 D. 571 E. 574

5. (#1 May 2003). Bruce deposits 100 into a bank account. His account is credited interest at a nominal rate of interest i convertible semiannually. At the same time, Peter deposits 100 into a separate account. Peter's account is credited interest at a force of interest of δ. After 7.25 years, the value of each account is 200. Calculate $(i - \delta)$.
   (A) 0.12% (B) 0.23% (C) 0.31% (D) 0.39% (E) 0.47%

6. (#23, Sample Test). At time 0, deposits of $10,000$ are made into each of Fund X and Fund Y. Fund X accumulates at an annual effective interest rate of 5 %. Fund Y accumulates at a simple interest rate of 8 %. At time t, the forces of interest on the two funds are equal. At time t, the accumulated value of Fund Y is greater than the accumulated value of Fund X by Z. Determine Z.
   A. 1625 B. 1687 C. 1697 D. 1711 E. 1721

7. (#24, Sample Test). At a force of interest $\delta_t = \frac{2}{k+2t}$.

   (i) a deposit of 75 at time $t = 0$ will accumulate to X at time $t = 3$;

   and

   (ii) the present value at time $t = 3$ of a deposit of 150 at time $t = 5$ is also equal to X.

   Calculate X.

   A. 105 B. 110 C. 115 D. 120 E. 125

8. (# 37, May 2000). A customer is offered an investment where interest is calculated according to the following force of interest:

$$\delta_t = \begin{cases} 0.02t & \text{if } 0 \leq t \leq 3 \\ 0.045 & \text{if } 3 < t \end{cases}$$

   The customer invests 1000 at time $t = 0$. What nominal rate of interest, compounded quarterly, is earned over the first four–year period?

   (A) 3.4% (B) 3.7% (C) 4.0% (D) 4.2% (E) 4.5%

9. (# 53, November 2000). At time 0, K is deposited into Fund X, which accumulates at a force of interest $\delta_t = 0.006t^2$. At time m, $2K$ is deposited into Fund Y, which accumulates at an annual effective interest rate of 10%. At time n, where $n > m$, the accumulated value of each fund is $4K$. Determine m.

   (A) 1.6 (B) 2.4 (C) 3.8 (D) 5.0 (E) 6.2

10. (# 45, May 2001). At time $t = 0$, 1 is deposited into each of Fund X and Fund Y. Fund X accumulates at a force of interest $\delta_t = \frac{t^2}{k}$. Fund Y accumulates at a nominal rate of discount of 8% per annum convertible semiannually. At time $t = 5$, the accumulated value of Fund X equals the accumulated value of Fund Y. Determine k .

    (A) 100 (B) 102 (C) 104 (D) 106 (E) 108

11. (# 49, May 2001). Tawny makes a deposit into a bank account which credits interest at a nominal interest rate of 10% per annum, convertible semiannually. At the same time, Fabio deposits 1000 into a different bank account, which is credited with simple interest. At the end of 5 years, the forces of interest on the two accounts are equal, and Fabio's account has accumulated to Z . Determine Z.

    (A) 1792 (B) 1953 (C) 2092 (D) 2153 (E) 2392

12. (# 1, May 2000). Joe deposits 10 today and another 30 in five years into a fund paying simple interest of 11% per year. Tina will make the same two deposits, but the 10 will be deposited n years from today and the 30 will be deposited $2n$ years from today. Tina's deposits earn an annual effective rate of 9.15%. At the end of 10 years, the accumulated

amount of Tina's deposits equals the accumulated amount of Joe's deposits. Calculate n.
(A) 2.0 (B) 2.3 (C) 2.6 (D) 2.9 (E) 3.2

13. (# 1, November 2001). Ernie makes deposits of 100 at time 0, and X at time 3. The fund grows at a force of interest $\delta_t = \frac{t^2}{100}$, $t > 0$. The amount of interest earned from time 3 to time 6 is X. Calculate X.
(A) 385 (B) 485 (C) 585 (D) 685 (E) 785

14. (# 24, November 2001). David can receive one of the following two payment streams:
(i) 100 at time 0, 200 at time n, and 300 at time $2n$
(ii) 600 at time 10
At an annual effective interest rate of i, the present values of the two streams are equal. Given $\nu^n = 0.75941$, determine i.
(A) 3.5% (B) 4.0% (C) 4.5% (D) 5.0% (E) 5.5%

15. (# 17, May 2003). An association had a fund balance of 75 on January 1 and 60 on December 31. At the end of every month during the year, the association deposited 10 from membership fees. There were withdrawals of 5 on February 28, 25 on June 30, 80 on October 15, and 35 on October 31. Calculate the dollar–weighted rate of return for the year.
(A) 9.0% (B) 9.5% (C) 10.0% (D) 10.5% (E) 11.0%

16. (#32, Sample Test). 100 is deposited into an investment account on January 1, 1998. You are given the following information on investment activity that takes place during the year:

	April 19, 1998	October 30, 1998
Value immediately prior to deposit	95	105
Deposit	$2X$	X

The amount in the account on January 1, 1999 is 115. During 1998, the dollar–weighted return is 0% and the time-weighted return is y. Calculate y.
(A) -1.5% (B) -0.7% (C) 0.0% (D) 0.7% (E) 1.5%

17. (# 27, November 2000). An investor deposits 50 in an investment account on January 1. The following summarizes the activity in the account during the year:

Date	Value Immediately Before Deposit	Deposit
March 15	40	20
June 1	80	80
October 1	175	75

On June 30, the value of the account is 157.50. On December 31, the value of the account is X . Using the time–weighted method, the equivalent annual effective yield during the first 6 months is equal to the (time-weighted) annual effective yield during the entire 1-year period. Calculate X.

(A) 234.75 (B) 235.50 (C) 236.25 (D) 237.00 (E) 237.75

18. (#31, May 2001). You are given the following information about an investment account:

Date	Value Immediately Before Deposit	Deposit
January 1	10	
July 1	12	X
December 31	X	

Over the year, the time–weighted return is 0%, and the dollar-weighted return is Y. Calculate Y.

(A) −25% (B) −10% (C) 0% (D) 10% (E) 25%

19. (#16, May 2000). On January 1, 1997, an investment account is worth $100,000$. On April 1, 1997, the value has increased to 103,000 and $8,000$ is withdrawn. On January 1, 1999, the account is worth $103,992$. Assuming a dollar weighted method for 1997 and a time weighted method for 1998, the annual effective interest rate was equal to x for both 1997 and 1998. Calculate x.

(A) 6.00% (B) 6.25% (C) 6.50% (D) 6.75% (E) 7.00%

20. (# 28, November 2001). Payments are made to an account at a continuous rate of $(8k+tk)$, where $0 \le t \le 10$. Interest is credited at a force of interest $\delta_t = \frac{1}{8+t}$. After 10 years, the account is worth $20,000$. Calculate k.

(A) 111 (B) 116 (C) 121 (D) 126 (E) 131

21. (# 2, November, 2000) The following table shows the annual effective interest rates being credited by an investment account, by calendar year of investment. The investment year method is applicable for the first 3 years, after which a portfolio rate is used:

Calendar year of original investment	i_1	i_2	i_3	calendar year of Portfolio rate	Portfolio Rate
1990	10%	10%	t%	1993	8%
1991	12%	5%	10%	1994	$t - 1$%
1991	8%	$t - 2$%	12%	1995	6%
1993	9%	11%	6%	1996	9%
1994	7%	7%	10%	1997	10%

An investment of 100 is made at the beginning of years 1990, 1991, and 1992. The total amount of interest credited by the fund during the year 1993 is equal to 28.40. Calculate t. (A) 7.00 (B) 7.25 (C) 7.50 (D) 7.75 (E) 8.00

22. (# 51, November, 2000) An investor deposits 1000 on January 1 of year x and deposits another 1000 on January 1 of year $x+2$ into a fund that matures on January 1 of year $x+4$. The interest rate on the fund differs every year and is equal to the annual effective rate of growth of the gross domestic product (GDP) during the 4–th quarter of the previous year. The following are the relevant GDP values for the past 4 years:
Year III Quarter IV Quarter

Year	Quarter III	Quarter III
$x-1$	800.0	808.0
x	850.0	858.5
$x+1$	900.0	918.0
$x+2$	930.0	948.6

What is the internal rate of return earned by the investor over the 4 year period?
(A) 1.66% (B) 5.10% (C) 6.15% (D) 6.60% (E) 6.78%

23. (#26, Sample Test). Carol and John shared equally in an inheritance. Using his inheritance, John immediately bought a 10-year annuity-due with an annual payment of 2500 each. Carol put her inheritance in an investment fund earning an annual effective interest rate of 9%. Two years later, Carol bought a 15-year annuity-immediate with annual payment of Z. The present value of both annuities was determined using an annual effective interest rate of 8%. Calculate Z.
A. 2330 B. 2470 C. 2515 D. 2565 E. 2715

24. (#27, Sample Test). Susan and Jeff each make deposits of 100 at the end of each year for 40 years. Starting at the end of the 41st year, Susan makes annual withdrawals of X for 15 years and Jeff makes annual withdrawals of Y for 15 years. Both funds have a balance of 0 after the last withdrawal. Susan's fund earns an annual effective interest rate of 8 %. Jeff's fund earns an annual effective interest rate of 10 %. Calculate $Y - X$.
A. 2792 B. 2824 C. 2859 D. 2893 E. 2925

25. (# 22, November 2000). Jerry will make deposits of 450 at the end of each quarter for 10 years. At the end of 15 years, Jerry will use the fund to make annual payments of Y at the beginning of each year for 4 years, after which the fund is exhausted. The annual effective rate of interest is 7% . Determine Y.
(A) 9573 (B) 9673 (C) 9773 (D) 9873 (E) 9973

26. (# 27, November 2001). A man turns 40 today and wishes to provide supplemental retirement income of 3000 at the beginning of each month starting on his 65th birthday. Starting today, he makes monthly contributions of X to a fund for 25 years. The fund earns a nominal rate of 8% compounded monthly. Each 1000 will provide for 9.65 of income at the beginning of each month starting on his 65th birthday until the end of his life. Calculate X.

(A) 324.73 (B) 326.89 (C) 328.12 (D) 355.45 (E) 450.65

27. (# 47, May 2000). Jim began saving money for his retirement by making monthly deposits of 200 into a fund earning 6% interest compounded monthly. The first deposit occurred on January 1, 1985. Jim became unemployed and missed making deposits 60 through 72. He then continued making monthly deposits of 200. How much did Jim accumulate in his fund on December 31, 1999?

(A) $53,572$ (B) $53,715$ (C) $53,840$ (D) $53,966$ (E) $54,184$

28. (# 12, November 2001). To accumulate 8000 at the end of $3n$ years, deposits of 98 are made at the end of each of the first n years and 196 at the end of each of the next $2n$ years. The annual effective rate of interest is i. You are given $(l+i)^n = 2.0$. Determine i.

(A) 11.25% (B) 11.75% (C) 12.25% (D) 12.75% (E) 13.25%

29. (# 34, November 2000). Chuck needs to purchase an item in 10 years. The item costs 200 today, but its price inflates 4% per year. To finance the purchase, Chuck deposits 20 into an account at the beginning of each year for 6 years. He deposits an additional X at the beginning of years 4, 5, and 6 to meet his goal. The annual effective interest rate is 10% . Calculate X .

(A) 7.4 (B) 7.9 (C) 8.4 (D) 8.9 (E) 9.4

30. (# 8, May 2003). Kathryn deposits 100 into an account at the beginning of each 4–year period for 40 years. The account credits interest at an annual effective interest rate of i. The accumulated amount in the account at the end of 40 years is X, which is 5 times the accumulated amount in the account at the end of 20 years. Calculate X.

(A) 4695 (B) 5070 (C) 5445 (D) 5820 (E) 6195

31. (# 33, May 2003). At an annual effective interest rate of i, $i > 0$, both of the following annuities have a present value of X:

(i) a 20–year annuity–immediate with annual payments of 55

(ii) a 30–year annuity–immediate with annual payments that pays 30 per year for the first 10 years, 60 per year for the second 10 years, and 90 per year for the final 10 years

Calculate X.

(A) 575 (B) 585 (C) 595 (D) 605 (E) 615

32. (# 17, May 2001). At an annual effective interest rate of i, $i > 0\%$, the present value of a perpetuity paying 10 at the end of each 3–year period, with the first payment at the end of year 6, is 32. At the same annual effective rate of i, the present value of a perpetuity–immediate paying 1 at the end of each 4-month period is X. Calculate X.

    (A) 38.8 (B) 39.8 (C) 40.8 (D) 41.8 (E) 42.8

33. (# 5, May 2001). A perpetuity–immediate pays X per year. Brian receives the first n payments, Colleen receives the next n payments, and Jeff receives the remaining payments. Brian's share of the present value of the original perpetuity is 40%, and Jeff's share is K. Calculate K.

    (A) 24% (B) 28% (C) 32% (D) 36% (E) 40%

34. (# 50, May 2001). The present values of the following three annuities are equal:

    (i) perpetuity–immediate paying 1 each year, calculated at an annual effective interest rate of 7.25%

    (ii) 50–year annuity-immediate paying 1 each year, calculated at an annual effective interest rate of $j\%$

    (iii) n–year annuity-immediate paying 1 each year, calculated at an annual effective interest rate of $j - 1\%$

    Calculate n .

    (A) 30 (B) 33 (C) 36 (D) 39 (E) 42

35. (# 14, May 2000). A perpetuity paying 1 at the beginning of each 6–month period has a present value of 20. A second perpetuity pays X at the beginning of every 2 years. Assuming the same annual effective interest rate, the two present values are equal. Determine X.

    (A) 3.5 (B) 3.6 (C) 3.7 (D) 3.8 (E) 3.9

36. (#29, Sample Test). Chris makes annual deposits into a bank account at the beginning of each year for 20 years. Chris' initial deposit is equal to 100, with each subsequent deposit $k\%$ greater than the previous year's deposit. The bank credits interest at an annual effective rate of 5%. At the end of 20 years, the accumulated amount in Chris' account is equal to 7276.35. Given $k > 5$, calculate k.

    A. 8.06 B. 8.21 C. 8.36 D. 8.51 E. 8.68

37. (# 5, November 2001). Mike buys a perpetuity–immediate with varying annual payments. During the first 5 years, the payment is constant and equal to 10. Beginning in year 6, the payments start to increase. For year 6 and all future years, the current year's payment is $K\%$ larger than the previous year's payment. At an annual effective interest rate of 9.2%, the perpetuity has a present value of 167.50. Calculate K, given $K < 9.2$.

    (A) 4.0 (B) 4.2 (C) 4.4 (D) 4.6 (E) 4.8

38. (# 9, May 2000). A senior executive is offered a buyout package by his company that will pay him a monthly benefit for the next 20 years. Monthly benefits will remain constant within each of the 20 years. At the end of each 12-month period, the monthly benefits will be adjusted upwards to reflect the percentage increase in the CPI. You are given:

    (i) The first monthly benefit is R and will be paid one month from today.

    (ii) The CPI increases 3.2% per year forever.

    At an annual effective interest rate of 6%, the buyout package has a value of $100,000$. Calculate R.

    (A) 517 (B) 538 (C) 540 (D) 548 (E) 563

39. (# 45, May 2003). A perpetuity-immediate pays 100 per year. Immediately after the fifth payment, the perpetuity is exchanged for a 25–year annuity–immediate that will pay X at the end of the first year. Each subsequent annual payment will be 8% greater than the preceding payment. Immediately after the 10th payment of the 25–year annuity, the annuity will be exchanged for a perpetuity-immediate paying Y per year. The annual effective rate of interest is 8%. Calculate Y.

    (A) 110 (B) 120 (C) 130 (D) 140 (E) 150

40. (# 51, May 2000). Seth deposits X in an account today in order to fund his retirement. He would like to receive payments of 50 per year, in real terms, at the end of each year for a total of 12 years, with the first payment occurring seven years from now. The inflation rate will be 0.0% for the next six years and 1.2% per annum thereafter. The annual effective rate of return is 6.3%. Calculate X.

    (A) 303 (B) 306 (C) 316 (D) 327 (E) 329

41. (# 22, May 2003). A perpetuity costs 77.1 and makes annual payments at the end of the year. The perpetuity pays 1 at the end of year 2, 2 at the end of year 3, ... n at the end of year $(n+1)$. After year $(n+1)$, the payments remain constant at n. The annual effective interest rate is 10.5%. Calculate n.

    (A) 17 (B) 18 (C) 19 (D) 20 (E) 21

42. (# 16, November 2001). Olga buys a 5–year increasing annuity for X. Olga will receive 2 at the end of the first month, 4 at the end of the second month, and for each month thereafter the payment increases by 2. The nominal interest rate is 9% convertible quarterly. Calculate X.

    (A) 2680 (B) 2730 (C) 2780 (D) 2830 (E) 2880

43. (# 26, May 2000). Betty borrows 19,800 from Bank X. Betty repays the loan by making 36 equal payments of principal at the end of each month. She also pays interest on the unpaid

balance each month at a nominal rate of 12%, compounded monthly. Immediately after the 16th payment is made, Bank X sells the rights to future payments to Bank Y. Bank Y wishes to yield a nominal rate of 14%, compounded semi-annually, on its investment. What price does Bank X receive?

(A) 9,792 (B) 10,823 (C) 10,857 (D) 11,671 (E) 11,709

44. (# 44, November 2000). Joe can purchase one of two annuities:

    Annuity 1: A 10–year decreasing annuity–immediate, with annual payments of $10, 9, 8, ..., 1$.

    Annuity 2: A perpetuity–immediate with annual payments. The perpetuity pays 1 in year 1, 2 in year 2, 3 in year 3, . . . , and 11 in year 11. After year 11, the payments remain constant at 11.

    At an annual effective interest rate of i, the present value of Annuity 2 is twice the present value of Annuity 1 . Calculate the value of Annuity 1.

    (A) 36.4 (B) 37.4 (C) 38.4 (D) 39.4 (E) 40.4

45. (# 20, November 2000). Sandy purchases a perpetuity–immediate that makes annual payments. The first payment is 100, and each payment thereafter increases by 10. Danny purchases a perpetuity–due which makes annual payments of 180. Using the same annual effective interest rate, $i > 0$, the present value of both perpetuities are equal. Calculate i.

    (A) 9.2% (B) 9.7% (C) 10.2% (D) 10.7% (E) 11.2%

46. (# 30, Sample Test). Scott deposits:

    1 at the beginning of each quarter in year 1;

    2 at the beginning of each quarter in year 2;

    $$\cdots\cdots$$

    $$\cdots\cdots$$

    8 at the beginning of each quarter in year 8.

    One quarter after the last deposit, Scott withdraws the accumulated value of the fund and uses it to buy a perpetuity-immediate with level payments of X at the end of each year. All calculations assume a nominal interest rate of 10% per annum compounded quarterly. Calculate X.

    A. 19.4 B. 19.9 C. 20.4 D. 20.9 E. 21.4

47. (# 7, May 2001). Seth, Janice, and Lori each borrow 5000 for five years at a nominal interest rate of 12%, compounded semi–annually. Seth has interest accumulated over the five years and pays all the interest and principal in a lump sum at the end of five years.

Janice pays interest at the end of every six-month period as it accrues and the principal at the end of five years. Lori repays her loan with 10 level payments at the end of every six-month period. Calculate the total amount of interest paid on all three loans.

(A) 8718 (B) 8728 (C) 8738 (D) 8748 (E) 8758

48. (#28, Sample Test). A loan of $10,000$ is to be amortized in 10 annual payments beginning 6 months after the date of the loan. The first payment, X, is half as large as the other payments. Interest is calculated at an annual effective rate of 5% for the first 4.5 years and 3% thereafter. Determine X.

    A. 640 B. 648 C. 656 D. 664 E. 672

49. (# 12, November 2002). Kevin takes out a 10–year loan of L, which he repays by the amortization method at an annual effective interest rate of i . Kevin makes payments of 1000 at the end of each year. The total amount of interest repaid during the life of the loan is also equal to L. Calculate the amount of interest repaid during the first year of the loan.

    (A) 725 (B) 750 (C) 755 (D) 760 (E) 765

50. (# 37, Sample Test). A loan is being amortized by means of level monthly payments at an annual effective interest rate of 8%. The amount of principal repaid in the 12–th payment is 1000 and the amount of principal repaid in the t–th payment is 3700. Calculate t.

    A. 198 B. 204 C. 210 D. 216 E. 228

51. (# 10, May 2000). A bank customer borrows X at an annual effective rate of 12.5% and makes level payments at the end of each year for n years.

    (i) The interest portion of the final payment is 153.86.

    (ii) The total principal repaid as of time $(n-1)$ is 6009.12.

    (iii) The principal repaid in the first payment is Y.

    Calculate Y.

    (A) 470 (B) 480 (C) 490 (D) 500 (E) 510

52. (# 24, May 2000). A small business takes out a loan of $12,000$ at a nominal rate of 12%, compounded quarterly, to help finance its start-up costs. Payments of 750 are made at the end of every 6 months for as long as is necessary to pay back the loan. Three months before the 9th payment is due, the company refinances the loan at a nominal rate of 9%, compounded monthly. Under the refinanced loan, payments of R are to be made monthly, with the first monthly payment to be made at the same time that the 9th payment under the old loan was to be made. A total of 30 monthly payments will completely pay off the loan. Determine R.

    (A) 448 (B) 452 (C) 456 (D) 461 (E) 465

53. (# 55, November 2000). Iggy borrows X for 10 years at an annual effective rate of 6%. If he pays the principal and accumulated interest in one lump sum at the end of 10 years, he would pay 356.54 more in interest than if he repaid the loan with 10 level payments at the end of each year. Calculate X.

(A) 800 (B) 825 (C) 850 (D) 875 (E) 900

54. (# 34, November 2000). An investor took out a loan of $150,000$ at 8% compounded quarterly, to be repaid over 10 years with quarterly payments of 5483.36 at the end of each quarter. After 12 payments, the interest rate dropped to 6% compounded quarterly. The new quarterly payment dropped to 5134.62. After 20 payments in total, the interest rate on the loan increased to 7% compounded quarterly. The investor decided to make an additional payment of X at the time of his 20th payment. After the additional payment was made, the new quarterly payment was calculated to be 4265.73, payable for five more years. Determine X .

(A) $11,047$ (B) $13,369$ (C) $16,691$ (D) $20,152$ (E) $23,614$

55. (# 37, May 2001). Seth borrows X for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the second year is 1076.82 and at the end of the third year is 559.12. Calculate the principal repaid in the first payment.

(A) 444 (B) 454 (C) 464 (D) 474 (E) 484

56. (# 13, May 2001). Ron has a loan with a present value of $a_{\overline{n}|}$. The sum of the interest paid in period t plus the principal repaid in period $t+1$ is X. Calculate X.

(A) $1+\frac{\nu^{n-t}}{i}$ (B) $1+\frac{\nu^{n-t}}{d}$ (C) $1+\nu^{n-t}i$ (D) $1+\nu^{n-t}d$ (E) $1+\nu^{n-t}$

57. (# 9, November 2001). A loan is amortized over five years with monthly payments at a nominal interest rate of 9% compounded monthly. The first payment is 1000 and is to be paid one month from the date of the loan. Each succeeding monthly payment will be 2% lower than the prior payment. Calculate the outstanding loan balance immediately after the 40th payment is made.

(A) 6751 (B) 6889 (C) 6941 (D) 7030 (E) 7344

58. (# 36, Sample Test). Don takes out a 10-year loan of L, which he repays with annual payments at the end of each year using the amortization method. Interest on the loan is charged at an annual effective rate of i. Don repays the loan with a decreasing series of payments. He repays 1000 in year one, 900 in year two, 800 in year three, ..., and 100 in year ten. The amount of principal repaid in year three is equal to 600. Calculate L.

A. 4070 B. 4120 C. 4170 D. 4220 E. 4270

59. (# 26, May 2001). Susan invests Z at the end of each year for seven years at an annual effective interest rate of 5%. The interest credited at the end of each year is reinvested at an annual effective rate of 6%. The accumulated value at the end of seven years is X . Lori invests Z at the end of each year for 14 years at an annual effective interest rate of 2.5%. The interest credited at the end of each year is reinvested at an annual effective rate of 3%. The accumulated value at the end of 14 years is Y . Calculate $\frac{Y}{X}$.
(A) 1.93 (B) 1.98 (C) 2.03 (D) 2.08 (E) 2.13

60. (# 26, May 2003). 1000 is deposited into Fund X, which earns an annual effective rate of 6%. At the end of each year, the interest earned plus an additional 100 is withdrawn from the fund. At the end of the tenth year, the fund is depleted. The annual withdrawals of interest and principal are deposited into Fund Y, which earns an annual effective rate of 9%. Determine the accumulated value of Fund Y at the end of year 10.
(A) 1519 (B) 1819 (C) 2085 (D) 2273 (E) 2431

61. (# 9, November 2000). Victor invests 300 into a bank account at the beginning of each year for 20 years. The account pays out interest at the end of every year at an annual effective interest rate of i% . The interest is reinvested at an annual effective rate of $\left(\frac{i}{2}\right)$%. The yield rate on the entire investment over the 20 year period is 8% annual effective. Determine i .
(A) 9% (B) 10% (C) 11% (D) 12% (E) 13%

62. (# 31, Sample Test). Jason deposits 3960 into a bank account at $t = 0$. The bank credits interest at the end of each year at a force of interest $\delta_t = \frac{1}{8+t}$. Interest can be reinvested at an annual effective rate of 7%. The total accumulated amount at time $t = 3$ is equal to X. Calculate X.
A. 5394 B. 5465 C. 5551 D. 5600 E. 5685

63. (#33, Sample Test). Eric deposits 12 into a fund at time 0 and an additional 12 into the same fund at time 10. The fund credits interest at an annual effective rate of i. Interest is payable annually and reinvested at an annual effective rate of $0.75i$. At time 20, the accumulated amount of the reinvested interest payments is equal to 64. Calculate i, $i > 0$.
A. 8.8% B. 9.0% C. 9.2% D. 9.4% E. 9.6%

64. (# 39, May 2000). Sally lends $10,000$ to Tim. Tim agrees to pay back the loan over 5 years with monthly payments payable at the end of each month. Sally can reinvest the monthly payments from Tim in a savings account paying interest at 6%, compounded monthly. The yield rate earned on Sally's investment over the five-year period turned out to be 7.45%, compounded semi-annually. What nominal rate of interest, compounded monthly, did Sally

charge Tim on the loan?

(A) 8.53% (B) 8.59% (C) 8.68% (D) 8.80% (E) 9.16%

65. (# 35, November 2001). At time $t = 0$, Sebastian invests 2000 in a fund earning 8% convertible quarterly, but payable annually. He reinvests each interest payment in individual separate funds each earning 9% convertible quarterly, but payable annually. The interest payments from the separate funds are accumulated in a side fund that guarantees an annual effective rate of 7%. Determine the total value of all funds at $t = 10$.

    (A) 3649 (B) 3964 (C) 4339 (D) 4395 (E) 4485

66. (# 4, May 2001). A 20–year loan of $20,000$ may be repaid under the following two methods:

    (i) amortization method with equal annual payments at an annual effective rate of 6.5%

    (ii) sinking fund method in which the lender receives an annual effective rate of 8% and the sinking fund earns an annual effective rate of j

    Both methods require a payment of X to be made at the end of each year for 20 years. Calculate j .

    (A) $j \leq 6.5\%$ (B) $6.5\% < j \leq 8.0\%$ (C) $8.0\% < j \leq 10.0\%$ (D) $10.0\% < j \leq 12.0\%$
    (E) $j > 12.0\%$

67. (# 34, Sample Test). A 10–year loan of $10,000$ is to be repaid with payments at the end of each year consisting of interest on the loan and a sinking fund deposit. Interest on the loan is charged at a 12% annual effective rate. The sinking fund's annual effective interest rate is 8%. However, beginning in the sixth year, the annual effective interest rate on the sinking fund drops to 6%. As a result, the annual payment to the sinking fund is then increased by X. Calculate X.

    A. 122 B. 132 C. 142 D. 152 E. 162

68. (# 35, Sample Test). Jason and Margaret each take out a 17-year loan of L. Jason repays his loan using the amortization method, at an annual effective interest rate of i. He makes an annual payment of 500 at the end of each year. Margaret repays her loan using the sinking fund method. She pays interest annually, also at an annual effective interest rate of i. In addition, Margaret makes level annual deposits at the end of each year for 17 years into a sinking fund. The annual effective rate on the sinking fund is 4.62%, and she pays off the loan after 17 years. Margaret's total payment each year is equal to 10% of the original loan amount. Calculate L.

    A. 4840 B. 4940 C. 5040 D. 5140 E. 5240

69. (# 48, November 2000). A 12–year loan of 8000 is to be repaid with payments to the lender of 800 at the end of each year and deposits of X at the end of each year into a sinking fund.

Interest on the loan is charged at an 8% annual effective rate. The sinking fund annual effective interest rate is 4% . Calculate X .

(A) 298 (B) 330 (C) 361 (D) 385 (E) 411

70. (# 6, November 2001). A 10–year loan of 2000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:
(i) Equal annual payments at an annual effective rate of 8.07%.
(ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective rate of i.
The sum of the payments under option (i) equals the sum of the payments under option (ii). Determine i.
(A) 8.75% (B) 9.00% (C) 9.25% (D) 9.50% (E) 9.75%

71. (# 15, May 2003). John borrows 1000 for 10 years at an annual effective interest rate of 10%. He can repay this loan using the amortization method with payments of P at the end of each year. Instead, John repays the 1000 using a sinking fund that pays an annual effective rate of 14%. The deposits to the sinking fund are equal to P minus the interest on the loan and are made at the end of each year for 10 years. Determine the balance in the sinking fund immediately after repayment of the loan.
(A) 213 (B) 218 (C) 223 (D) 230 (E) 237

72. (# 30, November, 2000). A 1000 par value 20–year bond with annual coupons and redeemable at maturity at 1050 is purchased for P to yield an annual effective rate of 8.25%. The first coupon is 75. Each subsequent coupon is 3% greater than the preceding coupon. Determine P.
(A) 985 (B) 1000 (C) 1050 (D) 1075 (E) 1115

73. (# 41, May 2001). Bill buys a 10-year 1000 par value 6% bond with semi-annual coupons. The price assumes a nominal yield of 6%, compounded semi-annually. As Bill receives each coupon payment, he immediately puts the money into an account earning interest at an annual effective rate of i . At the end of 10 years, immediately after Bill receives the final coupon payment and the redemption value of the bond, Bill has earned an annual effective yield of 7% on his investment in the bond. Calculate i .
(A) 9.50% (B) 9.75% (C) 10.00% (D) 10.25% (E) 10.50%

74. (# 31, November 2001). You have decided to invest in two bonds. Bond X is an n-year bond with semi-annual coupons, while bond Y is an accumulation bond redeemable in $\frac{n}{2}$ years. The desired yield rate is the same for both bonds. You also have the following information:
Bond X

- Par value is 1000.
- The ratio of the semi-annual bond rate to the desired semi-annual yield rate, $\frac{r}{i}$ is 1.03125.
- The present value of the redemption value is 381.50.

Bond Y

- Redemption value is the same as the redemption value of bond X.
- Price to yield is 647.80.

What is the price of bond X?

(A) 1019 (B) 1029 (C) 1050 (D) 1055 (E) 1072

75. (# 29, May 2000). A firm has proposed the following restructuring for one of its 1000 par value bonds. The bond presently has 10 years remaining until maturity. The coupon rate on the existing bond is 6.75% per annum paid semiannually. The current nominal semiannual yield on the bond is 7.40%. The company proposes suspending coupon payments for four years with the suspended coupon payments being repaid, with accrued interest, when the bond comes due. Accrued interest is calculated using a nominal semiannual rate of 7.40%. Calculate the market value of the restructured bond.

    (A) 755 (B) 805 (C) 855 (D) 905 (E) 955

76. (#42, May 2003). A 10,000 par value 10-year bond with 8% annual coupons is bought at a premium to yield an annual effective rate of 6%. Calculate the interest portion of the 7th coupon.

    (A) 632 (B) 642 (C) 651 (D) 660 (E) 667

77. (#43, May 2000). A 1000 par value 5–year bond with 8.0% semiannual coupons was bought to yield 7.5% convertible semiannually. Determine the amount of premium amortized in the 6-th coupon payment.

    (A) 2.00 (B) 2.08 (C) 2.15 (D) 2.25 (E) 2.34

78. (#38, Sample Test). Laura buys two bonds at time 0. Bond X is a 1000 par value 14-year bond with 10% annual coupons. It is bought at a price to yield an annual effective rate of 8%. Bond Y is a 14-year par value bond with 6.75% annual coupons and a face amount of F. Laura pays P for the bond to yield an annual effective rate of 8%. During year 6, the writedown in premium (principal adjustment) on bond X is equal to the writeup in discount (principal adjustment) on bond Y. Calculate P.

    A. 1415 B. 1425 C. 1435 D. 1445 E. 1455

79. (#39, Sample Test). A 1000 par value 18–year bond with annual coupons is bought to yield an annual effective rate of 5%. The amount for amortization of premium in the 10th year is 20. The book value of the bond at the end of year 10 is X. Calculate X.
A. 1180 B. 1200 C. 1220 D. 1240 E. 1260

80. (# 40, November, 2000). Among a company's assets and accounting records, an actuary finds a 15-year bond that was purchased at a premium. From the records, the actuary has determined the following:
(i) The bond pays semi-annual interest.
(ii) The amount for amortization of the premium in the 2nd coupon payment was 977.19 .
(iii) The amount for amortization of the premium in the 4th coupon payment was 1046.79.
What is the value of the premium?
(A) 17,365 (B) 24,784 (C) 26,549 (D) 48,739 (E) 50,445

81. (#48, May 2000). A corporation buys a new machine for 2000. It has an expected useful life of 10 years, and a salvage value of 400. The machine is depreciated using the sinking fund method and an annual effective rate of i. Depreciation is taken at the end of each year for the 10–year period. The present value of the depreciation charges over the 10–year period is 1000 at the annual effective rate i. Calculate i .
(A) 8.4% (B) 8.8% (C) 9.2% (D) 9.6% (E) 10.0%

82. (#42, November 2001). A copier costs X and will have a salvage value of Y after n years.
(i) Using the straight line method, the annual depreciation expense is 1000.
(ii) Using the sum of the years digits method, the depreciation expense in year 3 is 800.
(iii) Using the declining balance method, the depreciation expense is 33.125% of the book value in the beginning of the year.
Calculate X.
(A) 4250 (B) 4500 (C) 4750 (D) 5000 (E) 5250

83. (#45, November 2001). A manufacturer buys a machine for $20,000$. The manufacturer estimates that the machine will last 15 years. It will be depreciated using the constant percentage method with an annual depreciation rate of 20%. At the end of each year, the manufacturer deposits an amount into a fund that pays 6% annually. Each deposit is equal to the depreciation expense for that year. How much money will the manufacturer have accumulated in the fund at the end of 15 years?
(A) $29,663$ (B) $34,273$ (C) $36,329$ (D) $38,509$ (E) $46,250$

84. (#5, May 2003). A machine is purchased for 5000 and has a salvage value of S at the end of 10 years. The machine is depreciated using the sum-of-the-years-digits method. At

the end of year 4, the machine has a book value of 2218 . At that time, the depreciation method is changed to the straight-line method for the remaining years. Determine the new depreciation charge for year 8.

(A) 200 (B) 222 (C) 286 (D) 370 (E) 464

85. (#11, May 2001). Machine I costs X, has a salvage value of $X/8$, and is to be depreciated over 10 years using the declining balance method.
Machine II costs Y, has a salvage value of $X/8$, and is to be depreciated over 10 years using the sum–of–the–years digits method.
The total amount of depreciation in the first seven years for Machine I equals the total amount of depreciation in the first seven years for Machine II. Calculate $\frac{Y}{X}$.
(A) 0.94 (B) 0.99 (C) 1.04 (D) 1.09 (E) 1.14

86. (# 8, November 2000) An actuary is trying to determine the initial purchase price of a certain physical asset. The actuary has been able to determine the following:
(i) Asset is 6 years old.
(ii) Asset is depreciated using the sinking fund method, with an annual effective rate of 9%.
(iii) Book value of the asset after 6 years is 55,216.36.
(iv) The loss of interest on the original purchase price is at an annual effective rate of 9%.
(v) Annual cost of the asset is 11,749.22.
(vi) Annual maintenance cost of the asset is 3000.
What was the original purchase price of the asset?
(A) 72,172 (B) 107,922 (C) 112,666 (D) 121,040 (E) 143,610

87. (# 36, May 2003) Eric and Jason each sell a different stock short at the beginning of the year for a price of 800. The margin requirement for each investor is 50% and each will earn an annual effective interest rate of 8% on his margin account. Each stock pays a dividend of 16 at the end of the year. Immediately thereafter, Eric buys back his stock at a price of $(800 - 2X)$ and Jason buys back his stock at a price of $(800 + X)$. Eric's annual effective yield, i, on the short sale is twice Jason's annual effective yield. Calculate i.
(A) 4% (B) 6% (C) 8% (D) 10% (E) 12%

88. (# 27, May 2001) Jose and Chris each sell a different stock short for the same price. For each investor, the margin requirement is 50% and interest on the margin debt is paid at an annual effective rate of 6%. Each investor buys back his stock one year later at a price of 760. Jose's stock paid a dividend of 32 at the end of the year while Chris's stock paid no dividends. During the 1-year period, Chris's return on the short sale is i, which is twice

the return earned by Jose. Calculate i.

(A) 12% (B) 16% (C) 18% (D) 20% (E) 24%

2 Solutions

1. Since each account earns an annual effective discount rate of d, each account earns the same annual effective interest rate i. Bruce's interest in the 11–th year is $100((1+i)^{11}-(1+i)^{10}) = 100i(1+i)^{10}$. Robbie's interest in the 17–th year is $50((1+i)^{17}-(1+i)^{16}) = 50i(1+i)^{16}$. So, $100i(1+i)^{10} = 50i(1+i)^{16}$, $(1+i)^6 = 2$, $i = 12.24\%$ and $x = 100i(1+i)^{10} = 100(0.1224)(1+0.1224)^{10} = 38.85$.

2. Eric's interest is
$$x\left(1+\frac{i}{2}\right)^{16} - x\left(1+\frac{i}{2}\right)^{15} = x\left(1+\frac{i}{2}\right)^{15}\frac{i}{2}.$$
Mike's interest is $2xi\frac{1}{2} = xi$. So, $(1+\frac{i}{2})^{15} = 2$ and $i = 9.45988\%$.

3. The following series of payments has present value zero:

Contributions	10	20	-100
Time	0	15	30

Since the rate changes at 10, we find the present value of the payments at this time. We have at time 10, the following series of payments have the same value

Contributions	10
Time	0

Contributions	-20	100
Time	15	30

The present value at time 10 of the payments at time 15 and at time 30 is
$$-20(1.03)^{-10} + 100(1.03)^{-40} = -14.8818 + 30.6557 = 15.7738.$$
This equals the present value at time 10 of the initial deposit, i.e. $10\left(1-\frac{d}{4}\right)^{-40} = 15.7738$. So, $d = 4.5318\%$.

4. Let $i^{(2)}$ be the nominal semiannual rate of Jennifer's loan. The future values of the loans are
$$x\left(1+\frac{i^{(2)}}{2}\right)^{20} = 800 \text{ and } x\left(1+i^{(2)}\right)^{20} = 1120.$$
Dividing these equations
$$\left(\frac{1+i^{(2)}}{1+\frac{i^{(2)}}{2}}\right)^{20} = \frac{1120}{800} = 1.4.$$
So, $i^{(2)} = \frac{(1.4)^{1/20}-1}{1-\frac{(1.4)^{1/20}}{2}} = 0.034518$ and $x = \frac{1120}{(1+0.034518)^{20}} = 568.14$.

5. Since i is a nominal rate of interest convertible semiannually, $100(1+\frac{i}{2})^{(2)(7.25)} = 200$ and $i = 9.7928\%$. To find the force of interest, we solve for δ, $100e^{\delta(7.25)} = 200$, and get $\delta = 0.09560$. We have that $i - \delta = 0.097928 - 0.09560 = 0.0023 = 0.23\%$.

6. For Fund X, $A(t) = 10000(1+0.05)^t$ and $\delta_t = \frac{d}{dt}\ln A(t) = \ln(1.05)$. For Fund Y, $A(t) = 10000(1+(0.08)t)$ and $\delta_t = \frac{d}{dt}\ln A(t) = \frac{0.08}{1+0.08t}$. The two forces of interest are equal, when, $\ln(1.05) = \frac{0.08}{1+0.08t}$. The solution of this equation is $t = \frac{1}{\ln(1.05)} - \frac{1}{0.08} = 8$. For Fund X, $A(8) = 10000(1+0.05)^8 = 14774.55$. For Fund Y, $A(8) = 10000(1+(0.08)8) = 16400$. So, $Z = 16400 - 14774.55 = 1625.45$.

7. We have that

$$a(t) = e^{\int_0^t \delta_s\,ds} = \exp\left(\int_0^t \frac{2}{k+2s}\,ds\right) = \exp\left(\ln(k+2s)\Big|_0^t\right) = e^{\ln(k+2t)-\ln k} = \frac{k+2t}{k}.$$

The information says that

$$x = 75a(3) = \frac{75(k+6)}{k}, x = 150\frac{a(3)}{a(5)} = \frac{150(k+6)}{k+10}.$$

Dividing these two equations, $1 = \frac{k+10}{2k}$ and $k = 10$. So, $x = \frac{75(k+6)}{k} = \frac{75\cdot 16}{10} = 120$.

8. First, we find $a(4) = e^{\int_0^4 \delta_s\,ds}$:

$$\int_0^4 \delta_s\,ds = \int_0^3 (0.02)s\,ds + \int_3^4 (0.045)\,ds = \frac{(0.02)s^2}{2}\Big|_0^3 + (0.045)s\Big|_3^4 = 0.135.$$

and $a(4) = e^{0.135}$. Let $i^{(4)}$ be the nominal rate of interest compounded quarterly. Then, $a(4) = \left(1+\frac{i^{(4)}}{4}\right)^{4\cdot 4} = \left(1+\frac{i^{(4)}}{4}\right)^{16}$. So, $i^{(4)} = 4(e^{0.135/16}-1) = 0.033893 = 3.3893\%$.

9. We have that for Fund X,

$$a(t) = e^{\int_0^t \delta_s\,ds} = e^{\int_0^t (0.006)s^2\,ds} = \exp\left((0.002)s^3\Big|_0^t\right) = e^{(0.002)t^3} = e^{t^3/500}.$$

For Fund X, $4K = A(n) = A(0)a(n) = Ke^{n^3/500}$. So, $n = (500\cdot\ln 4)^{1/3} = 8.8499$. For Fund Y, $a(t) = (1+0.10)^t$. So, $4K = A(n) = \frac{a(n)}{a(m)}A(m) = (1+0.10)^{n-m}2K$. So, $n - m = \frac{\ln 2}{\ln 1.1} = 7.2725$. Hence, $m = 8.8499 - 7.2725 = 1.5775$.

10. For the Fund X, $a(5) = e^{\int_0^5 \delta_s\,ds} = e^{\int_0^5 \frac{s^2}{k}\,ds} = e^{125/(3k)}$. For the Fund Y, $a(5) = (1 - \frac{0.08}{2})^{-(2)\cdot 5} = (0.96)^{-10}$. From the equation, $e^{125/(3k)} = (0.96)^{-10}$, we get that $k = \frac{-125}{(3)(10)(\ln 0.96)} = 102.0691$.

11. For Tawny's bank account, $a(t) = \left(1+\frac{0.1}{2}\right)^{2t} = (1.05)^{2t}$ and the force of interest is $\delta_t = \frac{d}{dt}\ln a(t) = 2\ln(1.05)$. For Fabio's account, $a(t) = 1 + it$ and $\delta_t = \frac{i}{1+it}$. At time $t = 5$, $2\ln(1.05) = \frac{i}{1+i5}$. So, $i = \frac{2\ln(1.05)}{1-10\ln(1.05)} = 0.1905$. So,

$$Z = A(5) = A(0)(1 + it) = 1000 \cdot (1 + 5 \cdot (0.1905)) = 1952.75.$$

12. The value of Joe's deposits at time 10 is

$$10\,(1 + (0.11)(10-0)) + 30\,(1 + (0.11)(10-5)) = 21 + 46.5 = 67.5.$$

The value of Tina's deposits at time 10 is

$$10(1 + 0.0915)^{10-n} + 30(1 + 0.0915)^{10-2n}.$$

Call $x = (1.0915)^{-n}$. Then, $10(1 + 0.0915)^{10}x + 30(1 + 0.0915)^{10}x^2 = 67.5$. So, $3x^2 + x = (67.5) \cdot 10^{-1} \cdot (1.0915)^{-10} = 2.8123$. So, $(1.0915)^{-n} = x = \frac{-1\pm\sqrt{1^2+(4)(3)(2.8123)}}{(2)(3)} = 0.81579$ and $n = 2.3226$.

13. We have that

$$a(t) = e^{\int_0^t \delta_s\,ds} = e^{\int_0^t \frac{s^2}{100}\,ds} = e^{\frac{t^3}{300}}.$$

The accumulation of the investments at time $t = 3$ is $100a(3) + x$. The accumulation of the investments at time $t = 6$ is $\frac{a(6)}{a(3)}\,(100a(3) + x)$. The amount of interest earned from time 3 to time 6 is

$$x = \frac{a(6)}{a(3)}\,(100a(3) + x) - (100a(3) + x) = \frac{a(6) - a(3)}{a(3)} \cdot (100a(3) + x)\,.$$

Solving for x, we get that

$$\begin{aligned} x &= \tfrac{100\cdot a(3)(a(6)-a(3))}{2a(3)-a(6)} = \tfrac{100\cdot e^{\frac{3^3}{300}}\cdot(e^{\frac{6^3}{300}}-e^{\frac{3^3}{300}})}{2e^{\frac{3^3}{300}}-e^{\frac{6^3}{300}}} \\ &= \tfrac{(100)(1.094174)(2.054433-1.094174)}{(2)(1.094174)-2.054433} = 784.0595. \end{aligned}$$

14. Since the equations of value of the two streams of payments at time zero are equal,

$$100 + 200\nu^n + 300\nu^{2n} = 600\nu^{10}.$$

Using that $\nu^n = 0.75941$, we have that

$$600(1 + i)^{-10} = 600\nu^{10} = 100 + 200(0.75941) + 300(0.75941)^2 = 424.89.$$

So, $i = 3.51\%$.

15. We have that $V_0 = 75$ and $FV = 60$. We have deposits of 10 at times $\frac{1}{12}, \frac{2}{12}, \dots, \frac{12}{12}$, and withdrawals of $5, 25, 80, 35$, at respective times $\frac{2}{12}, \frac{1}{2}, \frac{9.5}{12}, \frac{10}{12}$. We have that

$$I = FV - V_0 - \sum_{j=1}^{n} C_j = 60 - 75 - (10)(12) + 5 + 25 + 80 + 35 = 10$$

and

$$\begin{aligned} & V_0 t + \textstyle\sum_{j=1}^{n}(t - t_j)C_j \\ = \; & (75)(1) + (10)\textstyle\sum_{j=1}^{12} \frac{12-j}{12} - 5\left(1 - \frac{1}{12}\right) - 25\left(1 - \frac{1}{2}\right) - 80\left(1 - \frac{9.5}{12}\right) - 35\left(1 - \frac{10}{12}\right) \\ = \; & 75 + \frac{10}{12}\frac{(11)(12)}{2} - 5\frac{10}{12} - 25\frac{1}{2} - 80\frac{2.5}{12} - 35\frac{2}{12} = 90.8347. \end{aligned}$$

So,

$$i = \frac{I}{V_0 t + \sum_{j=1}^{n}(t - t_j)C_j} = \frac{10}{90.8347} = 11\%.$$

16. Since the dollar–weighted return is 0%, $100 + 2x + x = 115$, which gives that $x = 5$. The time–weighted return y satisfies

$$1 + y = \frac{95}{100}\frac{105}{105}\frac{115}{110} = 0.993181.$$

So, $y = -0.006819 = -0.6819\%$.

17. According to the time–weighted method, the equivalent annual effective yield during the first 6 months satisfies

$$(1 + i)^{1/2} = \frac{40}{50} \cdot \frac{80}{60} \cdot \frac{157.5}{160}.$$

According to the time–weighted method, the annual effective yield satisfies

$$1 + i = \frac{40}{50} \cdot \frac{80}{60} \cdot \frac{175}{160} \cdot \frac{x}{250}.$$

So,

$$\left(\frac{40}{50} \cdot \frac{80}{60} \cdot \frac{157.5}{160}\right)^2 = \frac{40}{50} \cdot \frac{80}{60} \cdot \frac{175}{160} \cdot \frac{x}{250}$$

and

$$x = \frac{40 \cdot 80 \cdot (157.5)^2 \cdot 250}{50 \cdot 60 \cdot 160 \cdot 175} = 236.5$$

18. Since the time–weighted return is 0%, $\frac{12}{10}\frac{x}{12+x} = 1$. So, $x = 60$. The dollar–weighted return y satisfies

$$y = \frac{10 + 60 - 60}{(10)(1) + (60)(1/2)} = -0.25 = -25\%.$$

19. April 1 is time 0.25. Let A be the value of the investment on January 1, 1998. Using the dollar weighted method for 1997, $100,000(1+i) - 8,000(1+i(0.75)) = 92000 + 94000i = A$. Using the weighted method for 1998, $A(1 + i) = 103,992$. So, we must solve for i in the equation, $(92000 + 94000i)(1 + i) = 1039992$, which is equivalent to $94i^2 + 186i - 11.992 = 0$. So, $i = \frac{-186 \pm \sqrt{186^2 - (4)(94)(-11.992)}}{(2)(94)} = 0.06249905 = 6.249905\%$.

20. We have that $a(t) = e^{\int_0^t \frac{1}{8+s}\,ds} = e^{\ln(8+t)-\ln 8} = \frac{8+t}{8}$. The future value of the continuous cashflow is

$$20000 = \int_0^{10} C(s)\frac{a(10)}{a(s)}\,ds = \int_0^{10} k(8+s)\cdot\frac{8+10}{8+s}\,ds = \int_0^{10} 18k\,ds = 180k.$$

So, $k = \frac{20000}{180} = 111.11$.

21. The total amount of interest earned during the year 19993 is

$$\begin{aligned} &28.4 = (100)(1.10)(1.10)(1+t)(0.08) + (100)(1.12)(1.05)(0.10) + (100)(1.08)(t-0.03)(0.08) \\ = \; &(100)(1.10)(1.10)(0.08) + (100)(1.12)(1.05)(0.10) - (100)(1.08)(0.03)(0.08) \\ &+t\left((100)(1.10)(1.10)(0.08) + (100)(1.08)(0.08)\right) = 19.28 + 117.68t, \end{aligned}$$

and $t = \frac{28.4-19.28}{117.68} = 7.74983\%$.

22. The return in the investment is

$$100\left(\frac{808}{800}\right)^4\left(\frac{858.5}{850.0}\right)^4\left(\frac{918.0}{900.0}\right)^4\left(\frac{948.6}{930.0}\right)^4 + 100\left(\frac{918.0}{900.0}\right)^4\left(\frac{948.6}{930.0}\right)^4 = 2440.399.$$

So, $2440.399 = 1000(1+i)^4 + 1000(1+i)^2$ and $i = 6.78218856\%$.

23. The present value of John's inheritance is $2500\ddot{a}_{\overline{10}|0.08} = 18117.22$. 18117.22 is also the present value at time 0 of Carol's inheritance. Two years later, the value of Carol's inheritance is $18117.22(1.09)^2 = 21525.07$. To find z, we solve $za_{\overline{15}|8} = 21525.07$ to get $z = 2514.76$.

24. Susan's account at the end of 40 years is worth $100s_{\overline{40}|8\%} = 25905.63$. We find x solving $xa_{\overline{15}|8\%} = 25905.63$ and get $x = 3026.55$. Jeff's account at the end of 40 years is worth $100s_{\overline{40}|10\%} = 44259.25$. We find x solving $xa_{\overline{15}|10\%} = 44259.25$ and get $y = 5818.83$. The Jeff's withdrawals are of 5818.83. Finally, $5818.83 - 3026.55 = 2792.38$.

25. We have that $i^{(4)} = 6.8234\%$. The future value of Jerry's deposits at time 10 is $450s_{\overline{40}|6.8234/4} = 25513.22$. The future value of Jerry's deposits at time 15 is $25513.22(1.07)^5 = 35783.61$. Then, we find y solving the equation, $35783.61 = y\ddot{a}_{\overline{4}|7\%}$ to get $y = 9873.20$.

26. To get an income of \$1, the man needs $\frac{1000}{9.65}$. So, to get an income of \$3000, the man needs $\frac{(3000)\cdot(1000)}{9.65} = 310880.83$. We solve for x in the equation $x\ddot{s}_{\overline{(25)(12)}|8\%/12} = x\ddot{s}_{\overline{300}|0.33333\%} = 310880.83$ and we get $x = 324.72$.

27. Counting the time in months, with January 1, 1985 as time 1, the cashflow is

Contributions	200	⋯	200	0	⋯	0	200 ⋯	200	0
Time (in months)	1	⋯	59	60	⋯	72	73 ⋯	180	181

We have to find the future value of this cashflow at time 181. The value of the first 59 payments at time 60 is $200\ddot{s}_{\overline{59}|6\%/12} = 13754$. The value of this money 121 months later (on December 31, 1999) is $13754\left(1+\frac{0.06}{12}\right)^{121} = 25149.11$. The future value of the last 108 payments on December 31, 1999 is $200\ddot{s}_{\overline{108}|0.5\%} = 28690.72$. The total value of Jim's retirement account at the end is $25149.11 + 28690.72 = 53839.83$.

28. We have that

$$\begin{aligned} & 8000 = 98(1+i)^{2n}s_{\overline{n}|i} + 196s_{\overline{2n}|i} = 98(1+i)^{2n}\tfrac{(1+i)^n-1}{i} + 196\tfrac{(1+i)^{2n}-1}{i} \\ = \; & (98)2^2\tfrac{2-1}{i} + 196\tfrac{2^2-1}{i} = \tfrac{980}{i} \end{aligned}$$

So, $i = \frac{980}{8000} = 0.1225 = 12.25\%$.

29. The future value of the item at time 10 is $(200)(1.04)^{10} = 296.05$. The future value of Chuck's investments at time 10

$$(20)\ddot{s}_{\overline{6}|0.1}(1.1)^4 + x\ddot{s}_{\overline{3}|0.1}(1.1)^4 = 248.52 + 5.3308x.$$

So, $296.05 = 248.52 + 5.3308x$ and $x = 8.92$.

30. The cashflow is

Contributions	100	100	100	$\cdots$	100	0
Time	0	4	8	$\cdots$	36	40

The 4–year effective rate of interest is $(1+i)^4 - 1$. So, the accumulated amount in the account at the end of 40 years is

$$X = 100\ddot{s}_{\overline{10}|(1+i)^4-1} = 100\frac{(1+i)^4((1+i)^{40}-1)}{(1+i)^4-1},$$

and the accumulated amount in the account at the end of 20 years is

$$100\ddot{s}_{\overline{5}|(1+i)^4-1} = 100\frac{(1+i)^4((1+i)^{20}-1)}{(1+i)^4-1}.$$

So, $5 = (1+i)^{20} + 1$ and $(1+i)^4 = 1.3195$. So,

$$X = 100\frac{(1+i)^4((1+i)^{40}-1)}{(1+i)^4-1} = 100\frac{(1.3195)(4^2-1)}{1.3195-1} = 6194.68.$$

31. The cashflows are:

Contributions	55	55	55	$\cdots$	55
Time	1	2	3	$\cdots$	20

Contributions	30	$\cdots$	30	60	$\cdots$	60	90	$\cdots$	90
Time	1	$\cdots$	10	11	$\cdots$	20	21	$\cdots$	30

Since both annuities have the same present value:

$$55a_{\overline{20}|i} = 30a_{\overline{10}|i} + 60\nu^{10}a_{\overline{10}|i} + 90\nu^{20}a_{\overline{10}|i}.$$

Since

$$\frac{a_{\overline{20}|i}}{a_{\overline{10}|i}} = \frac{\frac{1-\nu^{20}}{i}}{\frac{1-\nu^{10}}{i}} = 1+\nu^{10},$$

we have that

$$55(1+\nu^{10}) = 30 + 60\nu^{10} + 90\nu^{20}$$

or $90\nu^{20} + 5\nu^{10} - 25 = 0$. This is equation is equivalent to $18\nu^{20} + \nu^{10} - 5 = 0$, whose solution is

$$\nu^{10} = \frac{-1 \pm \sqrt{1 + 5 \cdot 18 \cdot 4}}{2 \cdot 18} = \frac{1}{2}.$$

So, $(1+i)^{10} = 2$, $i = 7.1774\%$ and $x = 55a_{\overline{20}|7.1774\%} = 574.72$.

32. Let j be the rate of interest corresponding to a period of 3 years. Then, $1+j = (1+i)^3$. The present value of the perpetuity at time 3 years is $\frac{10}{j}$. The present value of the perpetuity at time 0 is $\frac{10}{j(j+1)}$. So, $j^2 + j - \frac{5}{16} = 0$, and $j = \frac{-1\pm\sqrt{1+(5/4)}}{2} = \frac{-1\pm 1.5}{2} = 0.25$. Hence, $i = (1+0.25)^{1/3} - 1 = 7.7217\%$. Then, $i^{(3)} = 7.5311\%$. The present value of a perpetuity–immediate paying 1 at the end of each 4–month period is $x = \frac{3}{i^{(3)}} = \frac{3}{0.075311} = 39.83$.

33. The total value of the perpetuity–immediate is $xa_{\overline{\infty}|} = \frac{x}{i}$. The present value of Brian' payments is $xa_{\overline{n}|i} = \frac{x(1-\nu^n)}{i}$. So, Brian's proportion of the perpetuity is $1 - \nu^n = 0.40$. So, $\nu^n = 0.6$. The present value of Jeff's payments is $\nu^{2n}x\frac{1}{i}$. So, Jeff's share is $\nu^{2n} = 0.36 = 36\%$.

34. The value of the first perpetuity is $a_{\overline{\infty}|} = \frac{1}{i} = \frac{1}{0.0725} = 13.7931$. The value of the second annuity is $13.7931 = a_{\overline{50}|j}$. Hence, $j = 7\%$. From the equation, $a_{\overline{n}|6} = 13.7931$, we get that $n = 30.1680$.

35. The present value of the first perpetuity is $20 = \frac{1+\frac{i^{(2)}}{2}}{\frac{i^{(2)}}{2}}$, where $i^{(2)}$ is the nominal rate of interest convertible semiannually. So, $i^{(2)} = \frac{2}{19} = 10.5263\%$. Let $i^{(0.5)}$ be the nominal rate of interest convertible twice a year. Then, $\left(1+\frac{i^{(2)}}{2}\right)^2 = \left(1+2i^{(0.5)}\right)^{0.5}$ and $i^{(0.5)} = 11.3869\%$. The value of the second perpetuity is $20 = x\frac{(1+2i^{(0.5)})}{2i^{(0.5)}} = \frac{1.2277}{0.2277}x$. So, $x = 3.71$.

36. The cashflow is

Payments	100	$100(1+r)$	$100(1+r)^2$	$\cdots$	$100(1+r)^{19}$
Time	0	1	2	$\cdots$	19

where $r = \frac{k}{100}$. Since $r > i$, the present value at time -1 of this cashflow is $\frac{100}{1+i} s_{\overline{20}|\frac{r-i}{1+i}}$. The accumulated value at time -1 of Chris' cashflow is $7276.35(1+i)^{-21}$. So, $s_{\overline{20}|\frac{r-i}{1+i}} = 72.7635(1+i)^{-20} = 27.4238$. Using the calculator, we get that $\frac{r-i}{1+i} = 0.032$ and $r = 0.05 + (1.05)(0.032) = 0.0836$. Hence, $k = 8.36$.

37. Le $r = \frac{k}{100}$. The cashflow of the perpetuity is

Payments	10	10	10	10	10	$10(1+r)$	$10(1+r)^2$	$10(1+r)^3$	$\cdots$
Time	1	2	3	4	5	6	7	8	$\cdots$

The present value at time 0 of this perpetuity is

$$167.5 = 10a_{\overline{4}|i} + 10\nu^4 \frac{1}{i-r} = 32.2555 + 7.0325 \frac{1}{0.092 - r}.$$

So, $r = 0.092 - \frac{7.0325}{167.5 - 32.2555} = 4\%$.

38. The nominal rate convertible monthly is $i^{(12)} = 5.8411\%$. The value of the payments which the senior executive accumulates at the end of the first year is $Rs_{\overline{12}|5.8411/12\%} = 12.3265R$. So, the cashflow is equivalent to

Payments	$12.3265R$	$12.3265R(1+r)$	$12.3265R(1+r)^2$	$\cdots$	$12.3265(1+r)^{19}$
Time	1	2	3	$\cdots$	20

where $r = 3.2\%$ and $i = 6\%$. The present value of the package is

$$100000 = 12.3265R \frac{1}{1+0.032} a_{\overline{20}|\frac{0.06-0.032}{1.032}} = 11.9443Ra_{\overline{20}|0.027132} = 182.5063R.$$

So, $R = 547.93$.

39. The value of the perpetuity immediately after the fifth payment is $\frac{100}{i} = 1250$. The payments of the 25–year annuity–immediate are

Inflows	x	$x(1.08)$	$x(1.08)^2$	$\cdots$	$x(1.08)^{24}$
Time	1	2	3	$\cdots$	25

Since $i = r$, the present value at time 0 of this annuity is $1250 = 25x(1.08)^{-1}$. Hence, $x = 50(1.08)$. The 15 last payments of the annuity are

Inflows	$50(1.08)^{11}$	$x(1.08)^{12}$	$\cdots$	$x(1.08)^{15}$
Time	1	2	$\cdots$	15

Since $i = r$, the present value at time 0 of this annuity is $(50)(15)(1.08)^{10} = 1619.19$. Hence, annual payment of the perpetuity is $1619.19(0.08) = 129.54$.

40. The cashflow is

Contributions	0	$\cdots$	0	50(1.012)	$50(1.012)^2$	$\cdots$	$50(1.012)^{11}$
time	0	$\cdots$	6	7	8	$\cdots$	18

So,

$$X = (1.063)^{-6}(50)a_{\overline{12}|\frac{0.063-0.012}{1.012}} = 34.6533a_{\overline{12}|5.0395\%} = 306.4765.$$

41. The cashflow is

Payments	0	1	2	$\cdots$	$n-1$	n	n	$\cdots$
Time	1	2	3	$\cdots$	n	$n+1$	$n+2$	$\cdots$

The present value at time 0 of the perpetuity is

$$77.1 = \nu\,(Ia)_{\overline{n}|i} + \nu^{n+1}n\frac{1}{i} = \nu\frac{a_{\overline{n}|i}(1+i) - n\nu^n}{i} + \frac{\nu^{n+1}n}{i} = \frac{a_{\overline{n}|i}}{i}.$$

So, $8.0955 = a_{\overline{n}|10.5\%}$ and $n = 19$.

42. The cashflow is:

Payments	2	4	6	$\cdots$	120
Time (in months)	1	2	3	$\cdots$	60

We have $i^{(4)} = 9\%$. So, $i = 9.3083\%$ and $i^{(12)} = 8.933\%$. We use the formula for the increasing annuities with $n = 60$ and $i = 8.933\%/12 = 0.74444\%$:

$$2\,(Ia)_{\overline{n}|i} = 2\cdot\frac{\ddot{a}_{\overline{n}|i} - n\nu^n}{i} = \frac{2(10.158740)}{0.00744} = 2729.21.$$

43. Since Betty pays 19800 into 36 payments, each monthly payment is $\frac{19800}{36} = 550$. Between the $j-1$–th and j–th month she owes $(37-j)550$. The interest in this quantity is $(37-j)5.50$. So, the cashflow for Betty is:

Payments	550 + (36)(5.5)	550 + (35)(5.5)	$\cdots$	550 + (1)(5.5)
Time	1	2	$\cdots$	36

For bank Y, the cashflow is

Payments	550 + (20)(5.5)	550 + (19)(5.5)	$\cdots$	550 + (1)(5.5)
Time	1	2	$\cdots$	20

For bank Y, we have $i^{(2)} = 14\%$. So, $i = 14.49\%$ and $i^{(12)} = 13.608312\%$. Taking $i = 13.608312\%/12 = 1.134026\%$ and $n = 20$, Bank X receives

$$\begin{aligned} & 550a_{\overline{n}|i} + 5.5\,(Da)_{\overline{n}|i} = 550a_{\overline{20}|1.134026\%} + \tfrac{5.5(20-a_{\overline{20}|1.134026\%})}{1.134026\%} \\ = \; & 9792.39 + \tfrac{5.5(20-17.80343)}{0.0113426} = 10857.72 \end{aligned}$$

44. The present value of Annuity 1 is

$$(Da)_{\overline{10}|i} = \frac{10 - a_{\overline{10}|i}}{i}.$$

The present value of Annuity 2 is

$$(Ia)_{\overline{10}|i} + \nu^{-10} 11 a_{\overline{\infty}|i} + = \frac{(1+i)a_{\overline{10}|i} - 10\nu^{10}}{i} + \frac{11\nu^{10}}{i} = \frac{(1+i)a_{\overline{10}|i}}{i} + \frac{\nu^{10}}{i} = \frac{1 + a_{\overline{10}|i}}{i}.$$

where we have used that $\nu^{10} = 1 - ia_{\overline{10}|i}$. Hence, $2\left(\frac{10-a_{\overline{10}|i}}{i}\right) = \frac{1+a_{\overline{10}|i}}{i}$ and $a_{\overline{10}|i} = 19/3$. So, $i = 9.30\%$, and the present value of Annuity 1 is $\frac{10-a_{\overline{10}|i}}{i} = \frac{10-(19/3)}{0.0930} = 39.42$.

45. Since the present value of the two perpetuities are equal, we have that

$$\frac{90}{i} + \frac{10(1+i)}{i^2} = \frac{180(1+i)}{i}.$$

So, $18i^2 + 8i - 1 = 0$ and $i = \frac{-4 \pm \sqrt{16+18}}{18} = 0.1017$.

46. The value of the Scott deposits in the first year are $\ddot{s}_{\overline{4}|2.5} = 4.2563$. So, the cashflow is equivalent to the following

Payments	4.2563	2(4.2563)	$\cdots$	8(4.2563)
	1	2	$\cdots$	8

We have that $i^{(4)} = 0.10$ and $i = 0.103813$. The value of this cashflow is

$$(4.2563)\,(Is)_{\overline{8}|0.103813} = (4.2563)\frac{\ddot{s}_{\overline{8}|0.103813} - 8}{0.103813} = (4.2563)\frac{12.7992 - 8}{0.103813} = 196.7657.$$

The perpetuity payment is $196.7657i = (196.7657)(0.103813) = 20.4269$.

47. Seth's interest is $5000((1.06)^{10} - 1) = 3954.23$. Janice's interest is $(10)(5000)(0.06) = 3000$. Lori's semiannual payment is $\frac{5000}{a_{\overline{10}|0.06}} = 679.34$. So, Lori's interest is $(10)(679.34) - 5000 = 1793.4$. The total amount of interest paid is $3954.23 + 3000 + 1793.4 = 8747.63$.

48. The casflow is

Contributions	x	$2x$	$\cdots$	$2x$	$2x$	$\cdots$	$2x$
Time	0.5	1.5	$\cdots$	4.5	5.5	$\cdots$	9.5

with interest rates changing at time 4.5. The present value of the series of payments is

$$\begin{aligned}10000 &= x(1.05)^{-0.5} + 2x(1.05)^{-0.5}a_{\overline{4}|0.05} + 2x(1.05)^{-4.5}a_{\overline{5}|0.03}\\ &= x(0.9759 + 6.9210 + 7.3539) = 15.2508x.\end{aligned}$$

So, $x = \frac{10000}{15.2508} = 655.70$.

49. The finance charge is $L = \frac{nL}{a_{\overline{n}|i}} - L$. So, $a_{\overline{n}|i} = \frac{n}{2}$, i.e. $a_{\overline{10}|i} = 5$,and $i = 15.0984\%$. The loan is $L = 1000a_{\overline{n}|i} = 1000a_{\overline{5}|i} = 5000$. The amount of interest paid during the first year of the loan is $Li = 754.92$.

50. Let P be the monthly payment. $i^{(12)} = 0.077208$ and $0.077208/12 = 0.006434$. The amount of principal repaid in the 12–th payment is $1000 = P(1+0.006434)^{-(n+1-12)}$ and the amount of principal repaid in the t–th payment is $3700 = P(1 + 0.006434)^{-(n+1-t)}$. From these two equations, $3.7 = (1 + 0.006423)^{t-12}$ and $t = 216$.

51. Let P be the installment payments. Let L be the loan. The interest portion of the final payment is $153.86 = P(1 - \nu) = P(1 - (1.125)^{-1})$. So, $P = 1384.74$. The total principal repaid as of time $(n - 1)$ is $6009.12 = L - Pa_{\overline{1}|0.125} = L - 1230.88$. So, $L = 7240$. The loan of $L = 7240$ is paid in annual payments of $P = 1384.74$ at an interest of 12.5%. So, $7240 = 1384.74a_{\overline{n}|0.125}$ and $n = 9$. The principal repaid in the first payment is $Y = P\nu^n = 1384.74(1.125)^{-9} = 479.73$.

52. We have that $i^{(4)} = 12\%$. So, $i = 12.550881\%$ and $i^{(2)} = 12.18\%$. The balance after the 8–th payment is $12000(1+0.0609)^8 - 750s_{\overline{8}|0.0609} = 11809.35$. The value of this outstanding balance 3 months later is $11809.35(1.03) = 12163.63$. Two months later, the outstanding balance is $12163.63(1 + \frac{0.09}{12})^2 = 12346.77$. Let P be the new payment of the loan, then $12346.77 = Pa_{\overline{30}|0.09/12}$ and $P = 461.12$.

53. Assume first that Iggy makes a payment p Iggy makes at the end of each year. Then, $x = pa_{\overline{10}|6\%} = p7.36$. So, $p = \frac{x}{7.36}$ an total amount of the payments is $10p = \frac{10x}{7.36}$. If he pays the principal and accumulated interest in one lump sum at the end of 10 years, he would pay $x(1 + 0.06)^{10} = 1.7908x$. Hence, the difference between the two payments is $356.54 = 1.7908x - \frac{10x}{7.36} = 0.4321x$. So, $x = 825.13$.

54. The outstanding balance of the loan after 20 payments is $5134.62a_{\overline{20}|0.06/4} = 88154.43$. The present value at time 20 months of the future payments of the customer is $4265.73a_{\overline{20}|0.07/4} = 71463.27$. So, $x = 88154.43 - 71463.27 = 16691.16$.

55. Let P be the amount of each payment. The outstanding loan balance at the end of the second year is $1076.82 = Pa_{\overline{2}|0.08}$. So, $P = 603.85$. The principal repaid in the first payment is $P\nu^n = 603.85(1.08)^{-4} = 443.85$.

56. The interest paid in period t is $1 - \nu^{n+1-t}$. The principal repaid in period $t+1$ is ν^{n-t}. So,

$$x = 1 - \nu^{n+1-t} + \nu^{n-t} = 1 + \nu^{n-t}(1-\nu) = 1 + \nu^{n-t}d.$$

57. The interest per period is $i = (9/12)\% = 0.75\%$. Let $r = -0.02$ and let $P = 1000$. The outstanding balance immediately after the 40th payment is

$$B_{40} = P(1+r)^{k-1}a_{\overline{n-k}|\frac{i-r}{1+r}} = 1000(0.98)^{39}a_{\overline{20}|\frac{0.0075+0.02}{0.98}} = 454.7963a_{\overline{20}|0.028061} = 6889.06.$$

58. We have that $600 = 100a_{\overline{8}|i}$. So, $i = 6.876\%$. The loan is $100\,(Da)_{\overline{n}|i} = 100 \cdot \frac{10-a_{\overline{10}|0.06876}}{0.06876} =$ 4269.84.

59. We may assume that $Z = 1$. Susan's accumulated value consists of her \$7 invested plus the cashflow for her interest. The cashflow of her interest payments are

Payments	0	0.05	2(0.05)	$\cdots$	6(0.05)
Time	1	2	3	$\cdots$	7

The accumulated value of this cashflow is

$$(0.05)\,(Is)_{\overline{6}|6\%} = (0.05)\left(\frac{(1+0.06)s_{\overline{6}|6\%}-6}{0.06}\right) = 1.1653.$$

So, Susan's accumulated value is $X = 8.1653$.

Lori's accumulated value consists of her \$14 invested plus the cashflow for her interest. The cashflow of her interest payments are

Payments	0	0.025	2(0.025)	$\cdots$	13(0.025)
Time	1	2	3	$\cdots$	14

The accumulated value of this cashflow is

$$(0.025)\,(Is)_{\overline{13}|3\%} = (0.025)\left(\frac{(1+0.03)s_{\overline{13}|3\%}-13}{0.03}\right) = 2.5719.$$

So, Susan's accumulated value is $Y = 16.5719$.

We have that $\frac{Y}{X} = 2.0296$.

60. The balances along time in the Fund X are:

Balance	1000	900	800	$\cdots$	0
Time	0	1	2	$\cdots$	10

Hence, the deposits at Fund Y are:

Balance	$100+(6)\cdot(10)$	$100+(6)\cdot(9)$	$100+(6)\cdot(8)$	$\cdots$	$100+(6)\cdot(1)$
Time	1	2	3	$\cdots$	10

The accumulated value of Fund Y at the end of year 10 is

$$\begin{aligned} & 100s_{\overline{10}|9\%}+6\,(Ds)_{\overline{10}|9\%}=100s_{\overline{10}|9\%}+\frac{6(10(1+0.09)^{10}-s_{\overline{10}|9\%})}{0.09} \\ = & \ 1519.29+\frac{6\cdot(8.4807)}{0.09}=2084.67. \end{aligned}$$

61. Since the yield rate on the entire investment over the 20 year period is 8% annual effective, the accumulated value at the end of 20 years is $300\ddot{s}_{\overline{20}|0.08}=14826.87$. The cahsflow of the investments is

Payments	$300i$	$2(300i)$	$3(300i)$	$\cdots$	$20(300i)$
Time	1	2	3	$\cdots$	20

So, the future value at time 20 of this cahsflow is

$$300i\,(Is)_{\overline{n}|i}=(300i)\frac{\ddot{s}_{\overline{20}|i/2}-20}{i/2}=(600)(\ddot{s}_{\overline{20}|i/2}-20).$$

So,

$$14826.87=(300)(20)+(600)(\ddot{s}_{\overline{20}|i/2}-20).$$

and $20826.87=600\ddot{s}_{\overline{20}|i/2}$. Hence, $i/2=5\%$ and $i=10\%$.

62. We have that $\int_0^t \delta_s\,ds=\int_0^t \frac{1}{8+s}\,ds=\ln(8+s)\Big|_0^t=\ln(8+t)-\ln 8$. So, $a(t)=e^{\int_0^t \delta_s}=e^{\ln(8+t)-\ln 8}=\frac{8+t}{8}$. The interest earned in the first year is $3960\frac{a(1)-a(0)}{a(0)}=3960(a(1)-1)=3960(\frac{9}{8}-1)=3960\cdot\frac{1}{8}=495$. The interest earned in the second year is $3960\cdot\frac{a(2)-a(1)}{a(1)}=3960\cdot\frac{\frac{10}{8}-\frac{9}{8}}{\frac{9}{8}}=3960\cdot\frac{1}{9}=440$. The interest earned in the third year is $3960\frac{a(3)-a(2)}{a(2)}=3960\cdot\frac{\frac{11}{8}-\frac{10}{8}}{\frac{10}{8}}=3960\frac{1}{10}=396$. Since the 3 interest payements are accumulated at rate of 7%, the accumulated amount of money at time $t=3$ is $3960+495(1.07)^2+440(1.07)+396=3960+566.72+470.8+396=5393.53$.

63. The cahsflow of interest is

Payments	$12i$	$12i$	$12i$	$\cdots$	$12i$	$24i$	$24i$	$\cdots$	$24i$
Time	1	2	3	$\cdots$	10	11	12	$\cdots$	20

The future value at time 20 of this cashflow is

$$
\begin{aligned}
& 64 = 12is_{\overline{20}|0.75i} + 12is_{\overline{10}|0.75i} \\
= \ & 12i\tfrac{(1+0.75i)^{20}-1}{0.75i} + 12i\tfrac{(1+0.75i)^{10}-1}{0.75i} \\
= \ & 16((1+0.75i)^{20}-1) + 16((1+0.75i)^{10}-1) = 16(1+0.75i)^{20} + 16(1+0.75i)^{10} - 32
\end{aligned}
$$

So, $(1+0.75i)^{20} + (1+0.75i)^{10} - 6 = 0$, $(1+0.75i)^{10} = \frac{-1\pm\sqrt{1+24}}{2} = \frac{-1\pm 5}{2} = 2$ and $i = 9.5698\%$.

64. The value of the end of the 5 years of Sally's investment is $10000\left(1+\frac{0.0745}{2}\right)^{10} = 14415.65$. Let P be the amount of the payments Sally receives from Tim. Then, $Ps_{\overline{60}|0.06/12} = 14415.65$. So, $P = 206.6167$. Let $i^{(12)}$ be the nominal rate of interest, compounded monthly, which Sally charged Tim on the loan. Then, $10000 = 206.6167a_{\overline{60}|i^{(12)}/12}$. So, $i^{(12)}/12 = 0.7333$ and $i^{(12)} = 8.801\%$.

65. A nominal rate of 8% convertible quarterly is equivalent to an effective annual rate of 8.2432%. The interest earned each year is $(2000)(0.082432) = 164.86$. A nominal rate of 9% convertible quarterly is equivalent to an effective annual rate of 9.3083%. The interest obtained from 164.86 invested at an effective annual rate is 9.3083% is $(164.86)(0.093083) = 15.3457$. So, the cashflow going into the account paying an annual effective rate of 7% is

Contributions	0	0	15.35	2(15.35)	$\cdots$	9(15.35)
Time	0	1	2	3	$\cdots$	10

The future value of this cashflow is

$$15.3457\,(Is)_{\overline{9}|0.07} = 15.3457\frac{\ddot{s}_{\overline{9}|0.07} - 9}{0.07} = 15.3457\frac{12.8165 - 9}{0.07} = 836.66.$$

The total value of all funds is $2000 + (10)(164.84) + 836.66 = 4485.06$.

66. Let P be the payment under the first option Then, $20000 = Pa_{\overline{20}|0.065}$ and $P = 1815.12$. The annual interest payment under the second option is $20000(0.08) = 1600$. So, the deposit into the sinking fund is $1815.12 - 1600 = 215.12$. So, $20000 = 215.12s_{\overline{20}|j}$ and $j = 14.18\%$.

67. The P be the initial amount of the payments to the sinking fund. Then $Ps_{\overline{10}|8\%} = 10000$ and $P = 690.29$. The present value at time $t = 5$ of the first 5 payments is $690.29s_{\overline{5}|8\%} = 4049.68$. The Q be the amount of the payments to the sinking fund in the last 5 years. Since the sinking fund has to accumulate to 10000, we have that

$$4049.68(1.06)^5 + Qs_{\overline{5}|6\%} = 10000.$$

and $Q = 812.59$. So, $X = 812.59 - 690.29 = 122.30$.

68. Margaret pays $(0.1)L - iL$ to the sinking fund each years. The value of sinking fund accumulates to 10000 in 17 years at 4.62%. So, $L = (0.1-i)Ls_{\overline{17}|0.0462}$ and $\frac{1}{0.1-i} = s_{\overline{17}|0.0462} = 25.0003$. So, $i = 6\%$. For Jason, $L = 500a_{\overline{17}|0.06} = 5238.63$.

69. We have that $(8000)(1.08)^{12} = 800s_{\overline{12}|8\%} + xs_{\overline{12}|4\%}$. So, $20145.36 = 15181.68 + 15.0258x$ and $x = 330.34$.

70. Let P be the amount of equal annual payments made under the first option. Then, $Pa_{\overline{10}|0.0807} = 2000$. So, $P = 299$. The total payments are 2990. Under the second option, at time j, the loan is $2000 - 200j = (200)(10-j)$. Hence, the payment at time j is $(200)(10-(j-1))i = (200)(11-j)i$. So, the total payments under the second option are

$$2990 = \sum_{j=1}^{10}(200 + (200)(11-j)i) = 2000 + 200i\sum_{j=1}^{10}(11-j) = 2000 + 200i\sum_{j=1}^{10} j = \\ = 2000 + 200i\frac{10\cdot 11}{2} = 2000 + 11000i.$$

So, $i = 9\%$.

71. We find P solving $100 = Pa_{\overline{10}|10\%}$, we get that $P = 162.75$. The deposits to the sinking fund are of 62.75. The value of the sinking fund after the last deposit is $62.75s_{\overline{10}|14\%} = 1213.33$. So, the balance in the sinking fund immediately after repayment of the loan is 213.33.

72. The price of the bond is the present value of all payments at the annual effective rate of 8.25%:

$$\begin{aligned} P &= \tfrac{75}{1.03}a_{\overline{20}|\frac{0.0825-0.03}{1.03}} + 1050(1.0825)^{-20} \\ &= 72.8156a_{\overline{20}|0.050970} + 215.10 = 900.02 + 215.10 = 1115.12. \end{aligned}$$

73. Since Bill got an effective annual rate of interest of 7% in his investment, Bill got at the end $1000(1.07)^{10} = 1967.15$. The coupon payments are $1000(.03) = 30$. So, $1967.15 = 30s_{\overline{20}|i^{(2)}/2} + 1000$ and $967.15 = 30s_{\overline{20}|i^{(2)}/2}$. So, $i^{(2)}/2 = 4.7597\%$ and $i = 9.7459\%$.

74. Let C be the redemption value of the bonds are. Let i be the semiannual effective interest rate. Let $\nu = (1+i)^{-1}$. Then, $381.50 = C\nu^{2n}$ and $647.80 = C\nu^n$. So, $\frac{381.50}{647.80} = \nu^n$ and $(1+i)^n = 1.6980$. The present value of the coupons in bond X is

$$Fra_{\overline{2n}|i} = Fr\frac{1-\nu^{2n}}{i} = (1000)(1.03125)(1-(1.6980)^{-2}) = 673.59.$$

So, the total present value of the bond X is $381.50 + 673.59 = 1055.09$.

75. We have that $F = 100$ and $2r = 6.75\%$. So, the coupon payment is $Fr = (1000)\frac{0.0675}{2} = 33.75$. Since the nominal rate of interest in the investment is 7.4% convertible semiannually, we have that the price of the bond is

$$(33.75)a_{\overline{20}|0.037} + (1000)(1.037)^{-20} = 954.63.$$

Since the company proposes to pay the interest according with the same the rate of interest, the value of the restructured bond is the same as the price of the old bond: 954.63.

76. We have that $F = C = 10000$, $Fr = 800$, $i = 6\%$ and $n = 10$. So,

$$I_7 = Ci + (Fr - Ci)(1 - \nu^{n+1-k}) = 600 + (800 - 600)(1 - (1.06)^{-4}) = 641.5813.$$

77. We have that $F = 1000$, $r = 0.04$, $i = 0.0375$ and $n = 10$. So, Hence, the amortization of the premium in the 6–th year is

$$(Fr - Ci)\nu^{n+1-k} = (1000)(0.04 - 0.0375)(1.0375)^{-5} = 2.079694.$$

78. For the bond X, $n = 14$, $C = F = 1000$, $r = 10\%$, and $i = 8\%$. So, the write down in premium is $(Fr-Ci)\nu^{n+1-k} = (1000)(0.10-0.08)(1.08)^{-9}$. For the bond Y, $n = 14$, $C = F$, $r = 6.75\%$, and $i = 8\%$. So, the write down in discount is $(Ci - Fr)\nu^{n+1-k} = F(0.08 - 0.0675)(1.08)^{-9}$. We have that $(1000)(0.10 - 0.08)(1.08)^{-9} = F(0.08 - 0.0675)(1.08)^{-9}$, $F = \frac{(1000)(0.10-0.08)}{(0.08-0.0675)} = 1600$ and

$$P = (1600)(0.0675)a_{\overline{14}|0.08} + (1600)(1 + 0.08)^{-14} = 1435.115.$$

79. We know that $F = 10$, $n = 18$, $i = 5\%$ and

$$20 = B_9 - B_{10} = (Fr - Ci)\nu^{n+1-k} = (1000)(r - 0.05)(1.05)^{-9}.$$

So, $r = 8.1027\%$. The book value at the end of year 10 is

$$B_{10} = Fra_{\overline{n-k}|i} + C\nu^{n-k} = (1000)(0.081027)a_{\overline{8}|0.05} + (1000)(1 + 0.05)^{-8} = 1200.53.$$

80. We know that $n = 30$. The amortization in the k–th payment is $B_{k-1} - B_k = (Fr - Ci)\nu^{n+1-k}$. So, $(Fr-Ci)\nu^{29} = 977.19$ and $(Fr-Ci)\nu^{27} = 1046.79$. Hence, $(1+i)^2 = \frac{1046.79}{977.19}$, $i = 3.5\%$. and $(Fr - Ci) = (1 + 0.035)^{29}977.19 = 2650$. The premium is

$$P - C = (Fr - Ci)a_{\overline{n}|i} = (2650)a_{\overline{30}|3.5\%} = 48739.$$

81. The depreciation charge in the k–th year is

$$D_k = \frac{(A - S)(1 + i)^{k-1}}{s_{\overline{n}|i}} = \frac{1600(1 + i)^{k-1}}{s_{\overline{10}|i}}.$$

The present value of the depreciation charges is

$$1000 = \sum_{k=1}^{10} \frac{1600(1+i)^{k-1}}{s_{\overline{10}|i}}(1 + i)^{-k} = \frac{16000}{s_{\overline{10}|i}(1+i)} = \frac{16000}{\ddot{s}_{\overline{10}|i}}.$$

So, $\ddot{s}_{\overline{10}|i} = 16$ and $i = 8.3946\%$.

82. We know neither X, nor Y, nor n. Since using the straight line method, the annual depreciation expense is 1000, we have that $\frac{X-Y}{n} = 1000$. Using the sum of the years digits method, the depreciation expense in year 3 is

$$800 = D_3 = \frac{(A-S)(n-2)}{S_n} = \frac{2(n-2)(X-Y)}{n(n+1)}.$$

Using the declining balance method, we get that $Y = X(1-0.33125)^n = X(0.66875)^n$. We have the equations

$$1000 = \frac{X-Y}{n},\ 800 = \frac{2(n-2)(X-Y)}{n(n+1)} \text{ and } Y = X(0.66875)^n.$$

Dividing the first equation over the second one, we get that

$$1.25 = \frac{1000}{800} = \frac{\frac{X-Y}{n}}{\frac{2(n-2)(X-Y)}{n(n+1)}} = \frac{n+1}{2(n-2)}$$

and $n = 4$. So, we have that $X - Y = 4000$. Using the third equation, $Y = X(0.66875)^4 = X(0.2)$. From the equations, $X - Y = 4000$ and $Y = X(0.2)$, we get that $X = 5000$ and $Y = 1000$.

83. The depreciation in the k–th year is

$$D_k = Ad(1-d)^{k-1} = 20000(0.2)(0.8)^{k-1} = 4000(0.8)^{k-1},$$

where $A = 20000$ and $d = 0.2$. We have that the cashflow into the fund is

Contributions	4000	$4000(0.8)^1$	$4000(0.8)^2$	$\cdots$	$4000(0.8)^{14}$
Time	1	2	3	$\cdots$	15

So, the accumulated value in the fund at the end of the 15 years is

$$\tfrac{4000}{0.8} a_{\overline{15}|\frac{1.06}{0.80}-1}(1+0.06)^{15} = 11982.79 a_{\overline{15}|0.325} = 36328.83.$$

84. According with the sum-of-the-years-digits method at the end of year 4, the machine has a book value of 2218. So,

$$2218 = S + \frac{(n-k)(n-k+1)}{n(n+1)}(A-S) = S\frac{6\cdot 7}{10\cdot 11}(5000-S) = 1909.09 + \frac{68}{110}S.$$

So, $S = \frac{110}{68}(2218 - 1909.09) = 499$. At the year 4, the machine has a value of 2218. The salvage value of the machine is 449. So, the new depreciation charge is $D_8 = \frac{A-S}{6} = \frac{2218-499}{6} = 286.38$ is per year.

85. For Machine I, $S = A(1-d)^n$ gives that $\frac{X}{8} = X(1-d)^{10}$. So, $1-d = (1/8)^{1/10} = 0.125^{0.1}$. Hence, $B_7 = A(1-d)^7 = X(0.125^{0.7})$. The total amount of depreciation in the first seven years is $A - B_7 = X - X(0.125^{0.7}) = X(1-(0.125)^{0.7})$. For Machine II, the total amount of depreciation in the first seven years is

$$\begin{aligned} D_1 + \cdots + D_7 &= \textstyle\sum_{j=1}^{7} \left(Y - \frac{X}{8}\right) \frac{2}{(10)(11)}(11-j) \\ = &\left(Y - \frac{X}{8}\right) \frac{1}{55}(10 + \cdots + 4) = \left(Y - \frac{X}{8}\right) \frac{49}{55}. \end{aligned}$$

So, $X(1-(0.125)^{0.7}) = \left(Y - \frac{X}{8}\right)\frac{49}{55}$ and $\frac{Y}{X} = 0.125 + \frac{55}{49}(1-(0.125)^{0.7}) = 0.9856$.

86. We have that $k = 6$, $i = j = 9\%$, $H = 11749.22$, $M = 3000$, $B_6 = 55216.36$. Hence, the formulas

$$B_k = A - \frac{(A-S)s_{\overline{k}|j}}{s_{\overline{n}|j}}, \ H = Ai + \frac{A-S}{s_{\overline{n}|i}} + M$$

become

$$55216.36 = A - \frac{(A-S)s_{\overline{6}|9\%}}{s_{\overline{n}|9\%}}, \ 11749.22 = A(0.09) + \frac{A-S}{s_{\overline{n}|9\%}} + 3000.$$

Solving for $\frac{(A-S)}{s_{\overline{n}|9\%}}$ in both equations

$$\frac{A - 55216.36}{s_{\overline{6}|9\%}} = \frac{(A-S)}{s_{\overline{n}|9\%}} = 11749.22 - A(0.09) - 3000$$

and

$$A = \frac{11749.22 - 3000 + \frac{55216.36}{s_{\overline{6}|9\%}}}{(0.09) + \frac{1}{s_{\overline{6}|9\%}}} = \frac{11749.22 - 3000 + \frac{55216.36}{7.523334565}}{(0.09) + \frac{1}{7.523334565}} = 72172.$$

87. For Eric, $P = 800 - (800 - 2X) = 2X$, $M = (800)(0.50) = 400$, $I = (400)(0.08) = 32$ and $D = 16$. Eric's net profit is $P + I - D = 2X + 32 - 16 = 2X + 16$. Eric's interest is $i = \frac{P+I-D}{M} = \frac{2X+16}{400} = \frac{X+8}{200}$. For Jason, $P = 800-(800+X) = -X$, $M = (800)(0.50) = 400$, $I = (400)(0.08) = 32$ and $D = 16$. Jason's net profit is $P+I-D = -X+32-16 = -X+16$. Jason's interest is $i = \frac{P+I-D}{M} = \frac{-X+16}{400}$. Solving for X in $\frac{X+8}{200} = 2\frac{-X+16}{400}$, we get that $X = 4$ and $i = \frac{X+8}{200} = \frac{4+8}{200} = 6\%$.

88. Let x be the price at which Jose and Chris each sell a different stock short. For Jose, $P = 760 - x$, $M = x(0.50)$, $I = x(0.50)(0.06) = (0.03)x$ and $D = 32$. Jose's yield is $\frac{x-760+(0.03)x-32}{(0.5)x}$. For Chris, $P = 760 - x$, $M = x(0.50)$, $I = x(0.50)(0.06) = (0.03)x$ and $D = 0$. Chris' yield is $i = \frac{x-760+(0.03)x}{(0.5)x}$. We have

$$i = \frac{x - 760 + (0.03)x}{(0.5)x} = 2 \cdot \frac{x - 760 + (0.03)x - 32}{(0.5)x}$$

or

$$2 - \frac{1520}{x} + 0.06 = 4 - \frac{3040}{x} + 0.12 - \frac{128}{x}$$

which implies $(-1520 + 3040 + 128)\frac{1}{x} = 4 + 0.12 - 2 - 0.06$ and $x = \frac{-1520+3040+128}{4+0.12-2-0.06} = 800$. The interest is $i = \frac{800-760+(0.03)(800)}{(0.5)(800)} = 16\%$.